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INTERNAL ASSESSMENT TEST – I

Sub:	DIGITAL SIGNAL PROCESSING							Code:	17EC52
Date:	06 / 09 / 2019	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE(D)/TCE

Answer any 5 full questions

		Marks	CO	RBT
1	Derive an expression for DFT and IDFT of a finite length sequence.	[10]	CO1	L2
2	Compute the 6-point DFT of the sequence $x[n] = [1,3,5,7]$. Plot the magnitude spectrum and the phase spectrum.	[10]	CO1	L2
3(a)	Compute the 4-point DFT of $x[n] = [1,2,3,4]$ using matrix method. Plot the magnitude spectrum and the phase spectrum.	[06]	CO1	L2
3(b)	Compute the IDFT of $X[k] = [9, -3 + j1.7321, -3 - j1.7321]$ using matrix method.	[04]	CO1	L2

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4	The first 5 samples of 8-point DFT of a real 8-point sequence are as follows. $X[k] = [36,\ -4 + 9.6569j,\ -4 + 4j,\ -4 + 1.6569j,\ -4]$. Determine the remaining samples of $X[k]$. Evaluate the following without explicitly determining $x[n]$. i) $x[0]$ ii) $x[4]$ iii) $\sum_{n=0}^{7} x[n]$ iv) $\sum_{n=0}^{7} \lvert x[n] \rvert^2$	[10]	CO1	L3
5	Derive the relationship between DFT and Z-transform of a finite length sequence $x[n], 0 \leq n \leq N-1$. Compute the Z-transform of the sequence $x[n] = [0.5, 0, 0.5, 0]$. Using Z-transform compute the DFT of $x[n]$.	[10]	CO1	L2
6(a)	Compute the DFT of the sequence $x[n] = 0.5^n, 0 \leq n \leq 3$ by evaluating the DFT of $x[n] = a^n, 0 \leq n \leq N-1$ and $0 < a < 1$.	[05]	CO1	L2
6(b)	Prove the periodicity and linearity properties of DFT.	[05]	CO1	L2

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6(b)	Prove the periodicity and linearity properties of DFT.	[05]	CO1	L2

Solutions

1 $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$X\left(\frac{2\pi}{N}k\right) = \sum_{l=-\infty}^{\infty} \sum_{n=ln}^{lN+N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$0 \le k \le N-1$

$= \sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi}{N}kn}$

$x_p(n) = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}$

$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi}{N}kn}$

$\therefore a_k = \frac{1}{N} X(k)$

$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}$

$0 \le n \le N-1$

$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$0 \le k \le N-1$

2

k	X(k)	|X(k)|	∠X(k)
0	16	16	0
1	$-7-6.9282j$	9.8489	−2.3613
2	$4+1.7321j$	4.3589	0.4086
3	-4	4	3.1416
4	$4-1.7321j$	4.3589	−0.4086
5	$-7+6.9282j$	9.8489	2.3613

3a

K	X(K)	|X(K)|	∠X(K)
0	10	10	0
1	$-2+2j$	2.8284	2.3562
2	-2	2	3.1416
3	$-2+2j$	2.8284	-2.3562

3b $x(n) = (1, 3, 5)$

4 $x(0) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)$ $\quad x(n) = (1,2,3,4,5,6,7,8)$

$= 1$

$x(4) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)(-1)^k$

$= 5$

$\sum_{n=0}^{7} x(n) = X(0)$

$= 36$

$\sum_{n=0}^{7} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{7} |X(k)|^2$

$= 204$

5 $X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}$

$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}Kn}$

$X(K) = X(z)\Big|_{z=e^{j\frac{2\pi}{N}K}}$

$x(n) = (0.5, 0, 0.5, 0)$

$X(z) = 0.5 + 0.5\, z^{-2}$

$X(k) = 0.5 + 0.5\, e^{-j\frac{2\pi}{4}k(+2)}$

$= 0.5 + 0.5\, e^{-j\pi k}$

$= (1, 0, 1, 0)$

6a $x(n) = a^n$

$X(k) = \sum_{n=0}^{N-1} \left(a e^{-j\frac{2\pi}{N}k}\right)^n$

$= \dfrac{1 - a^N}{1 - a e^{-j\frac{2\pi}{N}k}}$

$x(n) = 0.5^n, \quad 0 \le n \le 3$

$X(k) = \dfrac{1 - 0.5^4}{1 - 0.5 e^{-j\frac{2\pi}{4}k}}$

$= \dfrac{0.9375}{1 - 0.5 e^{-j\frac{\pi}{2}k}}$

6b $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$X(k+N) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}(k+N)n}$

$= X(k)$

$a x_1(n) + b x_2(n) \xleftrightarrow{DFT} a X_1(k) + b X_2(k)$