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Internal Assesment Test - III

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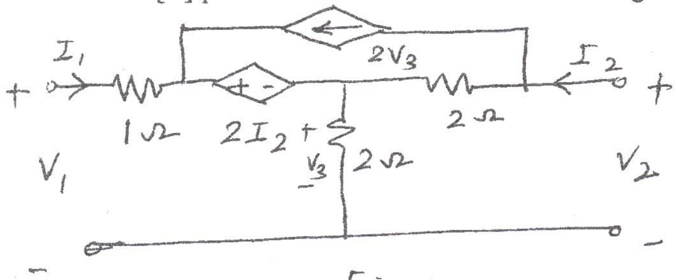
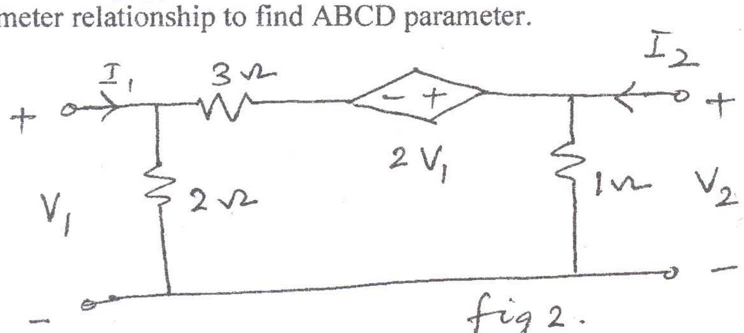
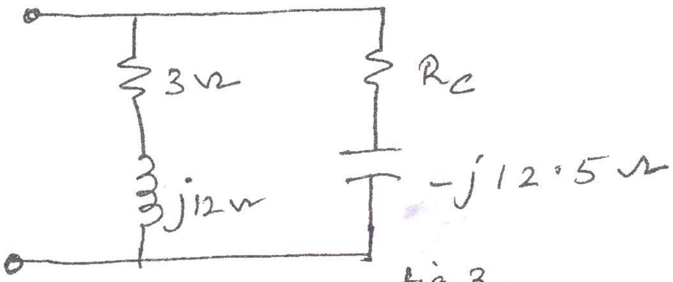
|---|---|---|---|---|---|---|---|---|---|
| Sub: | Electric Circuit Analysis | | | | | | | Code: | 18EE32 |
| Date: | 16/11/2019 | Duration: | 90 mins | Max Marks: | 50 | Sem: | 3 | Branch: | EEE(A) |
| Answer Any FIVE FULL Questions | | | | | | | | | |

| | | Marks | OBE CO | OBE RBT |
|---|---|---|---|---|
| 1. | Define [Y] parameter and draw the equivalent circuit of it.<br>Calculate the [Z] parameter of the circuit shown in fig1.<br><br>Fig 1. | [5+5] | CO4 | L1,L3 |
| 2. | Define [T] parameter.<br>Calculate the [Y] parameter of the circuit shown in fig2. Then use the parameter relationship to find ABCD parameter.<br><br>fig 2. | [4+6] | CO4 | L1,L3 |
| 3. | Prove that for series resonant circuit, the resonant frequency is the geometric mean of two half power frequencies.<br>Determine the value of $R_c$ for which the circuit of the fig3. resonates.<br><br>fig 3. | [5+5] | CO4 | L1,L3 |



P.T.O

P.~~R~~ (ccI)

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| | | | | |
|---|---|---|---|---|
| 4. | Derive expression for resonant frequency in series RLC circuit. A series RLC circuit has $R=4\Omega$, $L=1mH$ and $C=10\mu F$. Calculate Q factor, bandwidth, resonant frequency and half power frequencies. | [4+6] | CO4 | L3 |
| 5. | Find the equation of current if the switch is closed at t=0. Find also the voltage across L and R, the current at t=0.1 sec shown in fig5. | [10] | CO4 | L4 |

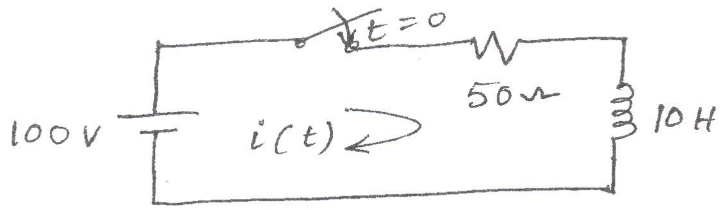


fig 5.

| | | | | |
|---|---|---|---|---|
| 6a. | Show the behaviour of R, L, C elements at the time of switching at t=0 both at t=0+ and t=∞. | [5] | CO3 | L1 |
| 6b. | Draw the dual network for the given network shown in fig6b. | [5] | CO3 | L3 |

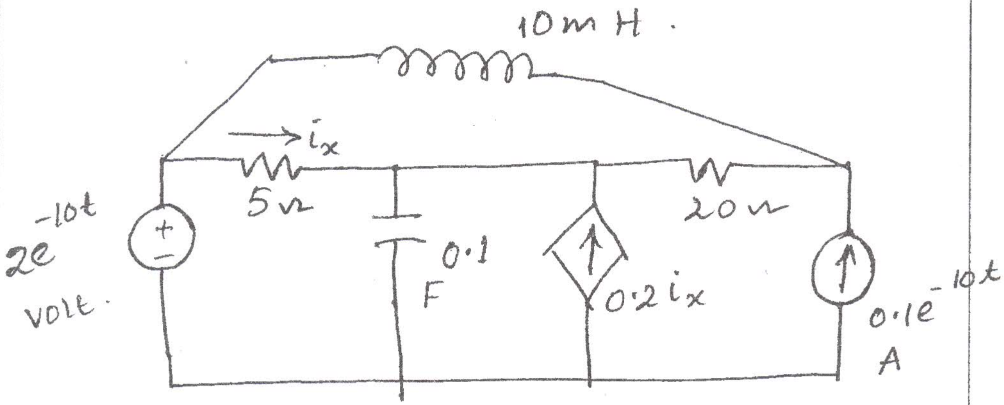


fig 6b.

— end —

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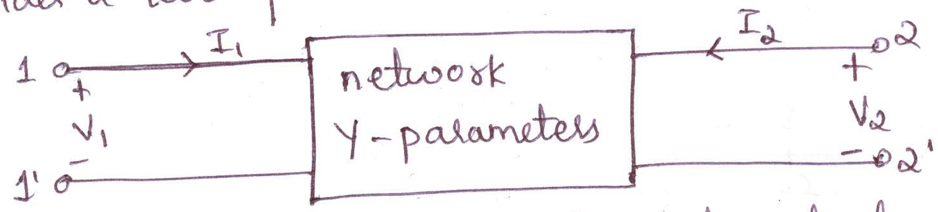
# Internal Assessment Test - III (Nov-2019)

1) a) <u>Y parameter model</u>

Consider a two port network as shown below




$[I] = [Y][V]$

Considering $I_1$ and $I_2$ to be dependant variables that depend on independant $V_1$ and $V_2$ we can write the following equations :

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Short circuit the input port ie) $V_1 = 0$.

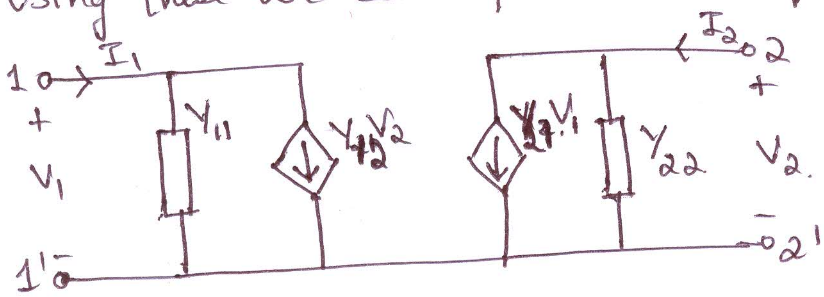
- $Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \Rightarrow$ short circuited transfer reverse admittance in S
- $Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \Rightarrow$ short circuited driving point output admittance in S or ℧.

Short circuit the output port ie) $V_2 = 0$

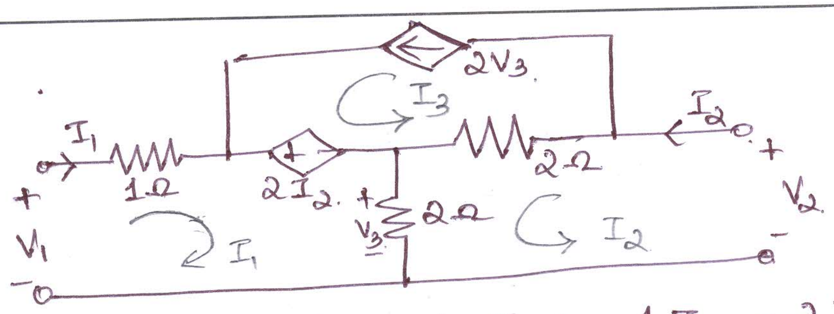
- $Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \Rightarrow$ short circuited driving point input admittance in S or ℧
- $Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \Rightarrow$ short circuited transfer forward admittance in S or ℧

These four parameters $Y_{11}, Y_{12}, Y_{21}, Y_{22}$ are called as admittance (Y) parameters / short circuit parameters

Using these we can represent an equivalent circuit as below :-



b)




- Applying KVL in mesh I :- $-1I_1 - 2I_2 - 2(I_1 + I_2) = -V_1$

  $-I_1 - 2I_2 - 2I_1 - 2I_2 = -V_1$

  $-3I_1 - 4I_2 = -V_1$

  $3I_1 + 4I_2 = V_1$ —— (1)

- Applying KVL in mesh II :- $-2(I_2 - I_3) - 2(I_1 + I_2) = -V_2$

  $-2I_2 + 2I_3 - 2I_1 - 2I_2 = -V_2$

  $-2I_1 - 4I_2 + 2I_3 = -V_2$

  $2I_1 + 4I_2 - 2I_3 = V_2$ —— (2)

- In mesh III constraint equation :- $I_3 = 2V_3$

  $V_3 = 2(I_1 + I_2) = 2I_1 + 2I_2$

  $\therefore I_3 = 2(2I_1 + 2I_2) = 4I_1 + 4I_2$ —— (3)

- Sub (3) in (2)

  $2I_1 + 4I_2 - 2(4I_1 + 4I_2) = V_2$

  $2I_1 + 4I_2 - 8I_1 - 8I_2 = V_2$

  $-6I_1 - 4I_2 = V_2$ —— (4)

- In (1) put $I_1 = 0$.

  $4I_2 = V_1$.

  $\cancel{\frac{I_2}{V_1}} = \frac{1}{4}$ $\frac{V_1}{I_2} = 4$

  $\boxed{Z_{12} = 4\,\Omega}$

  In (4) put $I_1 = 0$

  $-4I_2 = V_2$

  $-4 = \frac{V_2}{I_2}$

  $\boxed{Z_{22} = -4\,\Omega}$

In ① put $I_2 = 0$

$$3 I_1 = V_1$$

$$3 = \frac{V_1}{I_1}$$

$$\boxed{Z_{11} = 3\,\Omega}$$

In ④ put $I_2 = 0$

$$-6 I_1 = V_2$$

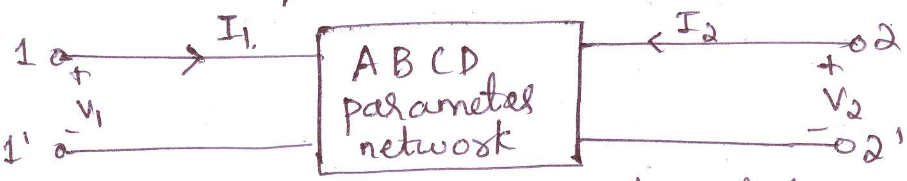
$$-6 = \frac{V_2}{I_1}$$

$$\boxed{Z_{21} = -6\,\Omega}$$

$$\therefore [Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ -6 & -4 \end{bmatrix} \Omega.$$

2.) <u>Transmission (T) parameters / ABCD parameters</u> :-

a) Consider a two port network as shown below:-




Considering $I_1$ and $V_1$ to be dependent on the independent variables $V_2$ and $I_2$, we can write the following equations

$$I_1 = AV_2 + B(-I_2)$$

$$V_1 = CV_2 + D(-I_2)$$

- When $V_2 = 0$

$$B = \left.\frac{I_1}{-I_2}\right|_{V_2=0} \Longrightarrow \text{Short circuit reverse current gain (no unit)}$$

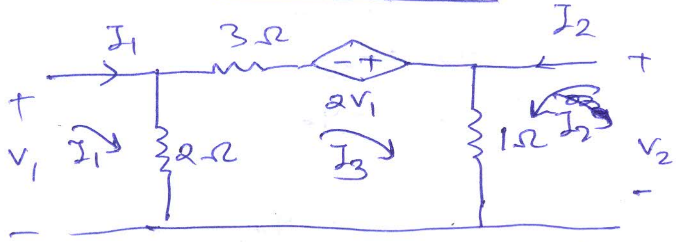
$$D = \left.\frac{V_1}{-I_2}\right|_{V_2=0} \Longrightarrow \text{Short circuit input transfer impedence } (\Omega)$$

- When $I_2 = 0$

$$A = \left.\frac{I_1}{V_2}\right|_{I_2=0} \longrightarrow \text{Open circuit output transfer admittance } (\mho) \text{ or } (S)$$

2. T- parameter is a representation of 2-port network using the A, B, C, D as its component.

Y-parameter model




To obtain Y-parameter.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Apply KVL in

$$V_1 - 2(I_1 - I_3) = 0$$
$$V_1 = 2I_1 - 2I_3 \quad (1)$$

$$-2(I_3 - I_1) - 3I_3 + 2V_1 - 1(I_3 + I_2) = 0$$
$$-2I_3 + 2I_1 - 3I_3 + 2V_1 - I_3 - I_2 = 0$$
$$\cancel{2V_1} = 2I_1 - I_2 - 6I_3 + 2V_1 = 0$$
$$-2V_1 = 2I_1 - I_2 - 6I_3 \quad (2)$$

$$V_2 - 1(I_2 + I_3) = 0$$
$$V_2 = I_2 + I_3 \quad (3)$$

At $V_2 = 0$, to find $Y_{11}$, $Y_{21}$

$$V_2 = I_2 + I_3$$
$$\boxed{I_2 = -I_3}$$

$$V_1 = 2I_1 - 2(-I_2)$$
$$V_1 = 2I_1 + 2I_2 \quad (1)$$

$$\frac{V_1 - 2I_1}{2} = I_2 \quad (1)$$

$$\frac{V_1 - 2I_2}{2} = I_1 \quad (1)$$

$$-2V_1 = \frac{2V_1 - 4I_2}{2} + 5I_2$$
$$-4V_1 = 2V_1 - 4I_2 + 10I_2$$
$$-6V_1 = 6I_2$$

$$\boxed{Y_{21} = \frac{I_2}{V_1} = -\frac{6}{6} = -1S}$$

$$-2V_1 = 2I_1 - I_2 - 6(-I_2)$$
$$-2V_1 = 2I_1 - I_2 + 6I_2$$
$$-2V_1 = 2I_1 + 5I_2$$
$$-2V_1 = 2I_1 + \frac{5V_1 - 10I_1}{2}$$
$$-4V_1 = 4I_1 + 5V_1 - 10I_1$$
$$-9V_1 = -6I_1$$

$$\boxed{Y_{11} = \frac{I_1}{V_1} = \frac{9}{6} = \frac{3}{2}S}$$

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To find $Y_{22}, Y_{12}$ , Take $V_1 = 0$

$0 = 2I_1 - 2I_3$

$\boxed{I_1 = I_3}$

$0 = 2I_1 - I_2 - 6I_1$

$0 = -4I_1 - I_2$

$4I_1 = -I_2$

$I_2 = -4I_1$

$V_2 = I_2 + I_1$

$V_2 = I_2 + \left(-\frac{1}{4}I_2\right)$

$V_2 = I_2\left(1 - \frac{1}{4}\right)$

$\frac{I_2}{V_2} = \boxed{Y_{22} = \frac{4}{3} S}$

$V_2 = -4I_1 + I_1$

$\boxed{Y_{12} = \frac{I_1}{V_2} = -\frac{1}{3} S}$

To obtain ABCD parameter.

$V_1 = AV_2 - BI_2$ , $I_2 = \frac{A}{B}V_2 - \frac{1}{B}V_1$ — ①

$I_1 = CV_2 - DI_2$ , $I_1 = CV_2 - D\left[\frac{A}{B}V_2 - \frac{1}{B}V_1\right]$

$I_1 = \left(C - \frac{AD}{B}\right)V_2 + V_1\left[\frac{D}{B}\right]$

$I_1 = \left(C - \frac{AD}{B}\right)V_2 + \frac{D}{B}V_1$ — ②

comparing eqn of Y-parameter.

$I_1 = \left(C - \frac{AD}{B}\right)V_2 + \frac{D}{B}V_1$

$I_2 = \frac{A}{B}V_2 - \frac{1}{B}V_1$

For Y

$I_1 = Y_{11}V_1 + Y_{12}V_2$

$I_2 = Y_{21}V_1 + Y_{22}V_2$

$Y_{11} = \frac{D}{B}$ , $Y_{12} = C - \frac{AD}{B}$

$Y_{21} = -\frac{1}{B}$ , $Y_{22} = \frac{A}{B}$

$B = \frac{-1}{Y_{21}} = \frac{-1}{-1} = 1$

$\boxed{B = 1\Omega}$

$Y_{22} = \frac{A}{B}$

$A = \frac{4}{3} \times 1$

$\boxed{A = \frac{4}{3}}$

$Y_{11} = \frac{D}{B}$

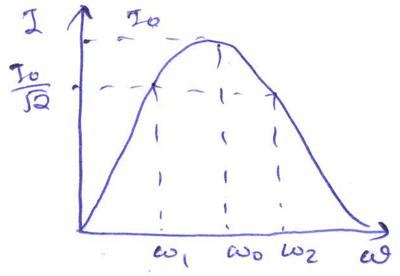
$D = \frac{3}{2}$

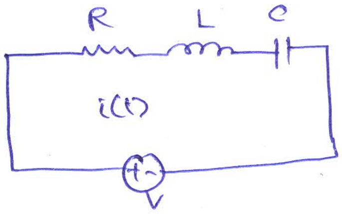
$Y_{12} = C - \frac{AD}{B}$

$C = -\frac{1}{3} + \frac{\frac{4}{3} \times \frac{3}{2}}{1}$

$= \frac{5}{3}$

3. In series resonant circuit, $\omega_0 = \sqrt{\omega_1 \omega_2}$.




For the circuit $P_{max} = I_0^2 R$

$$\frac{P_{max}}{2} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R$$

At half power of circuit

$$I = \frac{I_0}{\sqrt{2}}$$

$$I = \frac{V}{R\sqrt{2}} \quad -(1)$$

$$I = V / \sqrt{R^2 + (X_L - X_C)^2} \quad -(2)$$

equating

$$R\sqrt{2} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$2R^2 = R^2 + (X_L - X_C)^2$$

$$(X_L - X_C)^2 = R^2$$

$$X_L - X_C = \pm R$$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad -(A)$$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad -(B)$$

Add (A) + (B)

$$L[\omega_1 + \omega_2] - \frac{1}{C}\left[\frac{\omega_1 + \omega_2}{\omega_1 \omega_2}\right] = 0$$

$$L[\omega_1 \cancel{+} \omega_2] = \frac{1}{C}\left[\frac{\omega_1 \cancel{+} \omega_2}{\omega_1 \omega_2}\right]$$

$$\omega_1 \omega_2 = \frac{1}{LC}, \quad \text{wkt } \omega_0 = 1/\sqrt{LC}$$

$$\omega_0^2 = \omega_1 \omega_2$$

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

Hence geometric mean of 2 half power frequencies is $\omega_0$.

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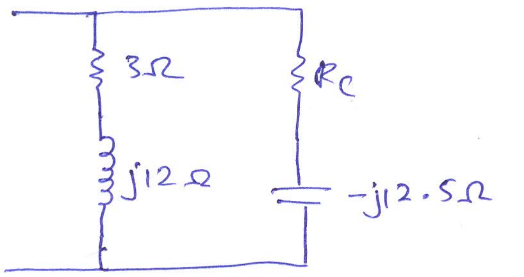



$$\omega_0 = \frac{1}{\sqrt{LC}} \cdot \left[\frac{R_L^2 - \frac{L}{C}}{R_C^2 - \frac{L}{C}}\right]^{1/2}$$

$$\frac{X_L}{R_L^2 + X_L^2} = \frac{X_C}{R_C^2 + X_C^2} \quad / \quad X_C = 12.5, \; X_L = 12$$

$$X_L(R_C^2 + X_C^2) = X_C(R_L^2 + X_L^2)$$

$$R_C^2 = \frac{X_C[R_L^2 + X_L^2] - X_L X_C^2}{X_L}$$

$$= \frac{12.5[3^2 + 12^2] - [12 \times 12.5^2]}{12}$$

$$R_C^2 = 3.125$$

$$R_C = \underline{1.7677\,\Omega}$$

$$-jX_C = -j12.5\,\Omega$$

$$= \frac{1}{jX_C} = \omega C$$

$$\cancel{jX_L = j12\,\Omega}$$

$$\cancel{\omega_0 L = X_L}$$

$$\cancel{L = 1}$$

$$-jX_C = -j12.5$$

$$X_C = 12.5$$

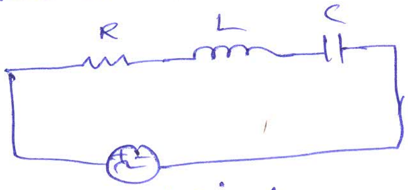
$$\frac{1}{\omega_0 C} = 12.5$$

$$C = \frac{1}{12.5\omega} \quad \text{— (1)}$$

$$\omega_0 L = 12$$

$$L = \frac{12}{\omega} \quad \text{— (2)}$$

4. series RLC circuit




$V_i = V_m \sin\omega t$

$$V = V_R + V_L + V_C$$

$$V = V_R + L\frac{di}{dt} + \frac{1}{C}\int V dt \quad , \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

For circuit to resonate

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}} \quad , \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Given, $R = 4\Omega$, $L = 1\,mH$, $C = 10\,\mu F$ , $\omega_0 = 10{,}000\ rad/s$, ~~$\omega_0 = 31622.77\ rad/s$~~

$$Q\ factor = \frac{\omega L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{31622.77 \times 10^{-3}}{4}$$

$$= 2.5$$

$$BW = \frac{R}{L} = \frac{4}{10^{-3}} = 4000\ rad/s.$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{10^{-3} \times 10^{-6}}} = 1591.549\,Hz$$

$$\omega_1 = \omega_0 - \frac{BW}{2} = 10{,}000 - \frac{4000}{2}$$

$$= 8000\ rad/s$$

$$\omega_2 = \omega_0 + \frac{BW}{2} = 10000 + \frac{4000}{2}$$

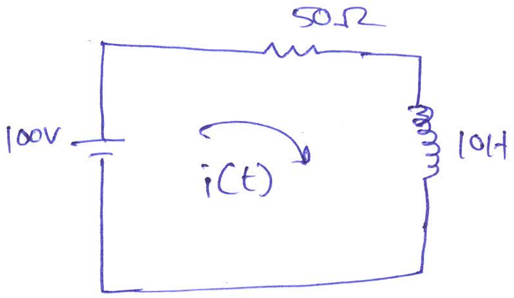
$$= 12000\ rad/s.$$

5) At $t=0$ switch is closed.

$t = 0^-$

$i(0^-) = 0 = i(0^+)$

At $t=0$




Apply KVL

$$100 = 50i(t) + 10\frac{di}{dt}$$

For circuit at any instant of time.

$$I(t) = \frac{V}{R}\left(1-e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R} = \frac{10}{50} = \underline{0.2}$$

$$I(t) = \frac{100}{50}\left(1-e^{-5t}\right)$$

$$= \underline{2\left(1-e^{-5t}\right)}$$

$$V_L = L\frac{di}{dt}$$

$$V_L = L \cdot -\frac{V}{R} \cdot e^{-t/\tau} \cdot -\frac{1}{\tau}$$

$$V_L = Ve^{-t/\tau}$$

At $t=0.1s$

$$I(0.1) = 2\left(1-e^{-0.1\times5}\right)$$

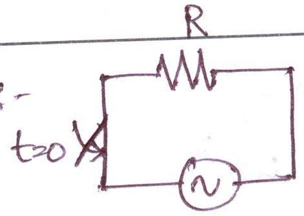
$$= \underline{0.7869A}$$

$$V_L = 100e^{-5t}$$

$$V_L(0.1) = 100e^{-5\times0.1}$$

$$= \underline{60.65V}$$

6) a). • A purely resistive circuit :-

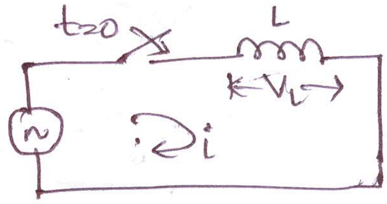



At $t=0^- \Rightarrow R = \frac{V}{I}$

At $t=0^+ \Rightarrow R = \frac{V}{I}$.

Since R is independent of t, Resistor behaves the same in both steady state $(t=\infty)$ and transient state $(t=0)$

• Considering a pure inductive circuit




Current through inductor

$$i_L = i = \frac{1}{L}\int i\,dt$$

$$i_L(0^-) = \frac{1}{L}\left[\int_{-\infty}^{t} i_L\,dt\right]$$

$$= \frac{1}{L}\left[\int_{-\infty}^{0} i_L\,dt + \int_{0}^{t} i_L\,dt\right]$$

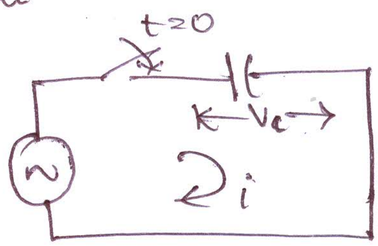
$$i_L(0^-) = \cancel{\tfrac{1}{L}}\ \frac{1}{L}\int_{-\infty}^{0} i_L\,dt + 0.$$

$\boxed{i_L(0^-) = i_L(0^+)} \Rightarrow$ Inductor current does not change instantaneously.

∴ At $t=0$, L acts as open circuit

At $t=\infty$, L acts as short circuit

• Considering a capacitive circuit




Voltage through capacitor

$$V_c = \frac{1}{C}\int V\,dt$$

$$V_c(0^-) = \frac{1}{C}\left[\int_{-\infty}^{t} V\,dt\right]$$

$$V_c(0^-) = \frac{1}{C}\int_{-\infty}^{0} V_c\,dt + \frac{1}{C}\int_{0}^{t} V_c\,dt$$

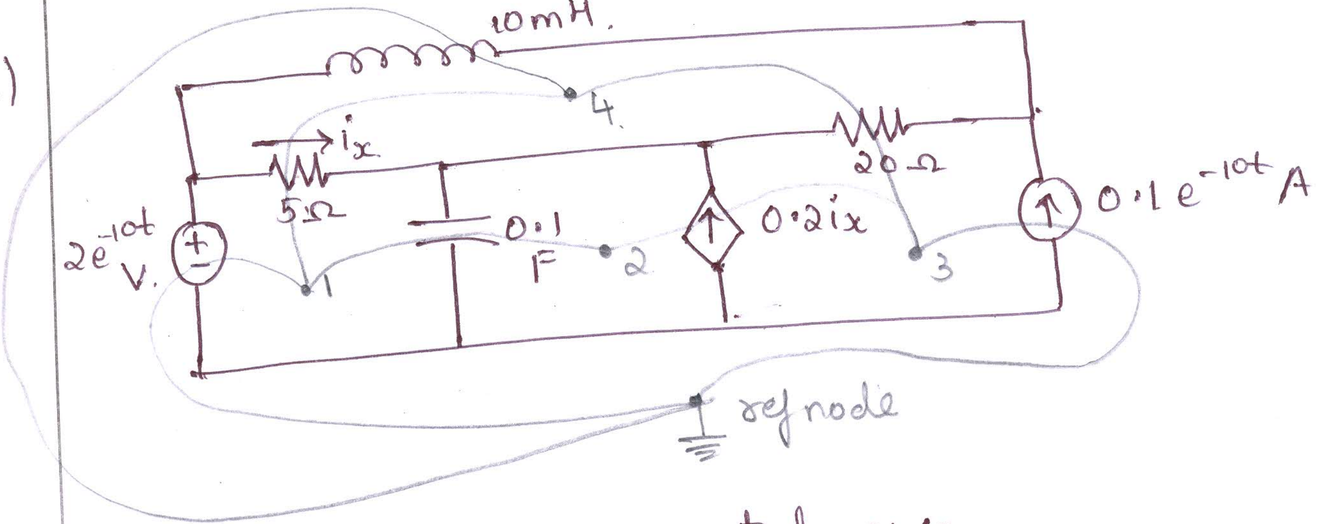
$$= \frac{1}{C}\int_{-\infty}^{0} V_c\,dt + 0$$

$\boxed{V_c(0^-) = V_c(0^+)}$ $\Rightarrow$ Capacitor voltage does not change instantaneously.

$\therefore$ At $t=0$, C acts as short circuit

At $t=\infty$, C acts as open circuit.

b)




Dual network is represented as :-