Kepler's First Law

1. The orbit of a satellite around Earth is elliptical with the centre of the Earth lying at one of the foci of the ellipse.

2. Eccentricity *(e)* is the ratio of the distance between the centre of the ellipse and either of its foci (= *ae*) to the semi-major axis of the ellipse *a*.

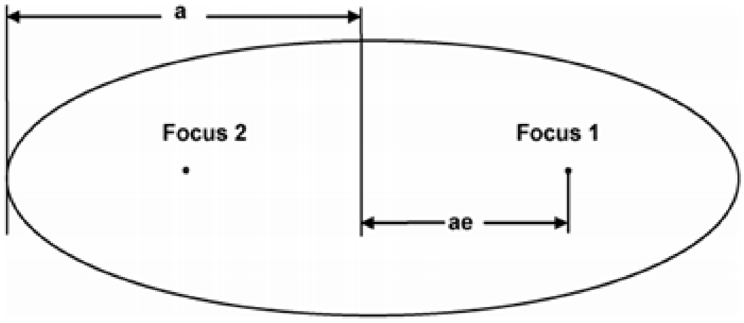


Fig: Kepler's first law

•The law of conservation of energy is valid at all points on the orbit.

•In the context of satellites, it means that the sum of the kinetic and the potential energy of a satellite always remain constant. The value of this constant is equal to $-Gm1m2/(2a)$, where

$m1$ = mass of Earth
$m2$ = mass of the satellite
a = semi-major axis of the orbit

$$\text{Kinetic energy} = \frac{1}{2}(m_2 v^2)$$

$$\text{Potential energy} = -\frac{Gm_1 m_2}{r}$$

$$\frac{1}{2}(m_2 v^2) - \frac{G m_1 m_2}{r} = -\frac{G m_1 m_2}{2a}$$

$$v^2 = G m_1 \left(\frac{2}{r} - \frac{1}{a}\right)$$

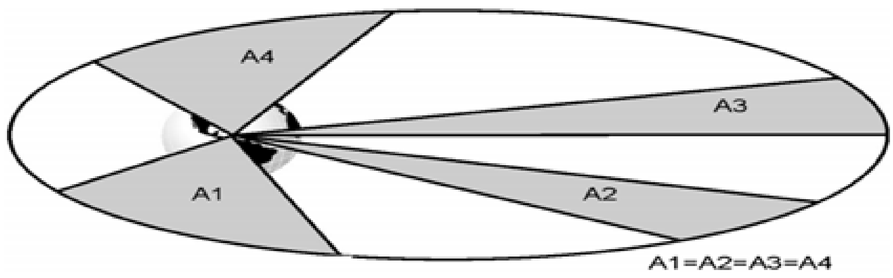
$$v = \sqrt{\left[\mu\left(\frac{2}{r} - \frac{1}{a}\right)\right]}$$

Kepler's Second Law

The line joining the satellite and the centre of the Earth sweeps out equal areas in the plane of the orbit in equal time intervals ie the rate (dA/dt) at which it sweeps area A is constant.

The rate of change of the swept-out area is given by

$$\frac{dA}{dt} = \frac{\text{angular momentum of the satellite}}{2m}$$



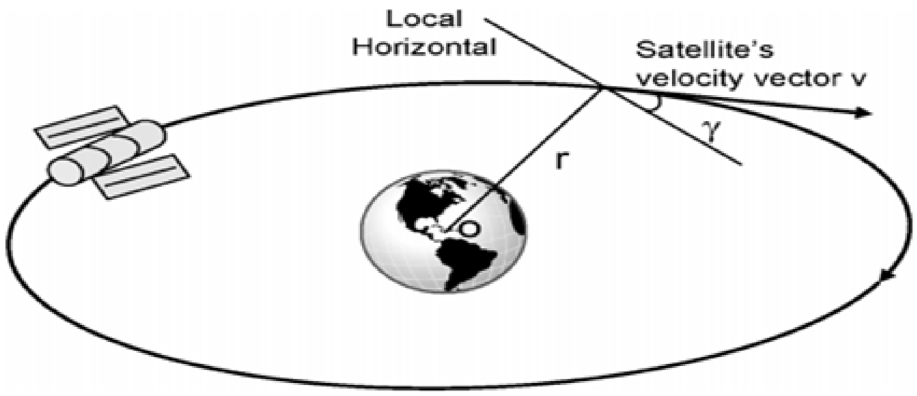


Fig: Satellite's position at any given time

Kepler's second law is also equivalent to the law of conservation of momentum, which implies that the angular momentum of the orbiting satellite given by the product of the radius vector and the component of linear momentum perpendicular to the radius vector is constant at all points on the orbit.

Kepler's Third Law

- The square of the time period of any satellite is proportional to the cube of the semi-major axis of its elliptical orbit.
- A circular orbit with radius *r* is assumed.
- A circular orbit is only a special case of an elliptical orbit with both the semi-major axis and semi-minor axis equal to the radius.

Equating the gravitational force with the centrifugal force gives

$$\frac{Gm_1m_2}{r^2} = \frac{m_2v^2}{r}$$

Replacing v by ωr in the above equation gives

$$\frac{Gm_1m_2}{r^2} = \frac{m_2\omega^2r^2}{r} = m_2\omega^2r$$

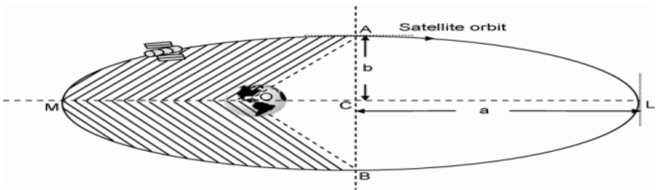
which gives $\omega^2 = Gm_1/r^3$. Substituting $\omega = 2\pi/T$ gives

$$T^2 = \left(\frac{4\pi^2}{Gm_1}\right)r^3$$

This can also be written as

$$T = \left(\frac{2\pi}{\sqrt{\mu}}\right)r^{3/2}$$

2. The satellite is moving in an elliptical orbit with its semi-major and semi-minor axes as a and b respectively and an eccentricity of 0.6. The satellite takes 3 hours to move from point B to point A. How much time will it take to move from point A to point B?



Sol.

Area of shaded region $= \dfrac{\pi ab}{2} - \dfrac{1}{2}\times(2b)\times OC$

$= 1.571\,\pi ab - b\times(ae)$

$= (1.571\pi)ab - (0.6)ab$

$= 0.97ab$ = Area swept by the satellite from B to A. —(1)

Area swept by the satellite from A to B

$= \pi ab - 0.97ab = 2.17ab$ —(2)

Taking ratio of (2) and (1), we get,

$$\frac{2.17ab}{0.97ab} = 2.23$$

$\therefore$ time taken for the satellite to move from point A to point B $= (2.23)\times$ (time taken from B to A)

$= 2.23 \times 3$ hours $= \boxed{6.71 \text{ hours}}$

3. A satellite is launched with an injection velocity v1 from a point above the surface of the earth at a distance P from the centre of the earth attains an elliptical orbit with an apogee distance A1. The same satellite when launched with an injection velocity v2 from the same perigee distance attains an elliptical orbit with an apogee distance A2. Derive the relationship between v1 and v2 in terms of P, A1 and A2.

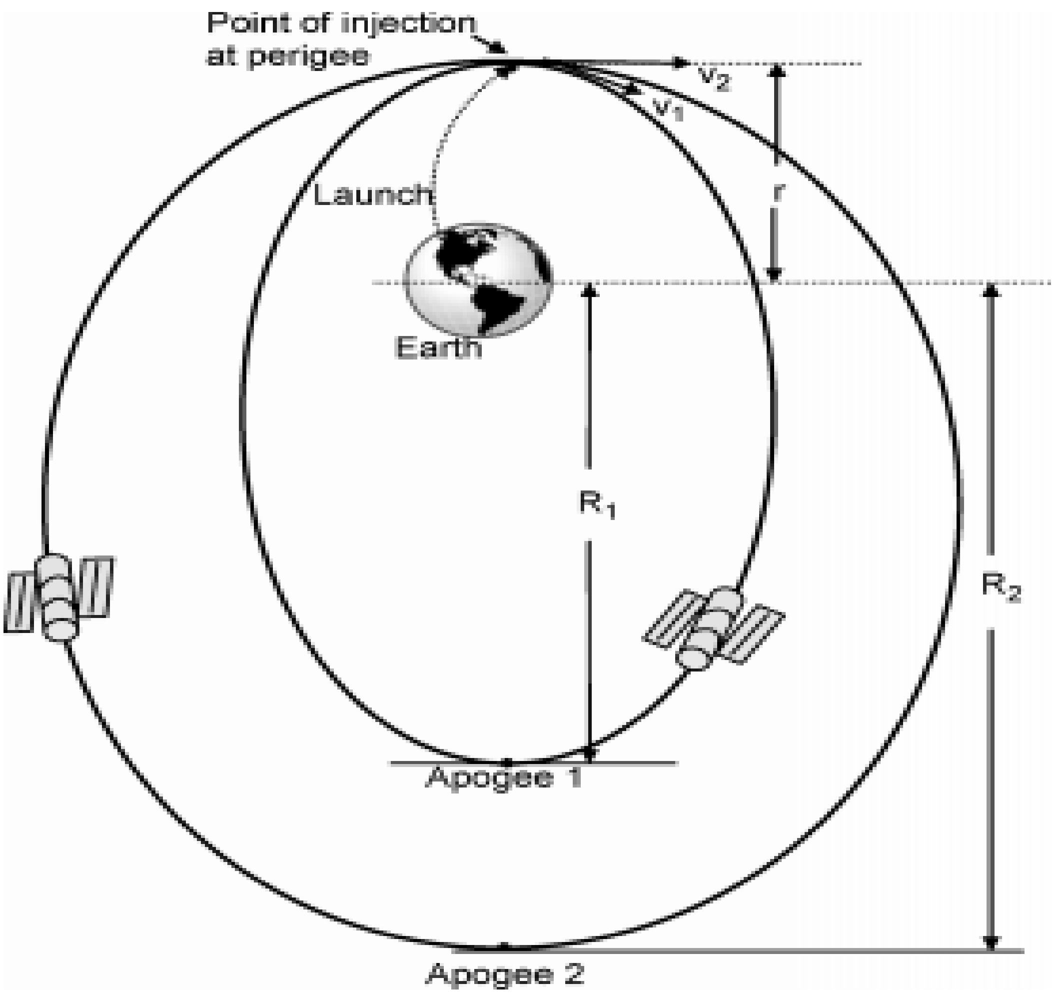
Sol.

$$v_1 = \sqrt{\left[(2\mu)\left(\frac{1}{r} - \frac{1}{R_1 + r}\right)\right]}$$

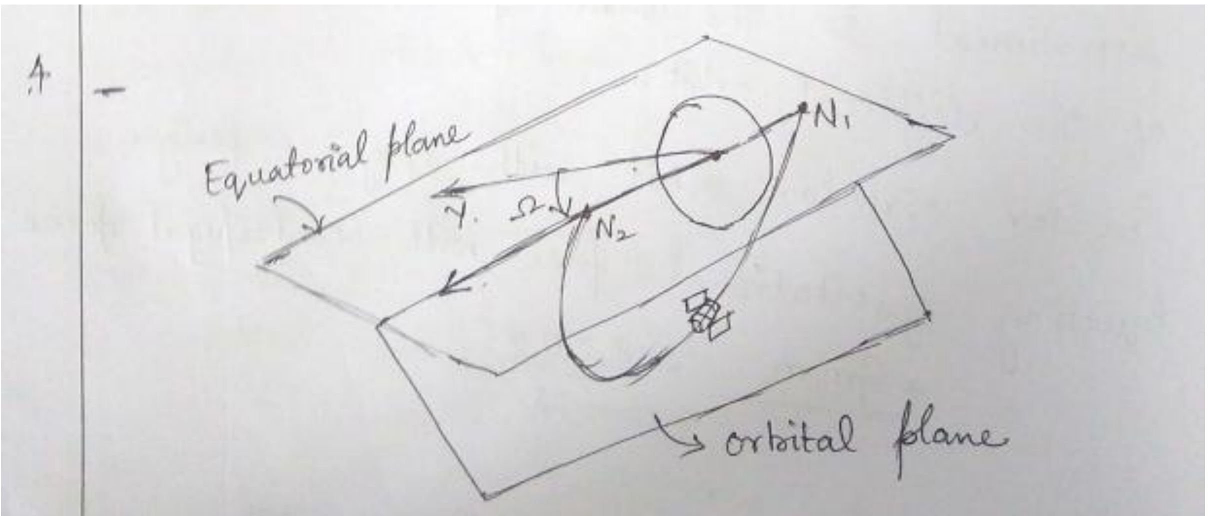
$$v_2 = \sqrt{\left[(2\mu)\left(\frac{1}{r} - \frac{1}{R_2 + r}\right)\right]}$$

Squaring the two expressions and then taking the ratio of the two yields

$$\left(\frac{v_2}{v_1}\right)^2 = \frac{1 + r/R_1}{1 + r/R_2}$$

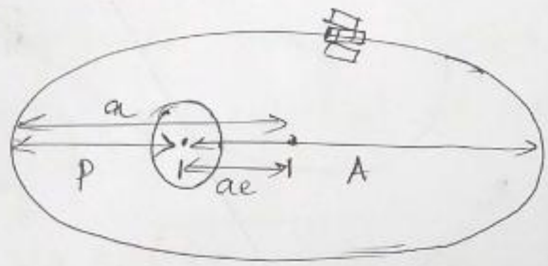


4.



N_1 – descending node
N_2 – ascending node
γ – direction of vernal equinox.

Right ascension of ascending node is the angle between the line of nodes, the line with the two nodes on it, and the direction of vernal equin
It is given by angle Ω measured from Υ to th
line of nodes.



Apogee is the point farthest from the centre of Earth, on the orbit.

Apogee distance $A = a(1+e)$

e - eccentricity

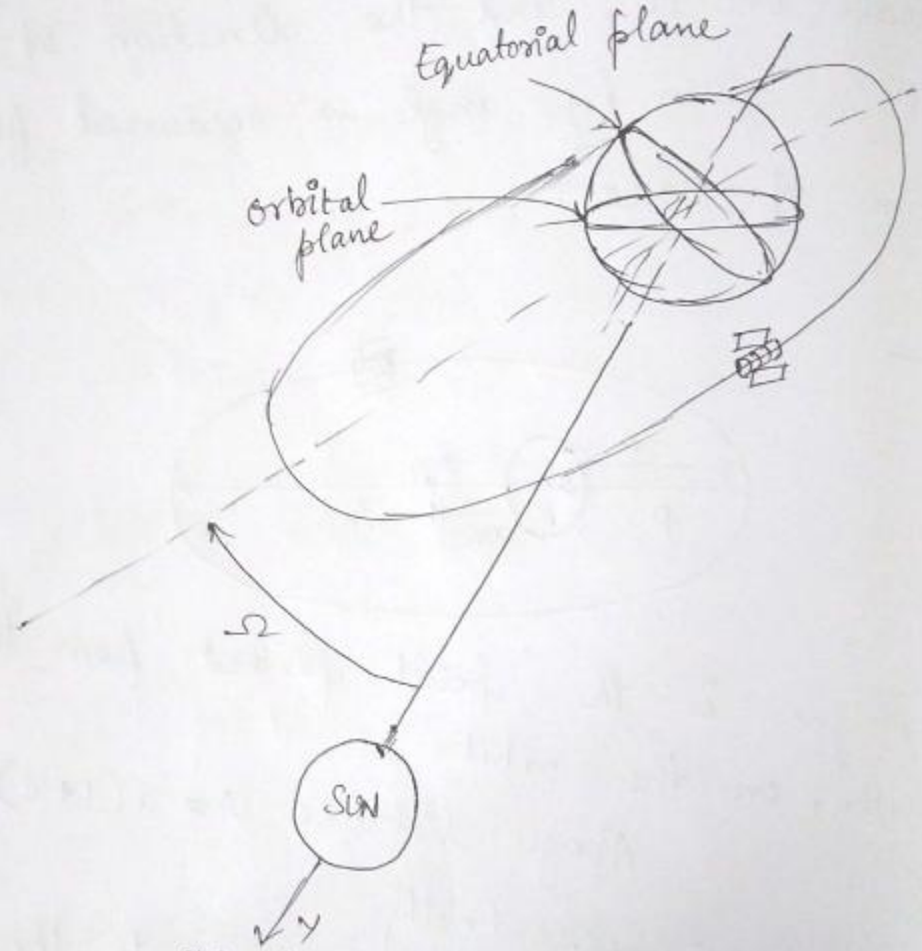
Perigee is the point nearest to the centre of Earth, on the orbit.

Perigee distance $P = a(1-e)$

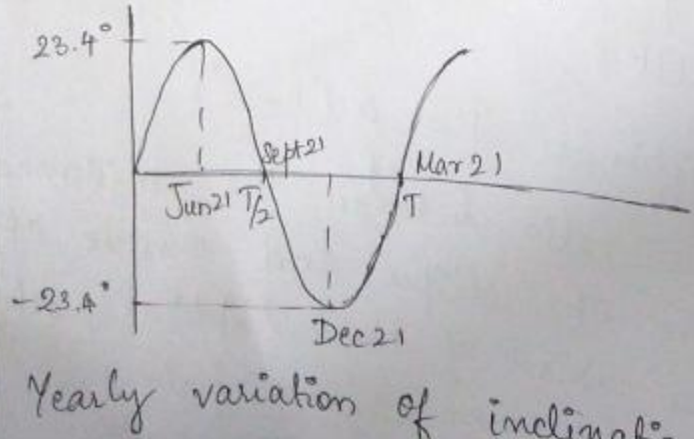
Eccentricity is the ratio ~~between~~ of the distance between the centre of ellipse and centre of Eart to the semi-major axis of the elliptical orbit.

$$e = \frac{\text{apogee} - \text{perigee}}{\text{apogee} + \text{perigee}} = \frac{\text{apogee} - \text{perigee}}{2a}$$

$$e = \frac{\sqrt{a^2-b^2}}{a}$$



Direction of vernal equinox



Yearly variation of inclination angle of Earth

Solstice occurs when the inclination angle reaches maximum, 23.4°.

June 20-21 : Summer equinox

December 21-22 : Winter equinox.

5.

Injection velocity and satellite trajectories.

It is explained using 3 cosmic velocity.

The velocity of satellite at perigee distance is given by

$$v_p = \sqrt{\frac{2\mu}{P} - \frac{2\mu}{A+P}}$$

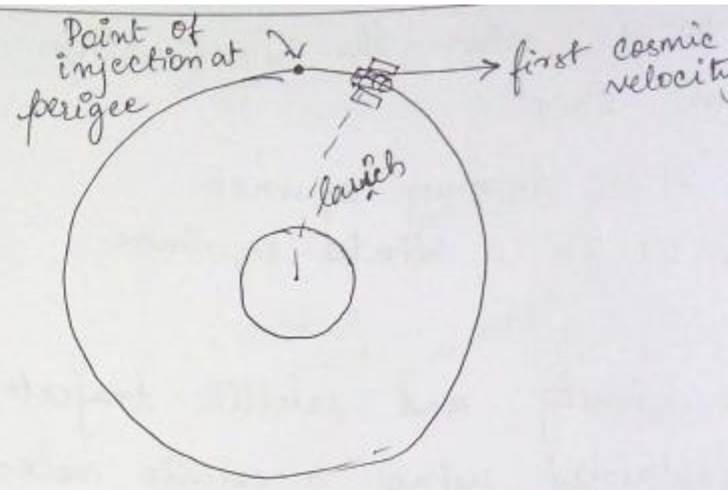
A - apogee distance

P - perigee distance

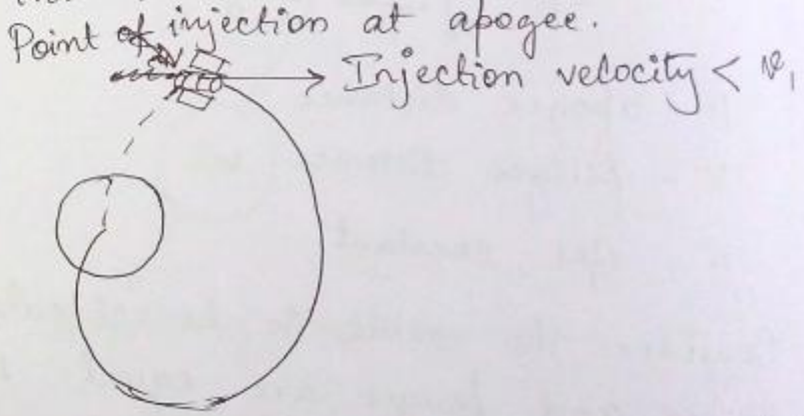
μ = GM - constant

Consider the orbit to be circular where the apogee and perigee are equal. A = r (radius of orbit).

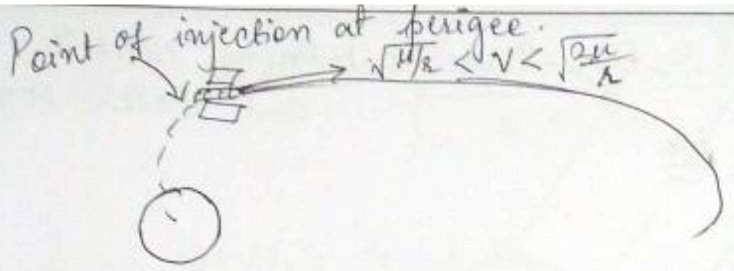
First cosmic velocity $v_1 = \sqrt{\frac{\mu}{r}}$



When the injection velocity is less than v_1, the path of orbit takes the ~~shape~~ ballistic trajectory where satellite ~~fo~~ falls back into the Earth.



When the injection velocity is greater than first cosmic velocity but less than second cosmic velocity, $v > \sqrt{\mu/r}$ but $v < \sqrt{2\mu/r}$, the orbit ta the shape of parabola.



When injection velocity becomes equal to $\sqrt{\frac{2\mu}{r}}$, path is elliptical.

When injection velocity is increased further, it can escape the solar system and this is the third cosmic velocity.

$$v = \cancel{2\mu}\sqrt{\frac{2}{\mu} - V_t^2\left(3-2\sqrt{2}\right)}$$

$$V = \sqrt{\frac{2\mu}{P} - \frac{2\mu}{A+P}} = \frac{v d \cos\gamma}{r}$$

6.

Types of Satellite Orbits

The satellite orbits can be classified on the basis of:

1. Orientation of the orbital plane
2. Eccentricity
3. Distance from Earth

Orientation of the Orbital Plane

The orbital plane of the satellite can have various orientations with respect to the equatorial plane of Earth. The angle between the two planes is called the angle of inclination of the satellite. On this basis, the orbits can be classified as
equatorial orbits, polar orbits and inclined orbits.

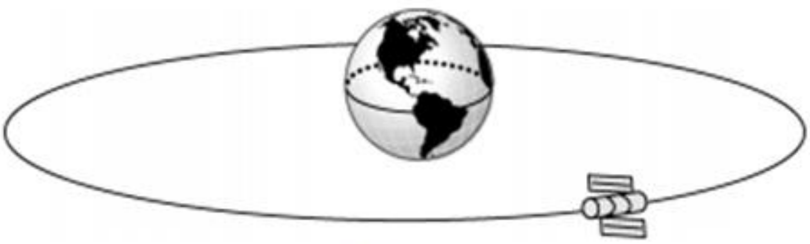


Fig: Equatorial orbit

Fig: Polar orbit

Fig: Prograde orbit

Fig: Retrograde orbit

*Orbit Types: **Distance** from Earth*

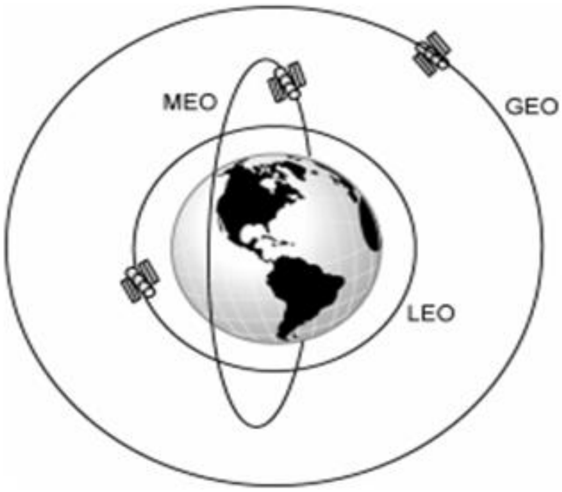


Fig: LEO, MEO and GEO orbits

Fig: Iridium constellation of satellites