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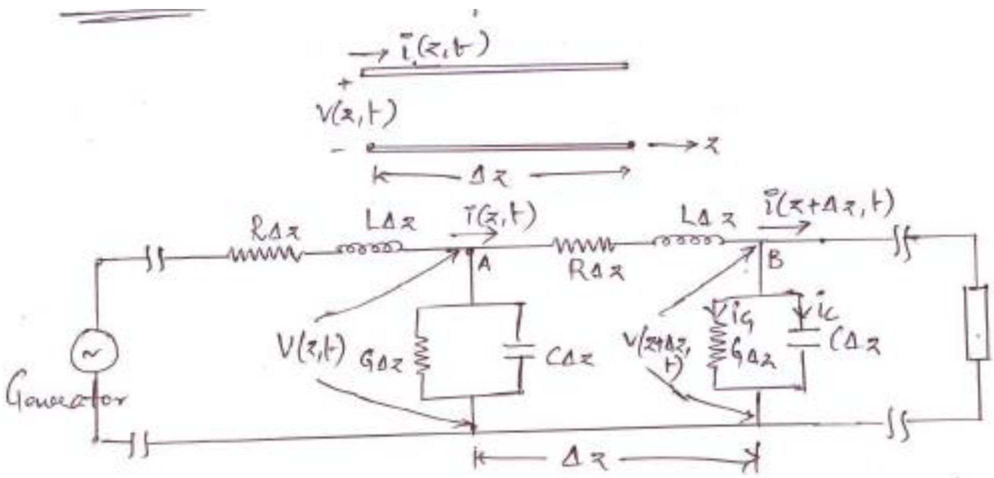
Internal Assesment Test - I

Sub:	Microwave and Antennas							Code:	15EC71
Date:	11/ 09 / 2019	Duration:	90 mins	Max Marks:	50	Sem:	7th	Branch:	ECE
Answer Any FIVE FULL Questions									

		Marks	OBE CO	OBE RBT
1.	Derive the expression for the voltage and current at any point along a uniform transmission line.	[10]	CO1	L2



2m

fig(1): Equivalent circuit of a two wire transmission line.

- Transmission line constants called distributed constants are spread along the entire length of the transmission line and can't be distinguished separately.

- The amount of inductance, capacitance, resistance depend on the length of line, size of conducting wires, spacing between wires and the dielectric between the wires.

- We can analyze a transmission line in terms of voltage, current, impedance and power along the line i.e., using distributed circuit method.

Applying Kirchoff's voltage law to loop (A):

$$v(z,t) - R\Delta z \cdot i(z,t) - L\Delta z \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0 \longrightarrow (1\cdot a)$$

$$\Rightarrow \lim_{\Delta z \to 0} \frac{v(z+\Delta z,t) - v(z,t)}{\Delta z} = \lim_{\Delta z \to 0} \left[-R\, i(z,t) - L\frac{\partial i(z,t)}{\partial t} \right]$$

$\Downarrow$ L'Hospital's rule

$$\Rightarrow \boxed{\frac{\partial v(z,t)}{\partial z} = -R\, i(z,t) - L\frac{\partial i(z,t)}{\partial t}} \longrightarrow (2\cdot a)$$

Applying Kirchoff's current law to node A:

$$i(z,t) - G\Delta z\, v(z+\Delta z,t) - C\Delta z \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0 \longrightarrow (1\cdot b)$$

$$\Rightarrow \lim_{\Delta z \to 0} \frac{i(z+\Delta z,t) - i(z,t)}{\Delta z} = \lim_{\Delta z \to 0} \left[G\, v(z+\Delta z,t) - C\frac{\partial v(z+\Delta z,t)}{\partial t} \right]$$

$$\Rightarrow \boxed{\frac{\partial i(z,t)}{\partial z} = -G\, v(z,t) - C\frac{\partial v(z,t)}{\partial t}} \longrightarrow (2\cdot b)$$

2m

Equations (2.a) & (2.b) are time domain form of transmission line equations.

For sinusoidal steady state condition, we can substitute $\frac{\partial}{\partial t}$ with $j\omega$. Thus, equations (1.a) and (1.b) can be written as:

$$(1\cdot a) \Rightarrow V(z) - (R + j\omega L)\, I(z)\,\Delta z - V(z+\Delta z) = 0$$

$$\Rightarrow \boxed{\frac{dV(z)}{dz} = -(R + j\omega L) I(z)} \longrightarrow (3\cdot a)$$

2m

and

$$(1\cdot b) \Rightarrow I(z) - (G + j\omega C)\Delta z\, V(z+\Delta z) - I(z+\Delta z) = 0$$

$$\Rightarrow \boxed{\frac{dI(z)}{dz} = -(G + j\omega C)\, V(z)} \longrightarrow (3\cdot b)$$

Differentiating (2.a) w.r.t z and (2.b) w.r.t t and combining the results will give transmission line equation in voltage form. This is found as:

(2a) $\Rightarrow$ differentiating w.r.t z : $\dfrac{\partial^2 v}{\partial z^2} = -R\dfrac{\partial i}{\partial z} - L\dfrac{\partial^2 i}{\partial z \partial t}$ $\longrightarrow$ (4.a)

(2b) $\Rightarrow$ diff. w.r.t t : $\dfrac{\partial^2 i}{\partial z \partial t} = -G\dfrac{\partial v}{\partial t} - C\dfrac{\partial^2 v}{\partial t^2}$ $\longrightarrow$ (4.b) 1m

Substituting (2.b) & (4.b) in (4.a) gives:

$$\frac{\partial^2 v}{\partial z^2} = -R\left[-Gv - C\frac{\partial v}{\partial t}\right] - L\left[-G\frac{\partial v}{\partial t} - C\frac{\partial^2 v}{\partial t^2}\right]$$

$$\Rightarrow \boxed{\frac{\partial^2 v}{\partial z^2} = RGv + (RC+LG)\frac{\partial v}{\partial t} + LC\frac{\partial^2 v}{\partial t^2}} \longrightarrow (5.a)$$

Similarly, differentiating (2.a) w.r.t 't' and (2.b) w.r.t 'z' and combining the results, we get:

$$\boxed{\frac{\partial^2 i}{\partial z^2} = RGi + (RC+LG)\frac{\partial i}{\partial t} + LC\frac{\partial^2 i}{\partial t^2}} \longrightarrow 5.b$$

Equations (5.a) and (5.b) are the final transmission line equations in vltg and current form.

Now, from equations (3.a) and (3.b), we can get the wave equations for $V(z)$ and $I(z)$. This is done by spatially differentiating both sides of (3.a) and substituting (3.b) for space derivative of current. 1m

i.e., (3.a) $\xrightarrow[\text{w.r.t } z]{\text{differentiating}}$ $\dfrac{d^2 V(z)}{dz^2} = -(R+j\omega L)\dfrac{dI(z)}{dz}$

Substituting (5.b) gives:

$$\frac{d^2V(z)}{dz^2} = (R+j\omega L)(G+j\omega C)\,V(z)$$

$$\Rightarrow \frac{d^2V(z)}{dz^2} = Z_s Y_p V(z)$$

Similarly, $\boxed{\frac{d^2I(z)}{dz^2} = Z_s Y_p I(z)}$

Let $\gamma^2 = Z_s Y_p$ = propagation constant

$$\Rightarrow \gamma = \sqrt{Z_s Y_p} = \sqrt{(R+j\omega L)(G+j\omega C)} = \alpha + j\beta$$

Thus, the eqns become:

$$\boxed{\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0} \longrightarrow (6.a)$$

$$\& \quad \boxed{\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0} \longrightarrow (6.b)$$

(6.a, 6.b) ⇒ Txn line eqns in the form of wave eqns

- The possible solution to equation (6.a) is:

$$\boxed{V = V_+ e^{-\gamma z} + V_- e^{+\gamma z} = V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z}} \quad (7.a)$$

The quantity βz = electrical length of the line (in radians)

- V_+ and V_- are complex quantities. 2m
- Term involving $e^{-j\beta z}$ shows the wave traveling in $+z$ direction and the term with factor $e^{j\beta z}$ is a wave going in $-z$ direction

Similarly, one solution to equation (6.b) is:

$$\boxed{I = I_+ e^{-\gamma z} + I_- e^{+\gamma z}} = \boxed{\frac{V_+}{Z_0} e^{-\gamma z} - \frac{V_-}{Z_0} e^{+\gamma z}}$$

$$\Rightarrow \boxed{I = Y_0 \left(V_+ e^{-\alpha z} e^{-j\beta z} - V_- e^{+\alpha z} e^{+j\beta z}\right)}$$

2. (a) Define reflection coefficient. Derive the equation for reflection coefficient at the load end. [07] CO1 L3

The reflection coefficient, which is designated by Γ is defined as

2m

$$\text{Reflection Coefficient} \equiv \frac{\text{Reflected voltage or current}}{\text{Incident voltage or current}}$$

$$= \frac{V_{ref}}{V_{inc}} = -\frac{I_{ref}}{I_{inc}}$$

The voltage to current relationship at the load point is fixed by the load impedance. The incident voltage and the current waves travelling along transmission line are given by,

$$V = V_+ e^{-\gamma z} + V_- e^{+\gamma z} \quad \text{——} (1.1)$$

$$I = I_+ e^{-\gamma z} + I_- e^{+\gamma z} \quad \text{——} (1.2)$$

in which the current wave can be expressed in terms of the voltage by,

$$I = \frac{V_+}{Z_0} e^{-\gamma z} - \frac{V_- e^{\gamma z}}{Z_0} \quad \text{——} (1.3)$$

If the line has a length of l, the voltage and current at the receiving end becomes,

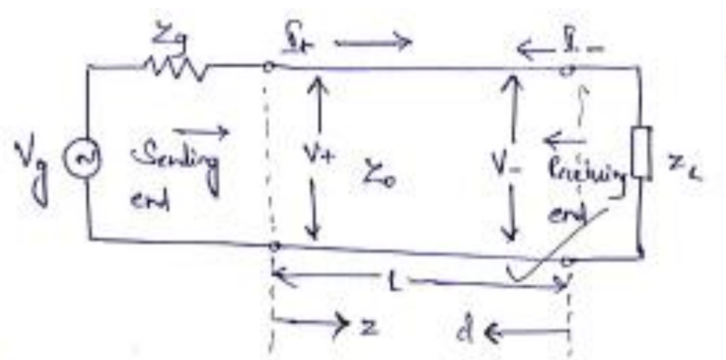
$$V_L = V_+ e^{\gamma l} + V_- e^{\gamma l} \quad \text{——} (1.4)$$

$$I_L = \frac{1}{Z_0} \left(V_+ e^{-\gamma l} - V_- e^{+\gamma l} \right) \quad \text{——} (1.5)$$

The ratio of the voltage to the current at the receiving end is the load impedance.

$$Z_L = \frac{V_L}{I_L} = Z_0 \frac{V_+ e^{-\gamma l} + V_- e^{+\gamma l}}{V_+ e^{-\gamma l} - V_- e^{+\gamma l}} \quad \text{——} (1.6)$$

(1.6)



Transmission line terminated in a load impedance.

Reflection coefficient, $\Gamma \equiv \dfrac{V_{ref}}{V_{inc}} = -\dfrac{I_{ref}}{I_{inc}}$ ——— (1.7) 3m

Reflection coefficient at the receiving end:

$$\Gamma_L = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{——— (1.8)}$$

If load impedance and/or the characteristic impedance are complex quantities, the reflection coefficient is generally a complex quantity that can be expressed as 1m

$$\Gamma_L = |\Gamma_L| e^{j\theta_L} \quad \text{——— (1.9)}$$

where $|\Gamma_L| \le 1$, θ_L is the phase angle between the incident and reflected voltages at the receiving end. It is usually called the phase angle of the reflection coefficient.

The general solution of the reflection coefficient at any point on the line, then corresponding to the incident and reflected waves at that point, each attenuated in the direction of its progress along the line. 1m

The generalized reflection coefficient is defined as,

$$\Gamma \equiv \frac{V_- e^{\gamma z}}{V_+ e^{-\gamma z}} \quad \text{——— (1.10)}$$

Let $z = l - d$. Then the reflection coefficient at some point located at a distance d from the receiving end is

$$\Gamma_d = \frac{V_- e^{\gamma(l-d)}}{V_+ e^{-\gamma(l-d)}} = \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}} e^{-2\gamma d} = \Gamma_L e^{-2\gamma d} \quad (1.11)$$

Reflection coefficient at that point can be expressed in terms of reflection coefficient at the receiving end as

$$\Gamma_d = \Gamma_L e^{-2\alpha d} e^{-j2\beta d} = |\Gamma_L| e^{-2\alpha d} e^{j(\theta_L - 2\beta d)} \quad (1.12)$$

2. (b) A twin wire transmission line in air has adjacent voltage maximas at 12.5 cm and 27.5 cm. What is the operating frequency of the line?

Distance between 2 maximas $= \frac{\lambda}{2}$ 2m

$\Rightarrow 27.5 - 12.5 = \frac{\lambda}{2}$

$(15 \times 2)\,cm = \lambda$

$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{30\,cm} = 1\,GHz \quad (1 \times 10^9)$ 1m

3. (a) What are standing waves? Draw the standing wave pattern for: [8] CO1 L2

(i) Open circuit termination (ii) Short Circuit termination

(ii) Matched termination

- In a mismatched terminated line, incident and reflected signals interfere to produce a standing wave pattern along the line.
- Equations (9a) & (9b) are the general solutions of tran line equation and consist of two waves traveling in opposite directions with unequal amplitude : 1m

$$\text{equ (9.a)} \Rightarrow V = V_+ e^{-\alpha z} e^{-j\beta z} + V_- e^{\alpha z} e^{j\beta z}$$

$$= V_+ e^{-\alpha z}\left[\cos(\beta z) - j\sin(\beta z)\right] + V_- e^{\alpha z}\left[\cos(\beta z) + j\sin(\beta z)\right]$$

$$\Rightarrow \boxed{V = (V_+ e^{-\alpha z} + V_- e^{\alpha z})\cos(\beta z) - j(V_+ e^{-\alpha z} - V_- e^{\alpha z})\sin(\beta z)} \quad (13.a)$$

With no loss, it is assumed that $V_+ e^{-\alpha z}$ and $V_- e^{\alpha z}$ are real.

Open-Circuit Case: When input approaches load end, current is zero there.
The mag. field collapses since current is zero there. This collapsing mag. field produces an electric field (from maxwell's eqn) which is added to the existing field & thus voltage at open circuit end is increased. This additional voltage gives rise to a wave which travels back to the sending end.
∴ at load end : $V_{inc} + V_{inc} \rightarrow$ doubled
↳ voltage doubling action

2m

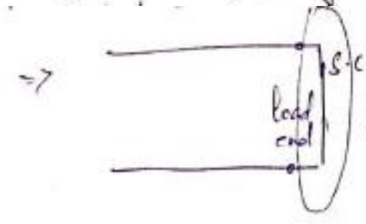
Z_R : load impedance $-1 \le \Gamma \le 1$

1m

$Z_R = Z_0$ (matched line)	$Z_R = 0$ (Short Circuit line)	$Z_R = \infty$ (Open Circuit line)
$\Gamma = 0$ (no reflection).	$\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0}$	$\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0}$
∴ Max power transferred from source to load.	$\Rightarrow \Gamma = -1$.	$\Rightarrow \Gamma = \frac{1 - Z_0/Z_R}{1 + Z_0/Z_R} = \frac{1-0}{1+0}$
	$\Rightarrow V_{ref} = -V_{inc}$	$\Rightarrow \Gamma = 1$
	⇒ entire voltage is reflected with magnitude inverse	$\Rightarrow V_{ref} = V_{inc}$ ⇒ entire voltage reflects

Short-Circuit Case: Electric field collapses which generates a magnetic field which is added to already existing magnetic field. $\frac{I_{ref}}{I_{inc}} = -1 \Rightarrow I_{ref} = I_{inc}$

$\Rightarrow$



$I_{inc} + I_{inc} = 2 I_{inc}.$

↳ Current Doubling Action.

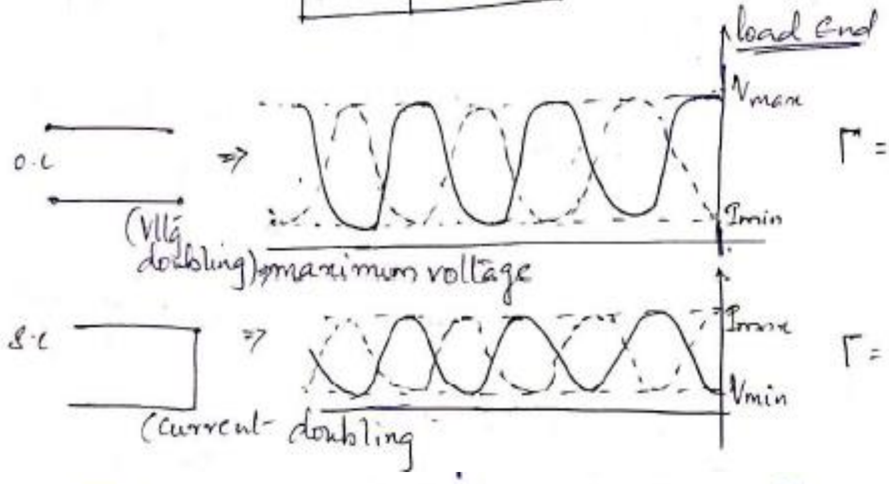
2m

Short-Circuit : $Z_R = 0$; $\Gamma = -1 \Rightarrow \rho = \frac{1+1}{1-1} = \infty$

Open-circuit : $Z_R = \infty$; $\Gamma = 1 \Rightarrow \rho = \infty$.

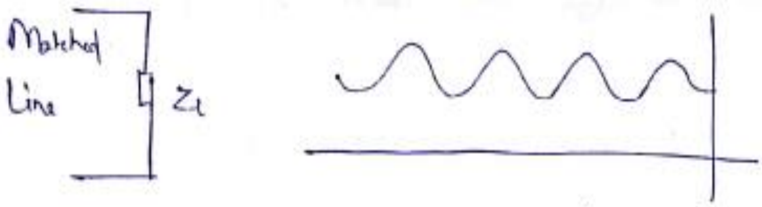
Matched line : $Z_R = Z_0$; $\Gamma = 0 \Rightarrow \rho = 1$. 2m

$\Longrightarrow \boxed{1 \le \rho \le \infty}$



$\Gamma = 1.$

$\Gamma = -1$



eg

eg $\Gamma = +ve \longrightarrow V_{max}$ near load

$\Gamma = -ve \longrightarrow V_{min}$ near. load

3. (b) Derive the relation between SWR and reflection coefficient.

- The ratio of the maximum of the standing wave pattern to the minimum is defined as the standing-wave ratio, designated by ρ.

$$\boxed{SWR\ (\rho) = \left|\frac{V_{max}}{V_{min}}\right| \cdot \left|\frac{I_{max}}{I_{min}}\right|} \quad (14)$$

1m

Two traveling wave components add in phase at some points and subtract at other points

Then, maximum amplitude is:

$$\Rightarrow \boxed{V_{max} = V_+ e^{-\alpha z} + V_- e^{\alpha z} = V_+ e^{-\alpha z}(1+|\Gamma|)} \quad (13.e)$$

which occurs at $\beta x = n\pi$; $n = 0, \pm 1, \pm 2, \ldots\ldots$

and minimum amplitude is:

$$\Rightarrow \boxed{V_{min} = V_+ e^{-\alpha z} - V_- e^{\alpha z} = V_+ e^{-\alpha z}(1-|\Gamma|)} \quad (13.f)$$

$$\rho = \frac{V_+ e^{-\alpha z}(1+|\Gamma|)}{V_- e^{-\alpha z}(1-|\Gamma|)} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

from the eqns of V_{max} and V_{min} [(13e) & (13.f)] we get:

$$\boxed{\rho = \frac{|V_{max}|}{|V_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}} \quad (15)$$

1m

4. A transmission line has the following primary constants per km of the line: R=8Ω/m, G=0.1µ℧/m, L=3.5mH/m, C=9nF/m. Calculate Zo, α, β, v_p and γ at w=50 * 10^6 rad/sec.	[10]	CO1	L3

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{8+j(5000\times 3.3\times10^{-3})}{(0.1\times10^{-6})+j(5000\times 9\times10^{-7})}}$$ 3m

$$= \sqrt{\frac{19.241\angle 65.43}{4.5\times10^{-5}\angle 89.87}} = 653.87\angle -12.22 = 639.05 - j138.40\ \Omega$$

$$\alpha = \frac{1}{2}\left[R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right] = 6.445\times10^{-3}\ Np/km$$ 3m

$$\beta = \omega\sqrt{LC} = 0.02606\ rad/km.$$ 1m

$$v_p = \frac{\omega}{\beta} = 178.174\ km/s$$

$$\gamma = \alpha + j\beta = (6.445\times10^{-3}) + j(0.02606)$$ 3m

5. Define transmission coefficients. Derive an expression for transmission coefficient in the transmission line . Also, derive the relationship between Γ and T. [10] CO1 L3

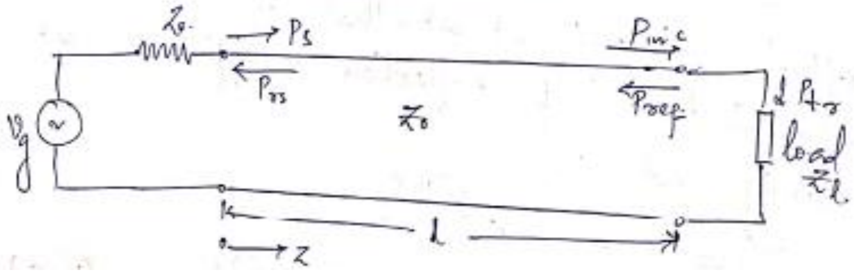
- A txn line terminated in its characteristic impedance Z_0 is called a properly terminated line. Otherwise, it is called improperly terminated line.
- According to principle of conservation of energy the incident power minus the reflected power must be equal to the power transmitted to the load.

$$1 - \Gamma_\ell^2 = \frac{Z_0}{Z_L} T^2$$ 2m

Transmission coefficient, T, is defined as:

$$T = \frac{\text{transmitted voltage or current}}{\text{incident voltage or current}}$$

$$\rightarrow \boxed{T \equiv \frac{V_{tr}}{V_{inc}} = \frac{I_{tr}}{V_{inc}}} \quad \text{---} (12.a)$$



Let the traveling wave at the receiving end be:

from (9.a) & (9.b)
$$\begin{cases} V_+ e^{-\gamma l} + V_- e^{\gamma l} = V_{tr} e^{-\gamma l} & \text{——— 12 (a)} \\ \frac{V_+}{Z_0} e^{-\gamma l} - \frac{V_-}{Z_0} e^{\gamma l} = \frac{V_{tr}}{Z_l} e^{-\gamma l} & \text{——— (b)} \end{cases}$$

from (a):

$\div$ (a) by $V_+ e^{-\gamma l}$, we get

$$\frac{V_{tr} e^{-\gamma l}}{V_+ e^{-\gamma l}} = 1 + \frac{V_- e^{\gamma l}}{V_+ e^{-\gamma l}}$$

$$\Rightarrow \boxed{T = 1 + \Gamma = 1 + \frac{Z_l - Z_0}{Z_l + Z_0} = \frac{2Z_l}{Z_l + Z_0}} \quad \text{——— (12.b)}$$

2m

The power carried by the two in the side of the incident & reflected waves is:

$$P_{in} = P_{inc} - P_{ref} = \frac{(V_+ e^{-\alpha l})^2}{2Z_0} - \frac{(V_- e^{\alpha l})^2}{2Z_0} \quad \text{——— (12.c)}$$

$$P_{avg} = \frac{1}{2} V I^* = \frac{|V|^2}{2Z_0}$$

$\therefore$ here $P_{inc} = \frac{|V_+ e^{-\gamma l}|^2}{2Z_0} = \frac{(V_+ e^{-\alpha l})^2}{2Z_0}$

The Power carried to the load by the transmitted waves is:

$$P_{tr} = \frac{(V_{tr} e^{-\alpha l})^2}{2Z_l}$$

2m

By setting $P_{in} = P_{tr}$, we have from (12.c)

$$\frac{(V_{tr} e^{-\alpha l})^2}{2Z_l} = \frac{(V_+ e^{-\alpha l})^2}{2Z_0} - \frac{(V_- e^{\alpha l})^2}{2Z_0}$$

$$\Rightarrow \left(\frac{V_{tr} e^{-\alpha l}}{V_+ e^{-\alpha l}}\right)^2 = \frac{Z_l}{Z_0}\left[1 - \left(\frac{V_- e^{\alpha l}}{V_+ e^{-\alpha l}}\right)^2\right]$$

$$\Rightarrow \boxed{T^2 = \frac{Z_l}{Z_0}\left(1 - \Gamma_l^2\right)} \quad \text{——— (12.d)}$$

2m

6 (a).	A transmission line has characteristic impedance of 50+j0.01 Ω and is terminated in a load impedance of 73-j42.5Ω. Calculate (i) Reflection coefficient (ii) SWR.	[07]	CO1	L2

i) Reflection Coefficient

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(73-j42.5)-(50+j0.01)}{(73-j42.5)+(50+j0.01)}$$

$$\Gamma_L = 0.2729 - j0.2517 = 0.371\angle -42.68^\circ$$ 3.5m

ii) SWR

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|} = 2.18$$ 3.5m

6. (b)	In air, a lossless transmission line of length (l) 50cm with L=10μH/m, C=40pF/m is operated at a 25 MHz frequency. What is the electrical length (βl) of the line?	[03]	CO2	L2

$$\beta = \omega\sqrt{LC} = 2\pi \cdot 25\times10^{6}\sqrt{40\times10^{-12}\times10\times10^{-6}} = \pi$$ 1m

$$f = 25\,MHz \Rightarrow \lambda = \frac{v_p}{f}, \quad v_p = \frac{1}{\sqrt{LC}}, \quad \beta = \frac{2\pi}{\lambda} = \frac{2\pi}{(v_p/f)} = \frac{2\pi f}{(1/\sqrt{LC})} = \pi$$ 2m

$$\text{ele. length} = \beta l = \pi\times50\times10^{-2} = \frac{\pi}{2}\,rad$$

7.	A 30 m long lossless line with Zo=50Ω operating at 2MHz is terminated with load Z_L = (60+j40) Ω. Find Γ and ρ.	[10]	CO2	2

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60+j40-50}{60+j40+50} = \frac{10+j40}{110+j40} = 0.35\angle 58^\circ$$ 5m

$$\rho = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.35}{1-0.35} = \frac{1.35}{0.65} = 2.07$$ 5m