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Internal Assesment Test - III

Sub:	Power System Analysis II							Code:	15EE71
Date:	22/10/2018	Duration:	90 mins	Max Marks:	50	Sem:	7	Branch:	EEE
Answer Any FIVE FULL Questions									

		Marks	OBE CO	OBE RBT
1	A 50 Hz synchronous generator having inertia constant H=5.2 MJ/MVA and xd'=0.3 pu is connected to an infinite bus through a double circuit line as shown in fig.The reactance of the connecting HT transformer is 0.2pu and reactance of each line is 0.4 pu. lEgl =1.2 pu and lVl =1.0 pu and Pe =0.8 pu. Plot the swing curve for sustained fault using point by point method if a 3 phase fault occurs at the middle of one of the transmission lines and is cleared by isolating the faulted line by 0.4sec	[10]	CO6	L3
2	Explain the Milne's predictor corrector method of solving transient stability equations	[10]	CO6	L2
3	Solve the question no 1 by Euler's method	[10]	CO6	L3
4	Illustrate clearly the steps involved in solving swing equation using Runge_ Kutta method for transient analysis.	[10]	CO6	L3
5	Explain the method of equal incremental cost for the economic operation of generators with transmission loss considered.	[10]	CO4	L2
6	A two bus system is shown if fig. If a load of 125 MW is transmitted from plant 1 to the load, a loss of 15.625 MW is incurred. Determine the generation schedule and the load demand if the cost of received power is rs 24 /MWhr. Solve the problem using coordination equations and the penalty factor method approach. The incremental production costs of the plants are $dF_1/dP_1=0.025\ P_1+15$ $dF_2/dP_2=0.05\ P_2+20$	[10]	CO3	L2

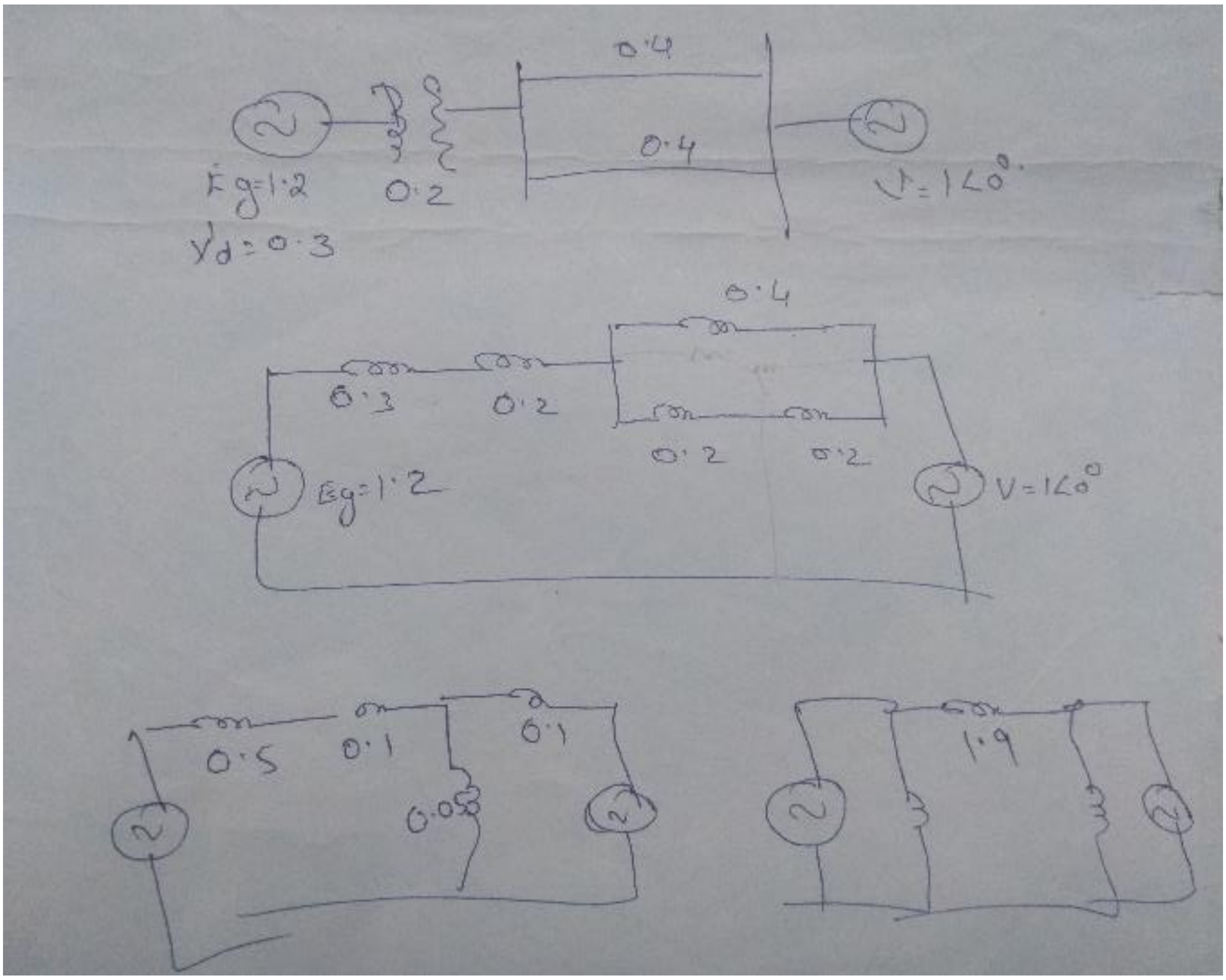
[10] CO4 L3

7 The fuel inputs per hour of plants 1 and 2 are given as
$F_1 = 0.2\ P_1^2 + 40\ P_1 + 120$ Rs per hr.
$F_2 = 0.25\ P_2^2 + 30\ P_2 + 150$ Rs per hr. Determine the economic operating schedule and the corresponding cost of generation if the maximum and minimum loading on each unit is 100 MW and 25 MW ,the demand is 180 MW and transmission losses are neglected. If the load is equally shared by both the units, determine the saving obtained by loading the units as per the incremental production cost.

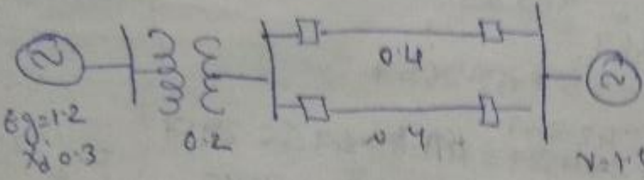
8 With a usual notation derive generalized transmission loss formula and B -coefficients

Solutions

1



(2) Pg. 393 Uma K



$X_1 = 0.7$

$P_1 = \frac{1.2 \times 1.0}{.7} = 1.714$

$Pe = 0.8 = P_m$

$0.8 = 1.714 \sin \delta_0$

$\therefore \delta_0 = 27.82^\circ$

During fault. $X_2 = 1.9$

$P_2 = \frac{1.2 \times 1}{1.9} = 0.63$

After fault.

$X_3 = 0.9$

$P_3 = \frac{1.2 \times 1.0}{0.9} = 1.33384$

At $t = 0$

At $t = 0^-$ $\quad Pa(0^-) = P_s - Pe = 0.8 - 1.714 \sin 27.82 = 0$

$t = 0^+$ $\quad Pa = 0.8 - .63 \sin 27.82 = 0.8 - .2940 = 0.506$

$$P_a = \frac{P_a(0^-) + P_a(0^+)}{2} = \frac{0 + .506}{2} = .253$$

$$\Delta\omega_1 = \frac{P_a \times \Delta t}{m} = \frac{.253 \times .05}{.00054} = 23.426$$

$\omega_1 = \omega_0 + \Delta\omega_1 = 0 + 23.426 = 23.426$

$\Delta\delta_1 = \Delta\omega_1 \times \Delta t = 23.426 \times .05 = 1.1713$

$\delta_1 = \delta_0 + \Delta\delta_1 = 27.82 + 1.1713 = 28.9913 \Rightarrow t = .05 \Rightarrow \delta = 28.99$, $\omega = 23.426$

$t=0.1$

$P_e = 0.63 \sin 28.99 = .30533$

$P_a = 0.8 - .30533 = .49467$

$\Delta\omega_2 = \frac{P_a}{m} \times \Delta t = \cancel{26.4587}\ 42.8657$

$\omega_2 = \omega_1 + \Delta\omega_2 = 23.426 + \cancel{26.4587}\ 42.8657 = \cancel{49.8847}\ 66.2917$

$\Delta\delta_2 = \Delta\omega_2 \times \Delta t = \cancel{26.4587}\ \cancel{42.8657}\ 66.2917 \times .05 = \cancel{1.3229}\ \cancel{2.1433} = 3.3146$

$\delta_2 = \delta_1 + \Delta\delta_2 = 28.99 + \cancel{1.3229}\ \cancel{2.1433}\ 3.3146 = \cancel{30.3129}\ \cancel{31.1332}\ 32.3046\ 32.3046$

$\omega_2 =$

$\delta_2 =$

$t=0.15$

$P_a = 0.8 - .63 \sin \cancel{31.1332}\ 32.3046 = 0.8 - \cancel{.3257}\ 0.3367 = \cancel{0.474}\ 0.4633$

$\Delta\omega_3 = \frac{\cancel{.4744}\ 0.4633}{m} \times \Delta t = \cancel{41.1005}\ 40.1473$

$\omega_3 = 66.2917 + \cancel{41.1005}\ 40.1473 = \cancel{107.3922}\ 106.4390$

$\Delta\delta_3 = \Delta\omega_3 \times .05 = \cancel{2.055}\ \cancel{5.3696}\ 5.3220$

$\delta_3 = \cancel{31.1332 + 2.05}\ 32.3046 + 5.3220 = \cancel{33.188}\ \cancel{35.64}\ 37.6266$

$\omega_3 =$

$\delta_3 =$

$t = 0.20$

$P_a = 0.8 - 0.63 \sin 37.6266 = 0.8 - 0.3846 = .4154$

$\Delta\omega_4 = \frac{.4154}{m} \times .05 = 35.99$

$\omega_4 = 106.4390 + 35.99 = 142.4355$

$\Delta\delta_4 = \omega_4 \times .05 = 7.1218$

$\delta_4 = 37.6266 + 7.1218 = 44.7484$

$\omega_4 = 142.4$

$\delta_4 = 44.7$

$t = 0.25$

$P_a = 0.8 - 0.63 \sin 44.7484 = 0.356483$

$\Delta\omega_5 = \frac{.356483}{m} \times .05 = 30.8911$

$\omega_5 = 142.4355 + 30.8911 = 173.3266$

$\Delta\delta_5 = \omega_5 \times .05 = 8.6663$

$\delta_5 = 44.7484 + 8.6663 = 53.4147$

$\omega_5 = 173.3$

$\delta_5 = 53.41$

Fault cleared in 2.5 cycles.

2.5 cycles ⇒ .05 sec.

$P_a(.05^-) \Rightarrow 0.8 - 0.63 \sin 28.99 = 0.8 - 0.3053 = 0.494$

$P_a(.05^+) \Rightarrow 0.8 - 1.33 \sin 28.99 = 0.8 - 0.6446 = 0.15$

$P_a = 0.32505$

$\Delta\omega_1 = \frac{.32505}{m} \times .05 = 28.16$

$\omega_1 = 23.42 + 28.16 = 51.5872$

$\Delta\delta_1 = 51.5872 \times .05 = 2.5794$

$\delta_1 = \cancel{23.42}\ 28.99 + 2.5794 = 31.5694$

t = 0.1

$P_a = 0.8 - 1.33 \sin 31.5694 = 0.8 - 0.69787 = 0.10213$

$\Delta\omega_2 = \frac{.10213}{m} \times .05 = 8.85008$

$\omega_2 = 51.5872 + 8.85008 = 60.4373$

$\Delta\delta_2 = 60.4373 \times .05 = 3.02186$

$\delta_2 = 34.5912 = 31.5694 + 3.02186$

continue →

Fault cleared in 6.25 cycles ⇒ 0.125 sec

t = 0.05 & t = 0.1 same as sustained fault.

t = 0.15 — take angle 37.62 (same as sustained fault)

$P_a = 0.8 - 1.333 \sin 37.62 = 0.8 - .810 = -0.010$

$\Delta\omega = \frac{-.010}{m} \times .05 = -.88302$

$\omega = 106.4390 - .88302 = 105.5471$

$\Delta\delta = 105.547 \times .05 = 5.277$

$\delta = 37.62 + 5.277 = 42.8978$ = this is at t = 0.2

2

Milne's Predictor Corrector Method

$$\frac{dx}{dt} = f_x(x,y,t) \quad \Big| \quad \frac{dy}{dt} = f_y(x,y,t)$$

with the known four consecutive previous values

$$x^p_{n+1} = x_{n-3} + \frac{4h}{3}\left[2x'_{n-2} - x'_{n-1} + 2x'_n\right]$$

$$y^p_{n+1} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

x' and y' are derivatives at the corresponding time

Then the corrected values are

$$x_{n+1} = x_{n-1} + \frac{h}{3}\left[x'_{n-1} + 4x'_n + x'_{n+1}\right]$$

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right]$$

where $x'_{n+1} = f_x\left[x^p_{n+1}, y^p_{n+1}, t_{n+1}\right]$

$$y'_{n+1} = f_y\left[x^p_{n+1}, y^p_{n+1}, t_{n+1}\right]$$

Milne's Predictor Corrector Method

~~$\delta_1' = 28.87$~~ ~~$\omega_1' = 0.8 - 0.63\sin\delta$~~

~~$\delta_2' = 32.03$~~

~~$\delta_3' = 2.161$~~

~~$\delta_4' = 2.067$~~

From Runga Kutta method.

$$t = 0\,sec \quad \left.\begin{aligned} \delta_1 &= 28.87 & \omega_1 &= 46.18 \\ \delta_2 &= 32.03 & \omega_2 &= 90.386 \\ \delta_3 &= 37.25 & \omega_3 &= 123.86 \\ \delta_4 &= 43.33 & \omega_4 &= 118.48 \end{aligned}\right\}$$

$\delta_1' = 46.18$

$\delta_2' = 90.386$

$\delta_3' = 123.86$

$\delta_4' = 118.48$

$$\omega_1' = \frac{0.8 - 0.63\sin 28.87}{.000544} = 911.43$$

$$\omega_2' = \frac{0.8 - 0.63\sin 32.03}{.000544} = \cancel{854.76}\ 803.04$$

$$\omega_3' = \frac{0.8 - 0.63\sin 37.25}{.000544} = \cancel{769.6}\ 725.588$$

$$\omega_4' = \frac{0.8 - \overset{1.333}{\cancel{0.63}}\sin 43.33}{.000544} = \cancel{69}\ \cancel{198.79}\ -199.15$$

$$\delta_5^P = \delta_1 + \frac{4\Delta t}{3}\left[2\delta_2' - \delta_3' + 2\delta_4'\right] = 28.87 + \frac{4\times.05}{3}\left[2\times 90.38 - 123.86 + 2\times 118.48\right]$$

$$= 78.37$$

$$\omega_5^P = \omega_1 + 4\times\frac{\Delta t}{3}\left[2\omega_2' - \omega_3' + 2\omega_4'\right]$$

$$= 46.18 + \frac{4\times.05}{3}\left[2\times 803.04 - 725.588 + 2\times \overset{-199.15}{\cancel{198.79}}\right] = 48.46$$

$\delta_5' = 78.37$

$$\omega_5' = \frac{0.8 - 1.333\sin 48.46}{.000577} = -342.7$$

$$\delta_5 = \delta_3 + \frac{\Delta t}{3}\left[\delta_3' + 4\delta_4' + \delta_5'\right]$$

$$= 37.25 + \frac{.05}{3}\left[123.86 + 4\times 118.48 + 78.37\right]$$

$$= 48.52$$

$$\omega_5 = \omega_3 + \frac{\Delta t}{3}\left[\omega_3' + 4\omega_4' + \omega_5'\right]$$

$$= 123.86 + \frac{.05}{3}\left[725.588 + 4\times \overset{-199.15}{\cancel{-198.79}} + {}^{-}342.7\right] = -$$

$$= \underline{\underline{116.98}}$$

$$\delta_5' = \omega_5 = 116.98$$

$$\omega_5' = \frac{.8 - 1.333 \sin 48.52}{.000577} = \underline{\underline{-344.30}}$$

3

Modified Eulers method

Consider two Simultaneous diff equ.

$$\frac{dx}{dt} = f_x(x,y,t)$$

$$\frac{dy}{dt} = f_y(x,y,t)$$

Starting with x_0, y_0, t_0 with a step size h.

$$D_x = f_x(x_0, y_0, t_0) = \left.\frac{dx}{dt}\right|_0$$

$$D_y = f_y(x_0, y_0, t_0) = \left.\frac{dy}{dt}\right|_0$$

$$\left.\begin{aligned} x^P &= x_0 + D_x h \\ y^P &= y_0 + D_y h \end{aligned}\right\} \text{ Predicted values.}$$

$$D_{xP} = \left.\frac{dx}{dt}\right|_P = f_x(x^P, y^P, t_1)$$

$$D_{yP} = \left.\frac{dy}{dt}\right|_P = f_y(x^P, y^P, t_1)$$

$$x_1 = x_0 + \left(\frac{D_x + D_{xP}}{2}\right) h$$

$$y_1 = y_0 + \left(\frac{D_y + D_{yP}}{2}\right) h$$

From swing equ.

$$\frac{d\delta}{dt} = \omega \quad \& \quad \frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{max}\sin\delta}{M}$$

$$\left.\frac{d\delta}{dt}\right|_0 = D_1 = \omega_0 \quad \left|\ \left.\frac{d\omega}{dt}\right|_0 = D_2 = \frac{P_m - P_{max}\sin\delta_0}{M}\right.$$

$$\delta^P = \delta_0 + D_1 \Delta t \quad \& \quad \omega^P = \omega_0 + D_2 \Delta t$$

$$\left.\frac{d\delta}{dt}\right|_P = D_{1P} = \omega^P \quad \& \quad \left.\frac{d\omega}{dt}\right|_P = D_{2P} = \frac{P_m - P_{max}\sin\delta^P}{M}$$

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$\delta_1 = \delta_0 + \left(\frac{D_1 + D_1P}{2}\right)\Delta t$ & $\omega_1 = \omega_0 + \left(\frac{D_2 + D_2P}{2}\right)\Delta t$

Same problem $X = \Delta U$ total reactance add

$\delta = \sin^{-1}\left(\frac{P_e}{P_{max}}\right)$ $P_{max} = \frac{EV}{X}$

$t = 0^-$, $\delta_0 = 27.8$ & $\omega_0 = 0$ $P_{max} = 1.714$

$t = 0^+$ $D_1 = \omega_0 = 0$

$D_2 = \frac{P_e - P_{max}\sin\delta_0}{m} = \frac{0.8 - 0.63\sin 27.8}{.000544} = 930.47$ $m = \frac{H}{\pi f}$

$\delta^P = \delta_0 + D_1\Delta t = 27.8$

$\omega^P = \omega_0 + D_2\Delta t = 46.52$

$D_1P = \omega_P = 46.52$

$D_2P = \frac{0.8 - 0.63\sin 27.8}{m} = 930.47$

$\delta_1 = 27.8 + \left[\frac{0 + 46.52}{2}\right] \times .05 = 28.9$

& $\omega_1 = 0 + \left[\frac{930.47 + 930.47}{2}\right] .05 = 46.52$

$t = 0.05 \Rightarrow \delta = 28.9$
$\omega = 46.52$

$D_1 = 46.52$

$D_2 = \frac{0.8 - 0.63\sin 28.9}{.000544} = 910.90$

$\delta^P = 28.9 + 46.52 \times .05 = 31.226$

$\omega^P = 46.52 + 910.9 \times .05 = 92.065$

$D_1P = 92.065$, $D_2P = \frac{0.8 - 0.63\sin 31.226}{.000544} = 870.2178$

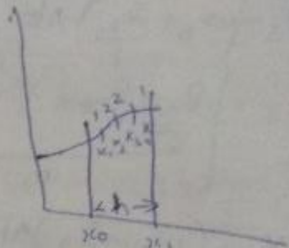
$\delta_1 = 28.9 + \left[\frac{46.52 + 92.065}{2}\right] \times .05 = \underline{32.36}$

$\omega_1 = 46.52 + \left[\frac{910.9 + 870.2178}{2}\right] \times .05 = \underline{91.048}$

After .5 sec fault is cleared then $P_{max} = 1.333$

4

Range kutta method



$$\frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting with x_0, y_0, t_0 with step size h.

$$x_1 = x_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$$k_1 = f_x(x_0, y_0, t_0)h$$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right)h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right)h$$

$$k_4 = f_x(x_0 + k_3, y_0 + l_3, t_0 + h)h$$

$$l_1 = f_y(x_0, y_0, t_0)h$$

$$l_2 = f_y\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right)h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right)h$$

$$l_4 = f_y(x_0 + k_3, y_0 + l_3, t_0 + h)h.$$

Two differential equ.

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{m} = \frac{P_m - P_{max}\sin\delta}{m}$$

$$k_1 = \omega_0 \Delta t$$

$$l_1 = \left[\frac{P_m - P_{max}\sin\delta_0}{m}\right]\Delta t$$

$$k_2 = \left(\omega_0 + \frac{l_1}{2}\right)\Delta t$$

$$l_2 = \left[\frac{P_m - P_{max}\sin\left(\delta_0 + \frac{k_1}{2}\right)}{m}\right]\Delta t$$

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$$K_3 = \left(\omega_0 + \frac{l_2}{2}\right)\Delta t$$

$$l_3 = \left\{\frac{P_m - P_{max}\sin\left(\delta_0 + \frac{K_2}{2}\right)}{m}\right\}\Delta t$$

$$K_4 = (\omega_0 + l_3)\Delta t$$

$$l_4 = \left[\frac{P_m - P_{max}\sin(\delta_0 + K_3)}{m}\right]\Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$\omega_1 = \omega_0 + \frac{1}{6}\left[l_1 + 2l_2 + 2l_3 + l_4\right]$$

Same Problem

1st interval $\delta_0 = 27.8$, $\omega_0 = 0$.

$K_1 = 0 \times .05 = 0$, $l_1 = \left[\frac{0.8 - 0.63\sin\delta_0}{.000544}\right].05 = 46.52$.

$K_2 = \left(0 + \frac{46.52}{2}\right).05 = \underline{1.163}$ $\quad l_2 = \left[\frac{0.8 - 0.63\sin\left(27.8 + \frac{0}{2}\right)}{.000544}\right] = 46.5$

$K_3 = \left(0 + \frac{46.52}{2}\right).05 = 1.163$ $\quad l_3 = \left[\frac{0.8 - 0.63\sin\left(27.8 + \frac{1.163}{2}\right)}{.000544}\right] = 46$

$K_4 = (0 + 46).05 = 2.3$ $\quad l_4 = \left(\frac{0.8 - 0.63\sin(27.8 + 1.163)}{.000544}\right).05 =$

$\delta_1 = 27.8 + \frac{1}{6}\left[0 + 2\times1.163 + 2\times1.163 + 2.3\right] = \underline{28.9}$

$\omega_1 = 0 + \frac{1}{6}\left[46.5 + 2\times46.52 + 2\times46 + 45.54\right] = \underline{46.18}$

$$k_1 = 46.18 \times .05 = 2.309 \qquad l_1 = \left[\frac{0.8 - 0.63 \sin 28.9}{.000544}\right].05 = 45.54$$

$$k_2 = \left(46.18 + \frac{45.54}{2}\right).05 = 3.4475 \qquad l_2 = \left[\frac{0.8 - 0.63 \sin\left(28.9 + \frac{2.309}{2}\right)}{.000544}\right].05 = 44.53$$

$$k_3 = \cancel{2.309}\left(46.18 + \frac{44.53}{2}\right).05 = 3.422 \qquad l_3 = \left(\frac{0.8 - 0.63 \sin\left(28.9 + \frac{3.4475}{2}\right)}{.000544}\right).05 = 44.64$$

$$k_4 = (46.18 + 44.04).05 = 4.5112 \qquad l_4 = \left(\frac{0.8 - 0.63 \sin(28.9 + 3.422)}{.000544}\right).05 = 42.56$$

$$\delta_2 = 28.9 + \frac{1}{6}\left[2.309 + 2 \times 3.4475 + 2 \times 3.422 + 4.5112\right] = 32.326$$

$$\omega_2 = 46.18 + \frac{1}{6}\left[45.54 + 2 \times 44.53 + 2 \times 44.04 + 42.56\right] = 90.386$$

5

$$\frac{dF_1}{dP_1} = 0.025P_1 + 15$$

$$\frac{dF_2}{dP_2} = 0.05P_2 + 20$$

Solution: Since the load is at bus 2 alone, therefore, the losses in t will not be affected by generator of plant 2.

$$\therefore \quad P_L = B_{11}P_1^2 \text{ as } B_{12} = B_{21} = 0 \text{ and } B_{22} = 0$$

$$\therefore \quad 15.625 = B_{11} \times 125^2$$

or

$$B_{11} = 0.001$$

Now coordination equation

$$\frac{dF_1}{dP_1} + \lambda \frac{\partial P_L}{\partial P_1} = \lambda$$

where $P_L = 0.001P_1^2$ or $\frac{dP_L}{dP_1} = 0.002P_1$

Substituting in the coordination equation for plant 1 we get

$$0.025P_1 + 15 + \lambda \cdot 0.002P_1 = \lambda$$

$$0.025P_1 + 0.048P_1 + 15 = 24$$

or

$$0.073P_1 = 9$$

or

$$P_1 = 123.28 \text{ MW}$$

and from the coordination equation for plant 2,

$$0.05P_2 + 20 = 24 \text{ or } P_2 = 80 \text{ MW}$$

$\therefore$ The transmission loss $P_L = 0.001 \times 123.28^2 = 15.19$ MW

$\therefore$ The load $P_D = 123.28 + 80 - 15.19 = 188.1$ MW

The solution using penalty factor is as follows: The penalty factor for plant 1 is

$$\frac{1}{1 - \dfrac{\partial P_L}{\partial P_1}} = \frac{1}{(1 - 0.002P_1)}$$

$$\therefore \quad \frac{dF_1}{dP_1}\frac{1}{1 - 0.002P_1} = 24$$

or

$$\frac{0.025P_1 + 15}{1 - 0.002P_1} = 24$$

$$\therefore \quad P_1 = 123.28 \text{ MW}$$

Similarly, since $\dfrac{dP_L}{dP_2}$ is zero, $\therefore$ $L_2 =$ unity, i.e. the incremental cost of received power equals the incremental cost of production.

$$\therefore \quad 0.05P_2 + 20 = 24 \text{ or } P_2 = 80 \text{ MW}$$

6

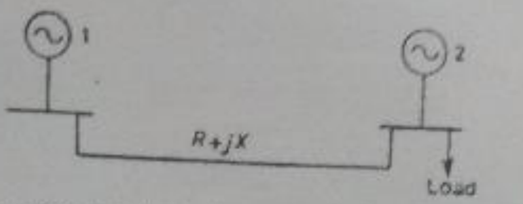

Fig. 19.3 Two identical generators connected through a transmission link

fore, the criterion of sharing load by equal incremental production cost does not hold good under such situation and a strategy must be evolved which takes into account the transmission losses also.

The optimal load dispatch problem including transmission losses is defined as

$$\text{Min } F_T = \sum_{n=1}^{n} F_n \tag{19.4}$$

$$\text{subject to } P_D + P_L - \sum_{n=1}^{n} P_n = 0 \tag{19.5}$$

where P_L is the total system loss which is assumed to be a function of generation and the other term have their usual significance.

Making use of the Lagrangian multiplier λ, the auxiliary function is given by

$$F = F_T + \lambda(P_D + P_L - \Sigma P_n)$$

The partial differential of this expression when equated to zero gives the condition for optimal load dispatch, i.e.

$$\frac{\partial F}{\partial P_n} = \frac{\partial F_T}{\partial P_n} + \lambda\left(\frac{\partial P_L}{\partial P_n} - 1\right) = 0$$

or

$$\frac{dF_n}{dP_n} + \lambda\frac{\partial P_L}{\partial P_n} = \lambda \tag{19.6}$$

Here the term $\frac{\partial P_L}{\partial P_n}$ is known as the incremental transmission loss at plant n and λ is known as the incremental cost of received power in Rs. per MWhr.

The equation (19.6) is a set of n equations with $(n+1)$ unknowns. Here n generations are unknown and λ is also unknown. These equations are known as coordination equations because they coordinate the incremental transmission losses with the incremental cost of production.

To solve these equations the loss formula equation (19.7) is expressed in terms of generations and is approximately expressed as

$$P_L = \sum_m \sum_n P_m B_{mn} P_n \tag{19.7}$$

where P_m and P_n are the source loadings, B_{mn} the transmission loss coefficients. The formula is derived under the following assumptions:

1. The equivalent load current at any bus remains a constant complex fraction of the total equivalent load current.
2. The generator bus voltage magnitudes and angles are constant.
3. The power factor of each source is constant.

The solution of coordination equation (19.6) requires the calculation of $\partial P_L/\partial P_n$ which is obtained from equation (19.7) as

$$\frac{\partial P_L}{\partial P_n} = 2\sum_m B_{mn}P_m \qquad (19.8)$$

Also

$$\frac{dF_n}{dP_n} = F_{nn}P_n + f_n \qquad (19.3)$$

∴ The coordination equations can be rewritten as

$$F_{nn}P_n + f_n + \lambda\,\Sigma\, 2B_{mn}P_m = \lambda \qquad (19.9)$$

Collecting all coefficients of P_n, we obtain

$$P_n(F_{nn} + 2\lambda B_{nn}) = -\lambda\Big(\sum_{m\neq n} 2B_{mn}P_m\Big) - f_n + \lambda$$

Solving for P_n we obtain

$$P_n = \frac{1 - \frac{f_n}{\lambda} - \sum_{m\neq n} 2B_{mn}P_m}{\frac{F_{nn}}{\lambda} + 2B_{nn}} \qquad (19.10)$$

To arrive at an optimal load dispatching solution, the simultaneous solution of the coordination equations along with the equality constraint (19.5) should suffice and any standard matrix inversion subroutine could be used. But, because of the fact that plants might go beyond their loading conditions, it becomes necessary to solve a new set of equations and thus by the process of elimination this could be done. This would be very time consuming in a large interconnected system. Therefore, an iterative procedure would be used. The iterative procedure involves a method of successive approximation which rapidly converge to the correct solution. The following steps are required for the iterative procedure:

1. Assume a suitable value of λ^0. This value should be more than the largest intercept of the incremental production cost of the various generators.
2. Calculate the generations based on equal incremental production cost.
3. Calculate the generation at all the buses using the equation

$$P_n = \frac{1 - \frac{f_n}{\lambda} - \sum_{m\neq n} 2B_{mn}P_m}{\frac{F_{nn}}{\lambda} + 2B_{nn}}$$

It is to be noted that the powers to be substituted on the right hand side during zeroth iteration correspond to the values as calculated in step 2. For subsequent iterations the values of powers to be substituted corres-

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(7)

$F_1 = 0.2 P_1^2 + 40 P_1 + 120$

$F_2 = 0.25 P_2^2 + 30 P_2 + 150.$

$P_D = 180\,MW \Rightarrow P_1 + P_2 = 180\,MW$

$$\frac{dF_1}{dP_1} = \frac{d}{dP_1}\left(0.2 P_1^2 + 40 P_1 + 120\right)$$

$$= 0.2 \times 2 P_1 + 40$$

$$\therefore \frac{dF_1}{dP_1} = 0.4 P_1 + 40$$

$$\frac{dF_2}{dP_2} = \frac{d}{dP_2}\left(0.25 P_2^2 + 30 P_2 + 150\right)$$

$$= 0.25 \times 2 P_2 + 30$$

$$\therefore \frac{dF_2}{dP_2} = 0.5 P_2 + 30$$

economic operating schedule,

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$0.4 P_1 + 40 = 0.5 P_2 + 30$$

$$0.4 P_1 - 0.5 P_2 = 30 - 40$$

$$0.4 P_1 - 0.5 P_2 = -10 \rightarrow \text{eq}^n (1)$$

$P_D = 180\,MW \Rightarrow \quad P_1 + P_2 = 180 \longrightarrow \text{eq}^n (2)$

CMR

Solving f^n ① + f^n ②.

$\therefore P_1 = 88.88\ MW = 88.9\ MW$

$\therefore P_2 = 91.11\ MW$

$F_1 = 0.2(88.9)^2 + 40(88.9) + 120 = \cancel{5256.640} = 5256.64$

$F_2 = 0.25(91.11)^2 + 30(91.11) + 150 = \cancel{4957.80} = 4957.8025$

Total cost of fuel $= F_1 + F_2 = \cancel{5256.640 + 4957.80}$ $5256.64 + 4957.8025$

$\therefore$ Total cost of fuel $= 10214.44$ Rs/hr. $= 10214.44$

load is equally shared by both units

$P_D = 180\ MW$ $\quad P_1 = P_2 = \frac{180}{2} = 90\ MW.$

P_1

$F_1 = 0.2(90)^2 + 40(90) + 120 = 5340$

$F_2 = 0.25(90)^2 + 30(90) + 150 = 4875$

Total cost of fuel $= F_1 + F_2 = 5340 + 4875$

$\therefore$ Total cost of fuel $= 10215$ Rs/hr

~~saving = 1021368~~ -

Savings $= 10215 - 10214.44 = 10215 - 10214.44$

$\therefore$ savings $= 0.56$ Rs/hr.

8

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig a

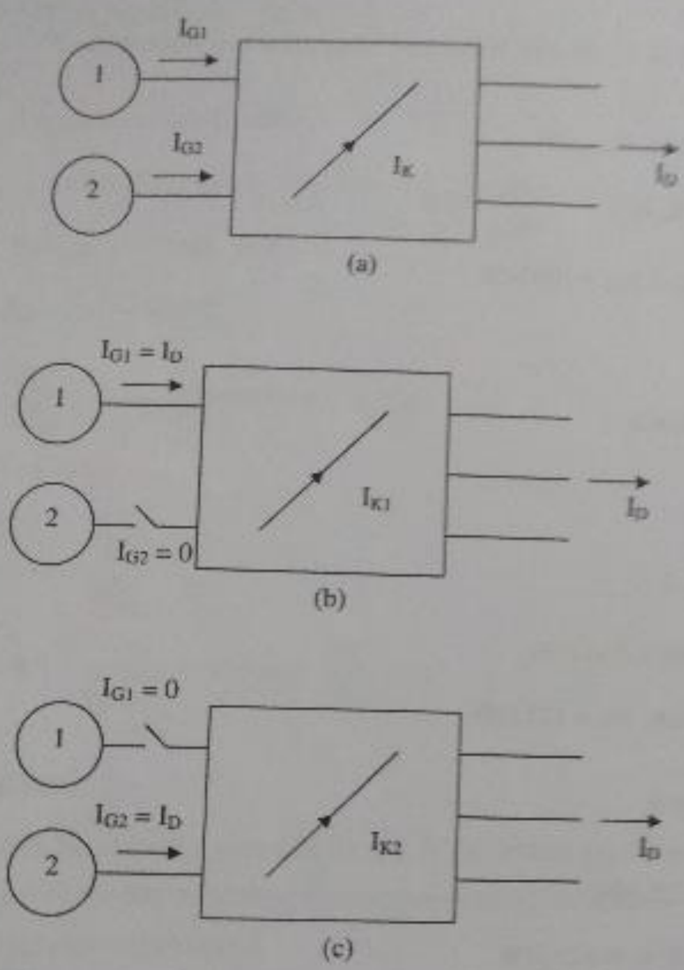

Fig Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{K1}. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

N_{K1} and N_{K2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1}\, I_{G1} + N_{K2}\, I_{G2}$$

where I_{G1}, I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D. Because of the assumptions made, I_{K1} and I_D have same phase angle, as do I_{K2} and I_D. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}|\angle\sigma_1 \text{ and } I_{G2} = |I_{G2}|\angle\sigma_2 .$$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} with respect to a common reference. We can write

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2\right)^2$$

$$= N_{K1}^2|I_{G1}|^2\left[\cos^2\sigma_1 + \sin^2\sigma_1\right] + N_{K2}^2|I_{G2}|^2\left[\cos^2\sigma_2 + \sin^2\sigma_2\right]$$
$$+ 2\left[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2\right]$$

$$= N_{K1}^2|I_{G1}|^2 + N_{K2}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where P_{G1}, P_{G2} are three phase real power outputs of plant1 and plant 2; V_1, V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K$$
$$+ \frac{P_{G2}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^2 R_K$$

$$P_L = P_{G1}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}^2 + \dots\dots\dots + \frac{P_{Gn}^2}{|V_n|^2 (\cos\phi_n)^2} \sum_K N_{Kn}^2 R_K$$
$$+ 2 \sum_{\substack{p,q=1 \\ p \neq q}}^{n} \frac{P_{GP} P_{Gq} \cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_k N_{KP} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n} \sum_{q=1}^{n} P_{Gp} B_{Pq} P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

Example 8