CMR
INSTITUTE OF
TECHNOLOGY

USN | | | | | | | | | | |

Internal Assesment Test – I

Sub:	Information Theory and Coding			Sec	*ALL*		Code:	17EC54
Date:	07 / 09 / 2019	Duration:	90 mins	Max Marks:	50	Sem: V	Branch:	ECE/TCE

	Answer Any FIVE FULL Questions	Marks	OBE	
			CO	RBT
1	Define *self-information*, *entropy* and *information rate*. Consider transmission of pictures in a black and white television, there are about 2.25 $Megapixels/frame$. For a good reproduction, 12 brightness levels are necessary. Assuming that all the levels are equally likely to occur, find the rate of transmission if one frame is transmitted in every $3sec$.	10	C504.1	L1
2	Mention different properties of entropy. Show that entropy is additive.	10	C504.1	L2
3	The state diagram of a Markov source is shown in the fig. 3. Show that $G_1 \geq G_2 \geq H$.	10	C504.1	L3

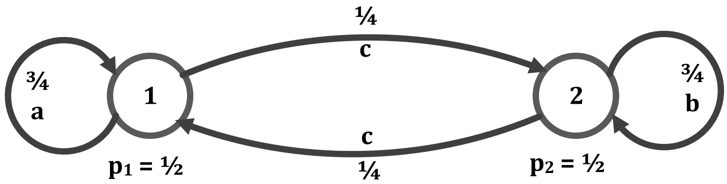


Fig. 3

4	Show that the Entropy of nth extension of a zero memory source is $H(S^n) = nH(S)$, if a source emits one of the Source Symbols S_1, S_2, S_3 with probabilities of $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ then show that $H(S^2) = 2H(S)$.		C504.2	L2
5	Apply Shannon's encoding algorithm to the following symbols and obtain the redundancy of the so formed code. If $S = \{\&, *, \%, @, !\}$ And $P = \left\{\frac{1}{8}, \frac{1}{16}, \frac{3}{16}, \frac{1}{4}, \frac{3}{8}\right\}$.	10	C504.2	L3
6	Given the symbols $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ with respective probabilities $P = \{0.02, 0.08, 0.1, 0.2, 0.2, 0.4\}$, construct a binary code by applying Shannon-Fano encoding procedure. Determine the code efficiency and redundancy of the so formed code. Draw the code tree for the same.	10	C504.2	L3
7	State of the markov source is as shown in fig. 7. Compute the source Entropy.	10	C504.1	L3

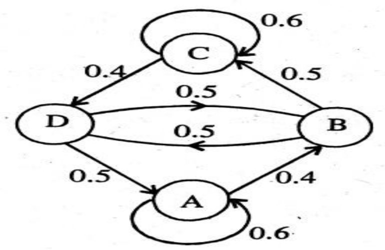


Fig. 7

CMR INSTITUTE OF TECHNOLOGY

USN | | | | | | | | | | |

Internal Assesment Test Scheme of Evaluation – I

Sub:	Information Theory and Coding		Sec	ALL				Code:	17EC54
Date:	07 / 09 / 2019	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE/TCE

	Answer Any FIVE FULL Questions	Marks	OBE CO	OBE RBT
1	Define *self-information*, *entropy* and *information rate*. Consider transmission of pictures in a black and white television, there are about 2.25 $Megapixels/frame$. For a good reproduction, 12 brightness levels are necessary. Assuming that all the levels are equally likely to occur, find the rate of transmission if one frame is transmitted in every $3sec$.	10	C504.1	L1
	Definition of each terms	1X3=3		
	No. of different frames $= \mathbf{12^{2.25X10^6}}$	1		
	Entropy $\boldsymbol{H(s) = 8.066 \text{ X } 10^6 \; bits/frame}$	2		
	Symbol rate $\boldsymbol{r_s = \frac{1}{3} \; frame/sec}$	2		
	Average information rate $\boldsymbol{R_s = 2.689 \text{ X } 10^6 \; bits/sec}$	2		
2	Mention different properties of entropy. Show that entropy is additive.	10	C504.1	L2
	Entropy is non negative	1		
	Entropy is symmetric	1		
	Entropy has boundaries	1		
	Entropy is additive	1		
	Proof for $\boldsymbol{H'(S) \geq H(S)}$	6		
3	The state diagram of a Markov source is shown in the fig. 3. Show that $G_1 \geq G_2 \geq H$.	10	C504.1	L3

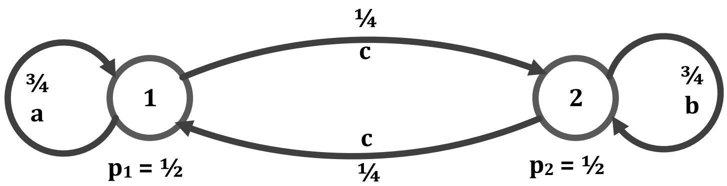


Fig. 3

		Marks
	$\boldsymbol{H_1 = 0.8113 \; bits/symbol}$	1
	$\boldsymbol{H_2 = 0.8113 \; bits/symbol}$	1
	$\boldsymbol{H = 0.8113 \; bits/symbol}$	1
	Code tree taking state 1 as initial state	1
	Code tree taking state 2 as initial state	1
	$\boldsymbol{G_1 = 1.56 \; bits/symbol}$	2
	$\boldsymbol{G_2 = 1.28 \; bits/symbol}$	2

$G_1 \geq G_2 \geq H$ 1

4 Show that the Entropy of nth extension of a zero memory source is $H(S^n) = nH(S)$, if a source emits one of the Source Symbols S_1, S_2, S_3 with probabilities of $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ then show that $H(S^2) = 2H(S)$. C504.2 L2

Proof for $H(S^n) = nH(S)$ 6

$H(S) = 1.5\ bits/sym$ 1

$H(S^2) = 3\ bits/sym$ 2

$H(S^2) = 2\ H(S)$ 1

5 Apply Shannon's encoding algorithm to the following symbols and obtain the redundancy of the so formed code. If $S = \{\&, *, \%, @, !\}$ And $P = \left\{\frac{1}{8}, \frac{1}{16}, \frac{3}{16}, \frac{1}{4}, \frac{3}{8}\right\}$. 10 C504.2 L3

Obtaining the Codes 5

Symbols	Prob	CW	l_i
!	$\frac{6}{16}$	00	2
@	$\frac{4}{16}$	01	2
%	$\frac{3}{16}$	101	3
&	$\frac{2}{16}$	110	3
*	$\frac{1}{16}$	1111	4

Average length, $L = 2.4375\ bits/sym$ 2

Entropy, $H(S) = 2.1085\ bits/sym$ 2

Efficiency, $\eta_s = 0.865$

Redundancy, $R_{\eta s} = 0.135$

$\eta_s = 86.5\%$; $R_{\eta s} = 13.5\%$ 1

6 Given the symbols $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ with respective probabilities $P = \{0.02, 0.08, 0.1, 0.2, 0.2, 0.4\}$, construct a binary code by applying Shannon-Fano encoding procedure. Determine the code efficiency and redundancy of the so formed code. Draw the code tree for the same. 10 C504.2 L3

Obtaining the codes 6

Symbols	Prob	CW	l_i
y_6	0.4	00	2
y_5	0.2	01	2
y_4	0.2	10	2
y_3	0.1	110	3
y_2	0.08	1110	4
y_1	0.02	1111	4

Average length, $L = 2.194\ bits/sym$ 1

Entropy, $H(S) = 2.3\ bits/sym$ 1

Efficiency, $\eta_s = 0.9539$

Redundancy, $R_{\eta s} = 0.0461$

$\eta_s = 95.39\%\ ;\ R_{\eta s} = 4.61\%$ 1

Code tree 1

7 State of the markov source is as shown in fig. 7. Compute the source Entropy. 10 C504.1 L3

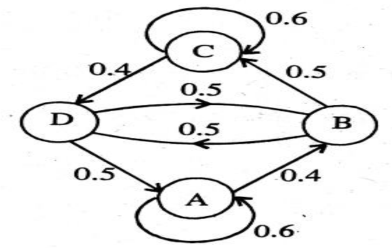


Fig. 7

$P(A) = P(C) = \frac{5}{18}$ 2

$P(B) = P(D) = \frac{4}{18}$ 2

$H_A = H_C = 0.971\ bits/sym$ 2

$H_B = H_D = 1\ bit/sym$ 2

$H = 0.9839\ bits/sym$ 2

1.

Define self-information, entropy and information rate. consider transmission of pictures in a black and white television, there are about 2.25 Megapixels | frame. For a good reproduction, 12 brightness levels are necessary. Assuming that all the levels are equally likely to occur, find the rate of transmission if one frame is transmitted in every 3sec.

$\Rightarrow$ i.) Self information:-

Let 'm_k' be a symbol for transmission at any instant of time with probability 'P_k', then amount of information (or) self information of 'm_k' is

$$\boxed{I_k = \log \frac{1}{P_k}}$$

ii.) Entropy:-

Suppose we have a source than emits one of the 'm' symbols in the statical independent manner

$$S = \{S_1, S_2, S_3 \cdots\cdots\cdots S_m\}$$

then; the average information content [entropy] is given by

$$\boxed{H(S) = \sum_{i=1}^{m} P_i \log_2 \frac{1}{P_i}} \text{ bits/sym}$$

Scanned with CamScanner

iii.) **Information rate :-**

if the source emits symbols at fixed time rate 'r_s' sym/sec. then; the average information rate will be

$$\boxed{R_s = r_s H(S)} \text{ bits/sec}$$

b.) → Total number of pixels per frame is

$= 2.25 \times 10^6$ pixels/frame

→ It is given that each element can have 12 brightness levels then;

total number different frames possible

$m = (12)^{2.25 \times 10^6}$ frames

Let us assume all the frames occur at equal probability then;

wkt;

entropy is given by $H(S)_{max}$

$H(S)_{max} = \log_2 m$

$= \log_2 (12)^{2.25 \times 10^6}$

$= 2.25 \times 10^6 \log_2 (12)$

$$\boxed{H(S)_{max} = 2.428 \text{ M bits/frames}}$$

and Given;

$r_s = 1/3$ frame/sec

then;

$R_s = r_s H(S)$

$R_s = \frac{1}{3} \times 2.428 \times 10^6$

$$\boxed{R_s = 809.38 \times 10^3 \text{ bits/sec}}$$

2. Mention different properties of entropy. Show that entropy is additive

⟹ "Properties of Entropy"

1.) Entropy function is continous for every independent variable 'P_k' in the interval (0,1)

2.) Entropy function is symmetrical of its arguments

$$H[P_k, (1-P_k)] = H[(1-P_k), P_k]$$

3.) Externmal property: [To show that entropy as boundaries]

- The lower bound for entropy is $H(S)=0$; this happens when one of the symbol $P_k = 1$, for any $k = 1, 2, \ldots m$.

$$\boxed{0 \leq H(S) \leq H(S)_{max}}$$

4.) Property of Additivity:-

Proof:-

Consider;

$$S = \{S_1, S_2, S_3 \ldots\ldots S_{q-1}, S_q\}$$

and along with probabilities;

$$P = \{P_1, P_2, P_3, \ldots\ldots P_{q-1}, P_q\}$$

consider; sample of 'S_q'

$S_a = \{ S_{a_1}, S_{a_2}, S_{a_3}, S_{a_4} \cdots\cdots S_{a_n} \}$

and their probabilities will be;

$P_a = \{ P_{a_1}, P_{a_2}, \cdots\cdots, P_{a_n} \}$

$$\sum_{j=1}^{n} P_{a_j} = P_{a_1} + P_{a_2} + \cdots\cdots + P_{a_n} = P_a \longrightarrow (1)$$

therefore;

$$H' = H[(P_1, P_2 \cdots\cdots, P_{a-1}), (P_{a_1}, P_{a_2} \cdots\cdots, P_{a_n})]$$

$$H' = \sum_{i=1}^{a-1} P_i \log_2 \frac{1}{P_i} + \sum_{j=1}^{n} P_{a_j} \log_2 \frac{1}{P_{a_j}}$$

$$H' = \sum_{i=1}^{a} P_i \log_2 \frac{1}{P_i} - P_a \log_2 \frac{1}{P_a} + \sum_{j=1}^{n} P_{a_j} \log_2 \frac{1}{P_{a_j}}$$

From eq^n

$$H' = \sum_{i=1}^{a} P_i \log_2 \frac{1}{P_i} - \sum_{j=1}^{n} P_{a_j} \log_2 \frac{1}{P_a} + \sum_{j=1}^{n} P_{a_j} \log_2 \frac{1}{P_{a_j}}$$

$$H' = \sum_{i=1}^{a} P_i \log_2 \frac{1}{P_i} + \sum_{j=1}^{n} P_{a_j} \log_2 \frac{P_a}{P_{a_j}}$$

$H' = H +$ some positive quantity

$\forall 1, 2, \cdots n$

$$\boxed{H' \geq H}$$ // proved

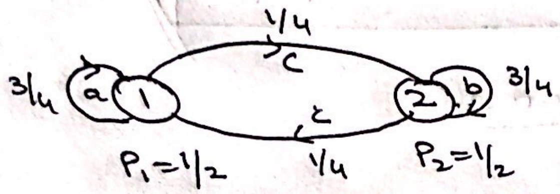
5.) <u>source</u> <u>efficiency</u>:-

$$\eta_s = \frac{H(s)}{H(s)_{max}}$$

redunency $R_{\eta_s} = 1 - \eta_s$

Both η_s and R_{η_s} is always expressed in percentage.

3. The state diagram of a markov source is shown in the fig. 3 show that $G_1 \geq G_2 \geq H$



$\Longrightarrow$

i.) State entropies (H_i);

wkt;

$$H_i = \sum_{j=1}^{n} P_{ij} \log_2 \frac{1}{P_{ij}} \text{ bits/sym}$$

Given

$n=2$; so; $i=1,2$

$\rightarrow$ for $i=1$;

$$H_1 = \sum_{j=1}^{2} P_{1j} \log_2 \frac{1}{P_{1j}}$$

$$H_1 = P_{11} \log_2 \frac{1}{P_{11}} + P_{12} \log_2 \frac{1}{P_{12}}$$

$$\boxed{H_1 = 0.81127 \text{ bits/sym}}$$

$\hookrightarrow$ for $i=2$;

$$H_2 = \sum_{j=1}^{2} P_{2j} \log_2 \frac{1}{P_{2j}} = P_{21} \log_2 \frac{1}{P_{21}} + P_{22} \log_2 \frac{1}{P_{22}}$$

$$\boxed{H_2 = 0.81127 \text{ bits/sym}}$$

ii.) Entropy of source [H]:-

wkt;

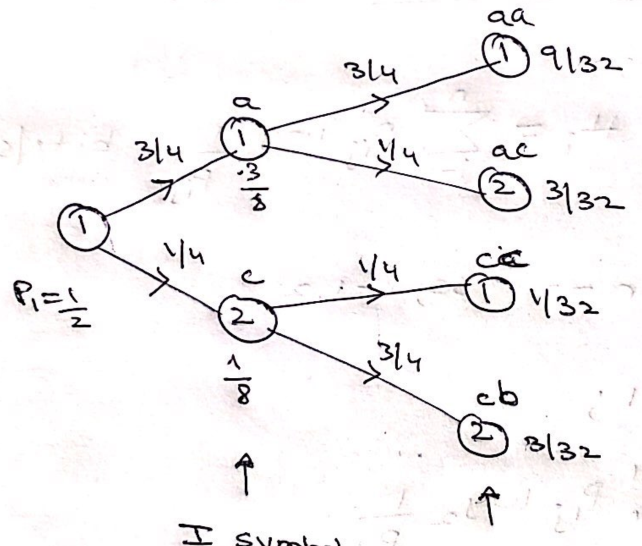
$$H = \sum_{i=1}^{n} P_i H_i \text{ bits/sym}$$

$$H = P_1 H_1 + P_2 H_2 = \frac{1}{2} \times 0.8113 + \frac{1}{2} \times 0.8113$$

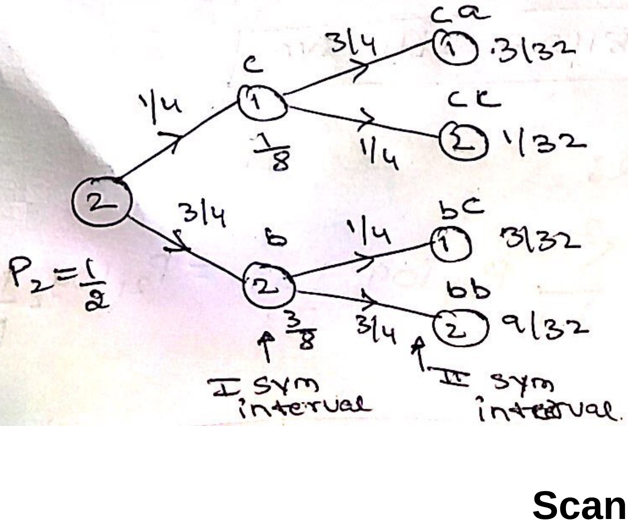
$$\boxed{H = 0.8113 \text{ bits/sym}}$$

iii.) Tree Diagram:-

→ Taking first symbol as reference;



→ Taking second symbol as reference;



I - symbol interval:-

sym	prob
a	3/8
b	3/8
c	2/8

$$G_N = \frac{1}{N} \sum_{\forall i} P(m_i) \log_2 \frac{1}{P(m_i)} \text{ bits/sym}$$

$$G_1 = P_a \log_2 \frac{1}{P_a} + P_b \log_2 \frac{1}{P_b} + P_c \log_2 \frac{1}{P_c}$$

$$\boxed{G_1 = 1.56 \text{ bits/sym}}$$

II - symbol interval:-

sym	prob
aa	9/32
ab	0
ac	3/32
ba	0
bb	9/32
bc	3/32
ca	3/32
cb	3/32
cc	2/32

$$G_2 = \frac{1}{2}\left[2\left(\frac{9}{32}\right) \log_2\left(\frac{32}{9}\right) + 4\left(\frac{3}{32}\right) \log_2\left(\frac{32}{3}\right) + \frac{2}{32} \log_2\left(\frac{32}{2}\right)\right]$$

$$\boxed{G_2 = 1.28 \text{ bits/sym}}$$

therefore;

$$\boxed{G_1 \geq G_2 \geq H}$$ // proved

4.) Show that the Entropy of the n^{th} extension of a zero memory source is $H(S^n) = nH(S)$, if a source emits one of the source symbols S_1, S_2, S_3 with probabilities of $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ then show that $H(S^2) = 2H(S)$

$\Longrightarrow$ a.) consider a sample the 's'

$$S = \{S_1, S_2\}$$

with probabilities;

$$P = \{P_1, P_2\}$$

$$\therefore P_1 + P_2 = 1$$

wkt;

Entropy $H(S) = \sum_{i=1}^{m} P_i \log_2 \frac{1}{P_i}$ bits/sym.

$$H(S) = \sum_{i=1}^{2} P_i \log_2 \frac{1}{P_i} = P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} \longrightarrow (1)$$

For II extension;

$$(\text{No of sym in II ext}) = \left(\begin{matrix}\text{No of sym in} \\ \text{basic src}\end{matrix}\right)^{ext}$$

then;

$S_1 S_1$ will occor prob $P_1 P_1 = P_1^2$

$S_1 S_2$ will occur prob $P_1 P_2 = P_1 P_2$

$S_2 S_1$ will occur prob $P_2 P_1 = P_1 P_2$

$S_2 S_2$ will occur prob $P_2 P_2 = P_2^2$

then;

$$P_1^2 + P_1 P_2 + P_1 P_2 + P_2^2 = 1$$

Therefore;

→ <u>Entropy for II extension</u> $H(S^2)$ will be

$$H(S^2) = \sum_{i=1}^{m^2} P_i \log_2 \frac{1}{P_i}$$

$$= P_1^2 \log_2 \frac{1}{P_1^2} + 2P_1 P_2 \log_2 \frac{1}{P_1 P_2} + P_2^2 \log_2 \frac{1}{P_2^2}$$

$$= 2P_1^2 \log_2 \frac{1}{P_1} + 2P_1 P_2 \log_2 \frac{1}{P_1} + 2P_1 P_2 \log_2 \frac{1}{P_2} + 2P_2^2 \log_2 \frac{1}{P_2}$$

$$H(S^2) = 2P_1 (P_1 + P_2) \log_2 \frac{1}{P_1} + 2P_2 (P_1 + P_2) \log_2 \frac{1}{P_2}$$

$$H(S^2) = 2P_1(P_1 + P_2) \log_2 \frac{1}{P_1} + 2P_2 (P_1 + P_2) \log_2 \frac{1}{P_2}$$

$$H(S^2) = 2\left[P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2}\right]$$

$$\boxed{H(S^2) = 2H(S)}$$ //

similary;

$$\boxed{H(S^n) = n\,H(S)}$$ // proved

b) Given;

$S = \{S_1, S_2, S_3\}$

with probabilities

$P = \{\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\}$

then;

$S_1 S_1$ will occur with prob $= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$S_1 S_2$ will occur with prob $= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$S_2 S_1$ will occurs with prob $= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$S_2 S_2$ will occurs with prob $= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$S_2 S_3$ will occurs with prob $= \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$

$S_3 S_1$ will occurs with prob $= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

$S_3 S_2$ will occurs with prob $= \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$

$S_3 S_3$ will occurs with prob $= \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$

Therefore;

$$H(S^2) = \frac{4}{4} \log_2 4 + \frac{4}{8} \log_2 8 + \frac{1}{16} \log_2 16$$

$$\boxed{H(S^2) = 3.75 \text{ bits/sym}}$$

but;

$$H(S) = P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} + P_3 \log_2 \frac{1}{P_3}$$

$$H(S) = \frac{2}{2} \log_2 2 + \frac{1}{4} \log_2 4$$

$$\boxed{H(S) = 1.5 \text{ bits/sym}}$$

Hence;

$$\boxed{H(S^2) = 2H(S) = 3.75} \text{ bits/sym}$$

proved

5. Apply shannon's encoding alogrithm to the following symbols and obtain the redundancy of the so formed code. if
$S = \{ \&, *, \%, @, ! \}$ and $P = \{ \frac{1}{8}, \frac{1}{16}, \frac{3}{16}, \frac{1}{4}, \frac{3}{8} \}$

Scanned with CamScanner

$\Rightarrow$ Shannon's encoding

$$P = \left\{ \frac{2}{16}, \frac{1}{16}, \frac{3}{16}, \frac{4}{16}, \frac{6}{16} \right\}$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$P_4 \quad P_5 \quad P_3 \quad P_2 \quad P_1$$

Step-1:-

Arranging in descending order of probabilites.

$$S = \{ !, @, \%, \&, * \}$$

with probabilites

$$P = \left\{ \frac{6}{16}, \frac{4}{16}, \frac{3}{16}, \frac{2}{16}, \frac{1}{16} \right\}$$

Step-2:- Assume the initial value of 'α' as '0'

$$\alpha_1 = 0$$

$$\alpha_2 = P_1 = \frac{6}{16} = (0.375)_{10}$$

$$\alpha_3 = \alpha_2 + P_2 = \frac{6}{16} + \frac{4}{16} = (0.625)_{10}$$

$$\alpha_4 = \alpha_3 + P_3 = \frac{10}{16} + \frac{3}{16} = (0.8125)_{10}$$

$$\alpha_5 = \alpha_4 + P_4 = \frac{13}{16} + \frac{2}{16} = (0.9375)_{10}$$

$$\alpha_6 = \alpha_5 + P_5 = \frac{15}{16} + \frac{1}{16} = (1)_{10}$$

Step-3:-

Determine the smallest integer value of 'l_i' using in equality

$$\boxed{l_i \geq \log_2 \frac{1}{P_i}} \text{ bits}$$

→ for i=1;

$$l_1 \geq \log_2 \frac{1}{P_1}$$

$$l_1 = \log_2 \frac{16}{6} = 1.415$$

$$\boxed{l_1 = 2 \text{ bits}}$$

→ for i=2;

$$l_2 \geq \log_2 \frac{1}{P_2}$$

$$l_2 = \log_2 \frac{16}{4}$$

$$\boxed{l_2 = 2 \text{ bits}}$$

→ for i=3;

$$l_3 \geq \log_2 \frac{1}{P_3}$$

$$l_3 \geq \log_2 \frac{16}{3}$$

$$l_3 = 2.415$$

$$\boxed{l_3 = 3 \text{ bits}}$$

→ for i=4;

$$l_4 \geq \log_2 \frac{1}{P_4}$$

$$l_4 = \log_2 \frac{16}{2}$$

$$\boxed{l_4 = 3 \text{ bits}}$$

→ for i=5;

$$l_5 \geq \log_2 \frac{1}{P_5}$$

$$l_5 = \log_2 16$$

$$\boxed{l_5 = 4 \text{ bits}}$$

Step-4 :-

Expansion Decimal to Binary

i.) $\alpha_1 = (0)_{10} = (00)_2$

ii.) $\alpha_2 = (0.375)_{10} = (0.011000...)_2$

iii.) $\alpha_3 = (0.625)_{10} = (0.10100...)_2$

iv.) $\alpha_4 = (0.8125)_{10} = (0.110100...)_2$

v.) $\alpha_5 = (0.9375)_{10} = (0.111100...)_2$

Step-5 :-

Table

sym	prob	code word	l_i
!	6/16	00	2
@	4/16	01	2
%	3/16	101	3
&	2/16	110	3
*	1/16	1111	4

i.) H(s) Entropy :-

$$H(s) = \sum_{i=1}^{5} P_i \log_2 \frac{1}{P_i} \quad \text{bits/sym}$$

$$H(s) = \frac{6}{16}\log_2\left(\frac{16}{6}\right) + \frac{4}{16}\log_2\left(\frac{16}{4}\right) + \frac{3}{16}\log_2\left(\frac{16}{3}\right)$$

$$+ \frac{2}{16}\log_2\left(\frac{16}{2}\right) + \frac{1}{16}\log_2 16$$

$$\boxed{H(s) = 2.1084 \text{ bits/sym}}$$

ii.) $L = \sum_{i=1}^{q} P_i l_i$ bits/sym

$$L = \sum_{i=1}^{5} P_i l_i$$

$$L = P_1 l_1 + P_2 l_2 + P_3 l_3 + P_4 l_4 + P_5 l_5$$

$$\boxed{L = 2.4375 \text{ bits/sym}}$$

iii.) efficiency (η_s):-

$$\eta_s = \frac{H(S)}{L} = \frac{2.1084}{2.4375}$$

$$\boxed{\eta_s = 0.8649} \simeq 86.49\%$$

iv.) Reducdency R_{η_s}:-

$$R_{\eta_s} = 1 - \eta_s$$

$$= 1 - 0.8649$$

$$\boxed{R_{\eta_s} = 0.1351} \simeq 13.51\%$$

6.) Given the symbols $S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ with respective probabilities $P = \{0.02, 0.08, 0.1, 0.2, 0.2, 0.4\}$, construct a binary code by applying shannon-fano encoding procedure. Determine the code efficiency and redundency of the so formed code. Draw the code tree for the same

$\Longrightarrow$ Given; to Apply "shannon-fano encoding"

$$S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$$

Scanned with CamScanner

with probabilities

$P = \{0.02, 0.08, 0.1, 0.2, 0.2, 0.4\}$

$\downarrow P_6 \quad \downarrow P_5 \quad \downarrow P_4 \quad \downarrow P_3 \quad \downarrow P_2 \quad \downarrow P_1$

Step-1:- ~~Arranging~~ Arranging in descending order of probability

$S = \{S_6, S_5, S_4, S_3, S_2, S_1\}$

with probabilities

$P = \{0.4, 0.2, 0.2, 0.1, 0.08, 0.02\}$

Step-2:-

Given;

q = no of symbol in source alphabet = 6

r = no of symbol in code alphabet = 2

$$\alpha = \frac{q-r}{r-1} = \frac{6-2}{2-1}$$

$$\boxed{\alpha = 4}$$

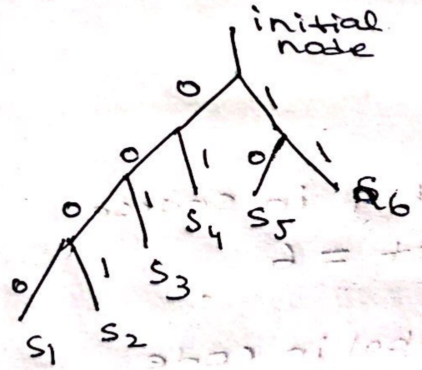
Step-3 to 6:-

sym	Prob	cw						
S_6	0.4	1	0.4	1				
S_5	0.2	1	0.2	0				
S_4	0.2	0	0.2	1				
S_3	0.1	0	0.1	0	0.1	1		
S_2	0.08	0	0.08	0	0.08	0	0.08	1
S_1	0.02	0	0.02	0	0.02	0	0.02	0

Table:-

sym	prob	cw	l_i
S_6	0.4	11	2
S_5	0.2	10	2
S_4	0.2	01	2
S_3	0.1	001	3
S_2	0.08	0001	4
S_1	0.02	0000	4

Code tree:-



$\rightarrow$ $$H(S) = \sum_{i=1}^{m} P_i \log_2 \frac{1}{P_i} \text{ bits/sym}$$

$$H(S) = 0.4 \log_2\left(\frac{1}{0.4}\right) + 2(0.2)\log_2\left(\frac{1}{0.2}\right) + 0.1\log_2\left(\frac{1}{0.1}\right) + 0.08 \log\left(\frac{1}{0.08}\right) + 0.02\log_2\left(\frac{1}{0.02}\right)$$

$$\boxed{H(S) = 2.21 \text{ bits/sym}}$$

$$\rightarrow L = \sum_{i=1}^{m} P_i \lambda_i$$

$$L = P_1 l_1 + P_2 l_2 + P_3 l_3 + P_4 l_4 + P_5 l_5 + P_6 l_6$$

$$L = 0.4(2) + 0.2(2) + 0.2(2) + 0.1(3) + 0.08(4) + 0.02(4)$$

$$\boxed{L = 2.3 \text{ bits/sym}}$$

$\rightarrow$ efficiency (η_s) :-

$$\eta_s = \frac{H(S)}{L} = \frac{2.21}{2.3}$$

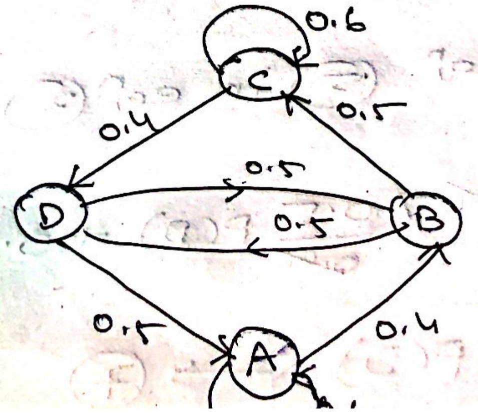
$$\boxed{\eta_s = 0.9608} \simeq 96.08\%$$

$\rightarrow$ Redundancy (R_{η_s}) :-

$$R_{\eta_s} = 1 - \eta_s$$

$$\boxed{R_{\eta_s} = 0.0392} \simeq 3.92\%$$

7. State of the markov source is as shown in fig 7. Compute the source Entropy



$\Longrightarrow$

i.) State probabilities will be;

$$P(A) = 0.6P(A) + 0.5P(D) \longrightarrow (1)$$

$$P(B) = 0.4P(A) + 0.5P(D) \longrightarrow (2)$$

$$P(C) = 0.5P(B) + 0.6P(C) \longrightarrow (3)$$

$$P(D) = 0.5P(B) + 0.4P(C) \longrightarrow (4)$$

From;

$\rightarrow$ eq^n (1)

$$0.4P(A) = 0.5P(D)$$

$$\boxed{P(A) = \frac{5}{4}P(D)} \longrightarrow (5)$$

$\rightarrow$ eq^n (3)

$$0.4P(C) = 0.5P(B)$$

$$\boxed{P(C) = \frac{5}{4}P(B)} \longrightarrow (6)$$

Substitute eq^n (5) in eq^n (2)

$$P(B) = \cancel{0.4} \times \frac{0.5}{\cancel{0.4}}P(D) + 0.5P(D)$$

$$\boxed{P(B) = P(D)} \longrightarrow (7)$$

then; substitute eq^n 7 in eq^n 6

$$\boxed{P(C) = \frac{5}{4} P(D)} \longrightarrow 8$$

w.k.t;

$$\sum_i P_i = 1$$

so;

$$P(A) + P(B) + P(C) + P(D) = 1 \longrightarrow 9$$

so, substitute eq^n 5, 6 and 8, 7 in eq^n 9 ; we get.

$$\frac{5}{4} P(D) + P(D) + \frac{5}{4} P(D) + P(D) = 1$$

$$\frac{5}{2} P(D) + 2P(D) = 1$$

$$5P(D) + 4P(D) = 2$$

$$9P(D) = 2$$

$$\boxed{P(D) = \frac{2}{9}}$$

then;

$$P(A) = \frac{5}{18}$$

$$P(B) = \frac{2}{9}$$

$$P(C) = \frac{5}{18}$$

Hence;

ii.) Entropy of each states $[H_i]$

wkt;

$$H_i = \sum_{j=1}^{n} P_{ij} \log_2 \frac{1}{P_{ij}} \quad \text{bits/sym}$$

→ for $i = A$;

$$H_A = \sum_{j=A}^{D} P_{Aj} \log_2 \frac{1}{P_{Aj}}$$

$$H_A = P_{AA} \log_2 \frac{1}{P_{AA}} + P_{AB} \log_2 \frac{1}{P_{AB}} + P_{AC} \log_2 \frac{1}{P_{AC}} + P_{AD} \log_2 \frac{1}{P_{AD}}$$

$$H_A = 0.6 \log_2 \left(\frac{1}{0.6}\right) + 0.4 \log_2 \frac{1}{0.4} + 0 + 0$$

$$\boxed{H_A = 0.97095 \text{ bits/sym}}$$

→ for $i = B$;

$$H_B = 0.5 \log_2 \frac{1}{0.5} + 0.5 \log_2 \frac{1}{0.5}$$

$$\boxed{H_B = 1 \text{ bits/sym}}$$

→ for $i = C$;

$$H_C = 0.6 \log_2 \frac{1}{0.6} + 0.4 \log_2 \frac{1}{0.4}$$

$$\boxed{H_C = 0.97095 \text{ bits/sym}}$$

→ for $i = D$;

$$H_D = 0.5 \log_2 \frac{1}{0.5} + 0.5 \log_2 \frac{1}{0.5}$$

$$\boxed{H_D = 1 \quad \text{bits/sym}}$$

⇒ Entropy of source:- [H]

$$H = \sum_{i=A}^{D} P_i H_i \quad \text{bits/sym}$$

$$H = P_A H_A + P_B H_B + P_C H_C + P_D H_D$$

$$H = \frac{5}{18}(0.97095) + \frac{2}{9}(1) + \frac{5}{18}(0.97095) + \frac{2}{9}(1)$$

$$\boxed{H = 0.983861 \quad \text{bits/sym}}$$