CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test – II

Sub: Digital Image Processing Code: 15EC72

Date: 12/ 10 / 2019 Duration: 90 mins Max Marks: 50 Sem: VII Branch: ECE: B,C

Answer any 5 questions.

Q. NO	Questions	Marks	OBE CO	OBE RBT
1	Explain the following with respect to image enhancement. a. Image Negatives b. Power law Transformations. c. Piece Wise Linear Transformation functions	 [2] [3] [5]	Co2	L1
2	Consider an image having intensity distribution as show in 1.It is desired to transform this histogram to that of specified histogram as given in table 2. Draw the transformation function and normalized histograms of image before and after matching.	[10]	Co2	L3
3	What is meant by Laplacian filter? Using the second derivative, develop a Laplacian mask for image sharpening. Also Explain high boost filtering.	[10]	Co2	L2
4	Explain all the noise probability density functions.	[10]	Co2	L1
5	What are the three methods of estimating the degradation function? Explain each of them.	[10]	Co2	L2
6	Explain briefly inverse filtering and Weiner filtering in image processing.	[10]	Co2	L1

Table 1:Intensity distribution

0	120
1	325
2	891
3	745
4	1021
5	1521
6	775
7	227

Table 2: Specified histogram levels

0	0
1	0
2	0.15
3	0.12
4	0.21
5	0.52
6	0
7	0

Scheme Of Evaluation

Internal Assessment Test II – Mar.2019

<table>
<tr><td>Sub:</td><td colspan="6">Digital Image Processing</td><td>Code:</td><td>15EC72</td></tr>
<tr><td>Date:</td><td>12/10/2019</td><td>Duration:</td><td>90mins</td><td>Max Marks:</td><td>50</td><td>Sem: VII</td><td>Branch:</td><td>ECE B & C</td></tr>
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Note: Answer Any Five Questions

<table>
<tr><th>Question #</th><th>Description</th><th>Marks Distribution</th><th>Max Marks</th></tr>
<tr><td rowspan="4">1</td><td>Explain the following with respect to image enhancement. a. Image Negatives b. Power law Transformations. c. Piece Wise Linear Transformation functions</td><td>-</td><td rowspan="4">10</td></tr>
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<tr><td>Power law transformation</td><td>3</td></tr>
<tr><td>Piece wise linear transformation</td><td>5</td></tr>
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<tr><td>7</td><td>0</td></tr>
</table>
</td><td>-</td><td rowspan="3">10</td></tr>
<tr><td>Histogram matching</td><td>7</td></tr>
<tr><td>Draw Transformation function, Histogram before and after matching</td><td>3</td></tr>
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<tr><td>Derivation</td><td>6</td></tr>
<tr><td>High boost filtering</td><td>3</td><td></td></tr>
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<tr><td>PDFs</td><td>4</td></tr>
<tr><td>Mathematical equations</td><td>4</td></tr>
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<tr><td>Experimentation</td><td>2</td></tr>
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<tr><td>Weiner Filtering</td><td>6</td></tr>
</table>

Solution:

Image Negatives

The negative of an image with gray levels in the range [*0, L - 1*] is obtained by using the negative transformation shown in Fig.3.3,which is given by the expression:

$$s = L - 1 - r$$

Reversing the intensity levels of an image in this manner produces the equivalent of a photographic negative. This type of processing is particularly suited for enhancing white or gray detail embedded in dark regions of an image, especially when the black areas are dominant in size.

Log Tramsformations

The general forrn of the log transformation shown in Fig.3.3 is given by:

$$s = c \log(1 + r)$$

where c is a constant, and it is assumed that $r \geq 0$. The shape of the log curve in Fig.3.3 shows that this transformation maps a narrow range of low gray-level values in the input image into a wider range of output levels. The opposite is true of higher values of input levels. We would use a transformation of this type to expand the values of dark pixels in an image while compressing the higher-level values. The opposite is true of the inverse log transformation. The log function has the important characteristic that it compresses the dynamic range of images with large variations in pixel values.

Application: An application in which pixel values have a large dynamic range is the Fourier spectrum. The spectrum values may range from 0 to 10^6 or higher. It is no problem for a computer to process numbers of such kind. But image display systems generally will not be able to reproduce faithfully such a wide range of intensity values. The net effect is that a significant degree of detail will be lost in the display of a typical Fourier spectrum.

Power - Law Transformations

Power-law transformations have the basic form:

$$s = cr^{\gamma}$$

where c and γ are positive constants. Sometimes the equation is written as $s = c(r + \epsilon)^{\gamma}$ to account for an offset (offset is a measurable output when the input is zero). Plots of *s* versus *r* for various values of γ are show in Fig.3.6. Power-law curves with fractional values of γ map a narrow range of dark input values into a wider range of output values, with the opposite being true for higher values of input levels. Unlike the log function, here a family of possible transformation curves can be obtained simply by varying γ. Curves generated with values of γ > 1 have exactly the opposite effect as those generated with values of γ <1. The above equation reduces to the identity transformation when $c = \gamma = 1$.

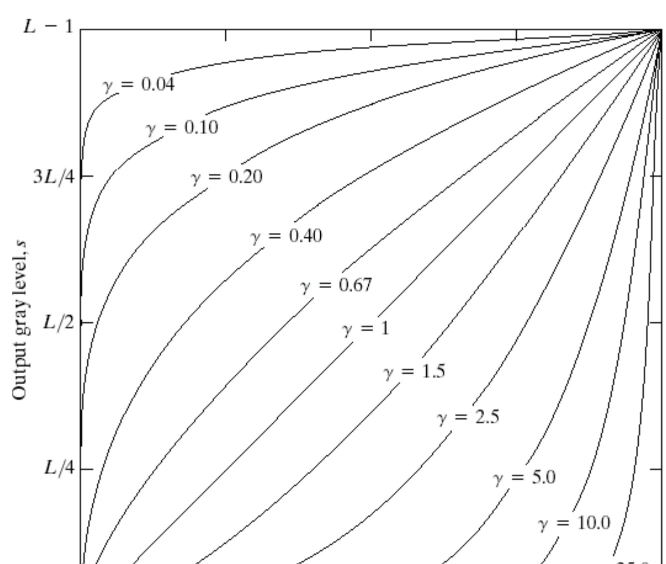


FIGURE 3.6 Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$ in all cases).

Applications

1. Gamma correction:

A variety of devices used for image capture, printing, and display respond according to a power law. The exponent in the power law equation is referred to as gamma. The process used to correct this power-law response phenomena is called gamma correction. For example, cathode ray tube (CRT) devices have an intensity-to voltage response that is a power function. If the value of γ = 2.5, then such display systems would tend to produce images that are darker than intented.

Gamma correction is important in displaying an image accurately on a computer screen is of concern. Images that are not corrected properly can look either bleached out or too dark .

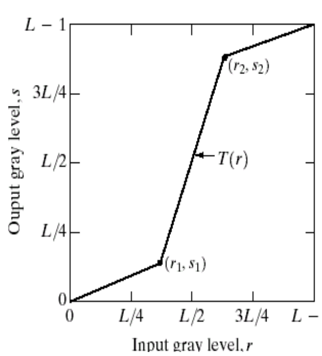
Piecewise-Linear Transformation Functions

The principal advantage of piecewise linear functions is that the form of piecewise functions can be arbitrarily complex. The principal disadvantage on piece-wise functions is that their specification requires considerably more user input.

Contrast stretching

One of the simplest principle piecewise linear functions is a contrast-stretching transformation. Low—contrast images can result from poor illumination, lack of dynamic range in the imaging sensor, or even wrong setting of a lens aperture during image acquisition. The idea behind contrast stretching is to increase the dynamic range of the gray levels in the image being processed.

Figure 3.l0(a) shows a typical transformation used for contrast stretching. The locations of points (r_1, s_1) and (r_2, s_2) control the shape of the transformation function.



If $r_1 = s_1$, and $r_2 = s_2$, the transformation is a linear function that produces no changes in gray levels.

If $r_1 = r_2$, $s_1=0$ and $s_2 = L - 1$, the transformation becomes a *thresholding function* that creates a binary image.

Intermediate values of (r_1, s_1) and (r_2, s_2) produce various degrees of spread in the gray levels of the output image, thus affecting its contrast. In general, $r_1 \leq r_2$ and $s_1 \leq s_2$, is assumed so that the function is single valued and monotonically increasing. This condition $r_1 \leq r_2$ preserves the order of gray levels thus preventing the creation of intensity artefacts in the processed image.

Gray-level slicing

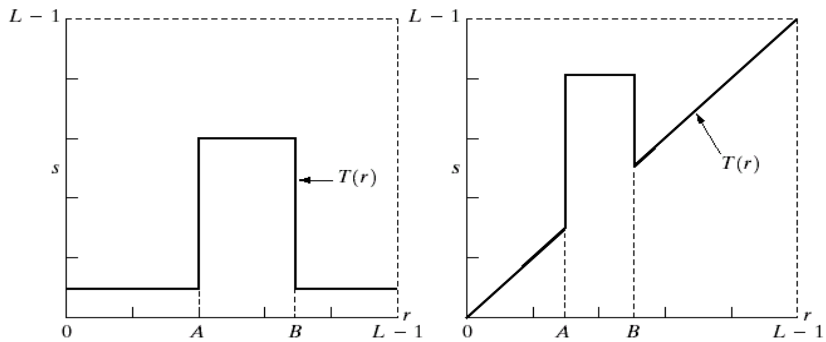
Highlighting a specific range of gray levels in an image often is desired.

Applications include enhancing features such as masses of water in satellite image and enhancing flaws in X-my images.

There are several ways of doing level slicing.

One approach is to display a high value for all gray levels in the range of interest and a low value for all other gray levels. This transformation shown in Fig.3.11 (a) produces a binary image.

The second approach, based on the transformation shown in Fig.3.11 (b),brightens the desired range of gray levels but preserves the background in the image. Figure 3.11(c) shows a gray scale image, and Fig.3.11(d) shows the result of using the transformation in Fig.3.11(a).



Bit-plane slicing

lnstead of highlighting gray-level ranges, highlighting the contribution made to total image appearance by specific bits might be desired. Suppose that each pixel in an image is represented by 8 bits. Imagine that the image is composed of eight 1-bit planes, ranging from bit - plane 0 for the least significant bit to bit-plane 7 for the most significant bit. In terms of 8- bit bytes, plane 0 contains all the lowest order bits in the bytes comprising the pixels in the image and plane 7 contains all the high-order bits.

Figure 3.12 illustrates these ideas, and Fig.3. 14 shows the various bit planes for the image shown in Fig. 3.13.

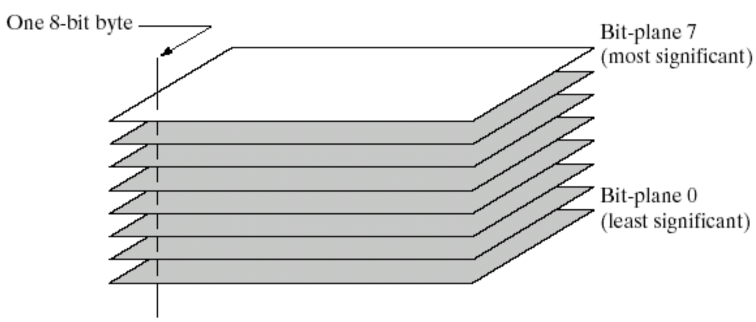


FIGURE 3.12 Bit-plane representation of an 8-bit image.

The higher-order bits contain the majority of the visually significant data. The other bit planes contribute to more subtle details in the image.

Separating *a digital image into its bit planes is useful for analyzing the relative importance played by each bit of the image, a process that aids in determining the adequacy of the number of bits used to quantize each pixel*. Also, this type of decomposition is useful for image compression.

2.

Intensity Distribution of 3-bit image

r_k	n_k	$P_r(r_k) = \frac{n_k}{MN}$	$S_k = (L-1)\sum_{j=0}^{k} p_r(r_j)$	Round off S_k

0	120	0.0213	0.1493	0
1	325	0.0578	0.5538	1
2	891	0.1584	1.6626	2
3	745	0.1324	2.5897	3
4	1021	0.1815	3.8603	4
5	1521	0.2704	5.7531	6
6	775	0.1378	6.7175	7
7	227	0.0404	7	7

MN= 5625

Intensity distribution after equalization

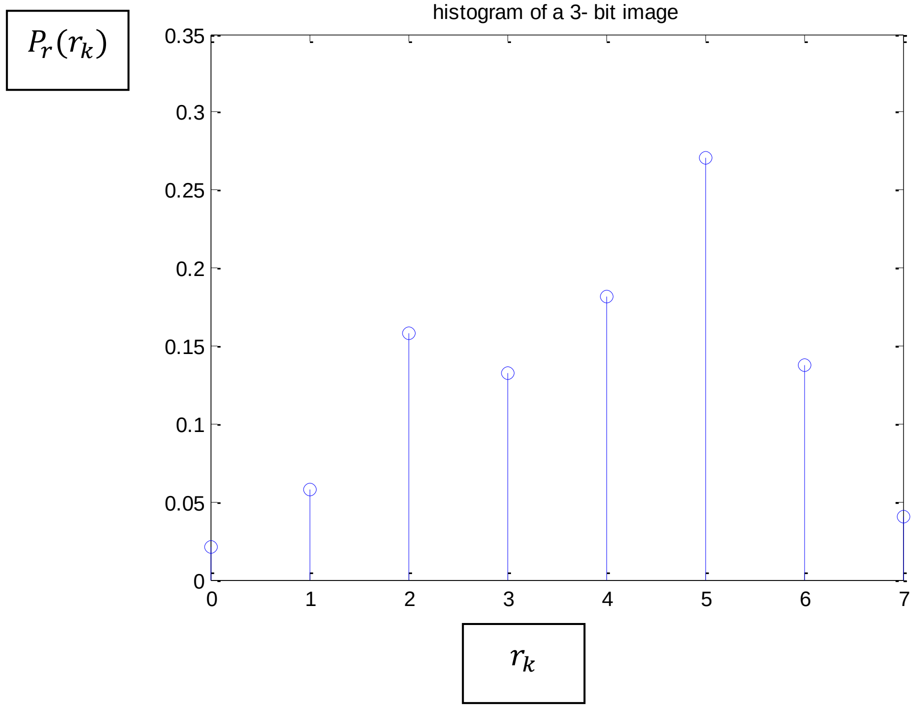
S_k	n_k
0	120
1	325
2	891
3	745
4	1021
5	0
6	1521
7	1002

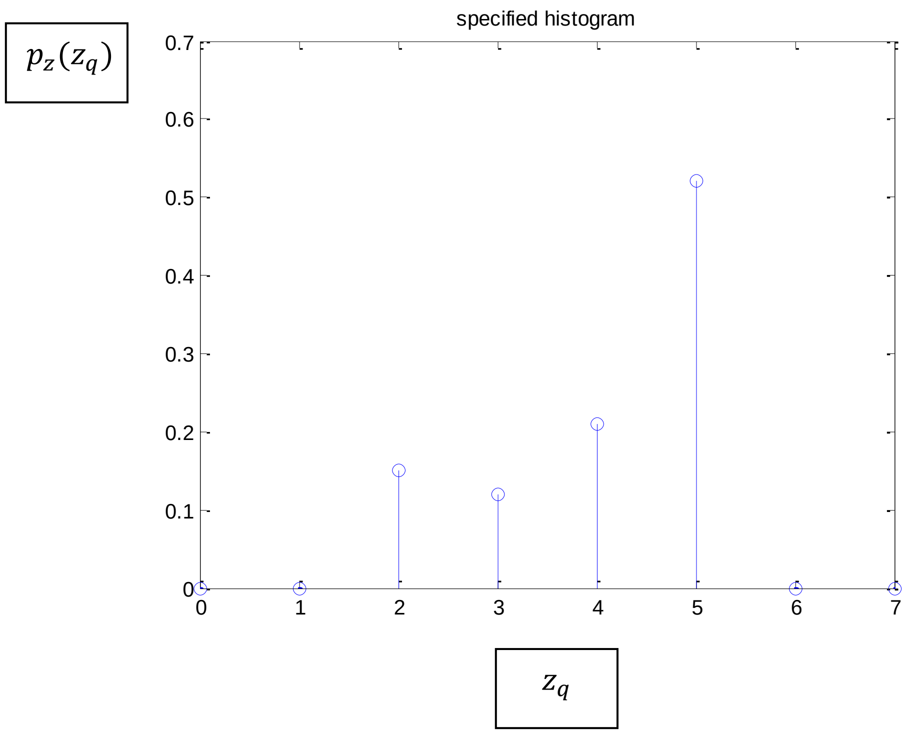
Intensity distribution of specified histogram

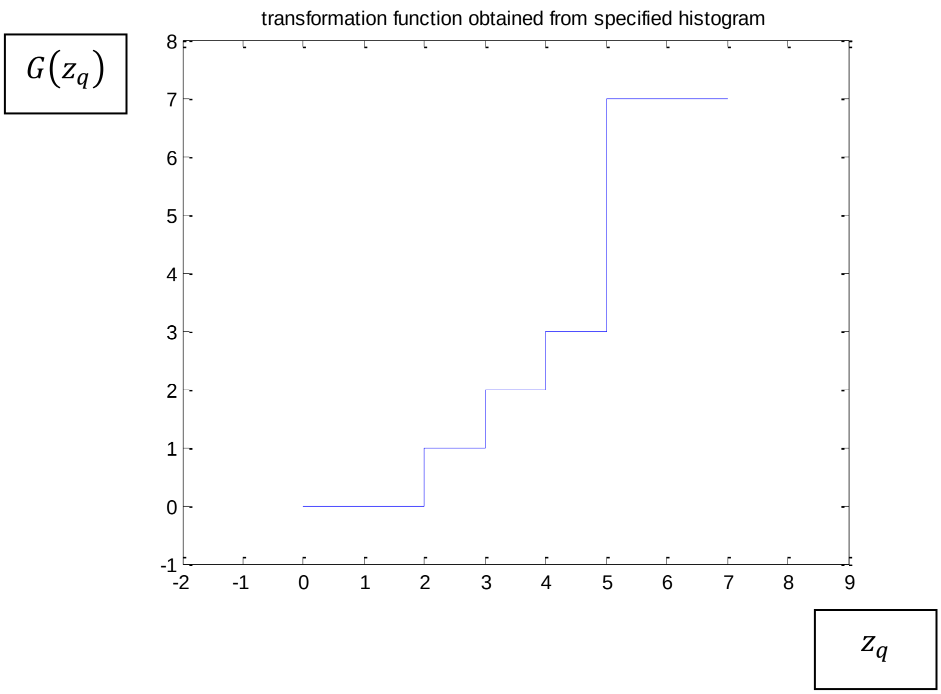
z_q	$p_z(z_q)$	$G(z_q)$	round off $G(z_q)$
0	0	0	0
1	0	0	0
2	0.15	1.05	1
3	0.12	1.89	2
4	0.21	3.36	3
5	0.52	7	7
6	0	7	7
7	0	7	7

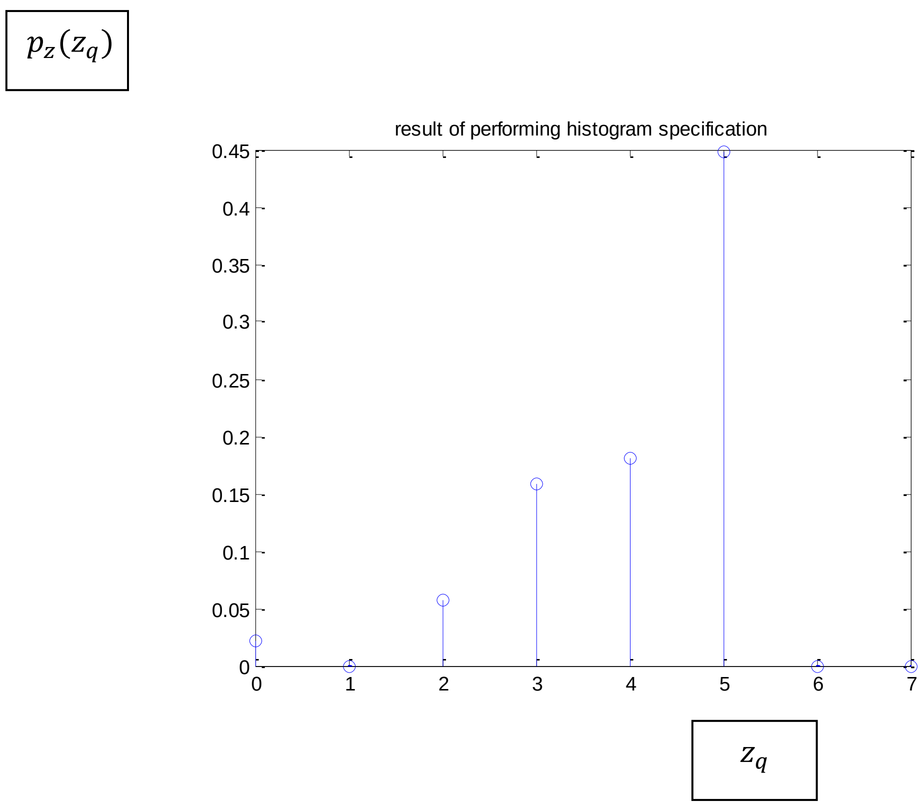
Intensity Distribution after specification

$z_q = G^{-1}(s_k)$	n_k
0	120
1	0
2	325
3	891
4	1021
5	2523
6	0
7	0


histogram of a 3- bit image
$P_r(r_k)$
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0 1 2 3 4 5 6 7
r_k


specified histogram
$p_z(z_q)$
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 1 2 3 4 5 6 7
z_q





3. The simplest isotropic derivative operator is the *Laplacian,* which for a function *f(x,y)* of two variables is defined as:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

It can be shown that

$$\Im\left[\frac{d^n f(x)}{dx^n}\right] = (ju)^n F(u)$$

{ NOTE: $\Im$ represents DFT and $\Im^{-1}$ represents IDFT }

It follows that

$$\begin{aligned}\Im\left[\frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}\right] &= (ju)^2 F(u,v) + (jv)^2 F(u,v) \\ &= -(u^2+v^2)F(u,v)\end{aligned}$$

* The expression on the left side is the Laplacian of $f(x,y)$ and we have

$$\Im[\nabla^2 f(x,y)] = -(u^2+v^2)F(u,v)$$

* Laplacian can be implemented in frequency domain by the filter

$$H(u,v) = -(u^2+v^2)$$

Assume that the origin of $F(u,v)$ is centered by performing the operation $f(x,y)(-1)^{x+y}$ before taking the transform

Center of filter function can be shifted by

$$H(u,v) = -((u-M/2)^2 + (v-N/2)^2)$$

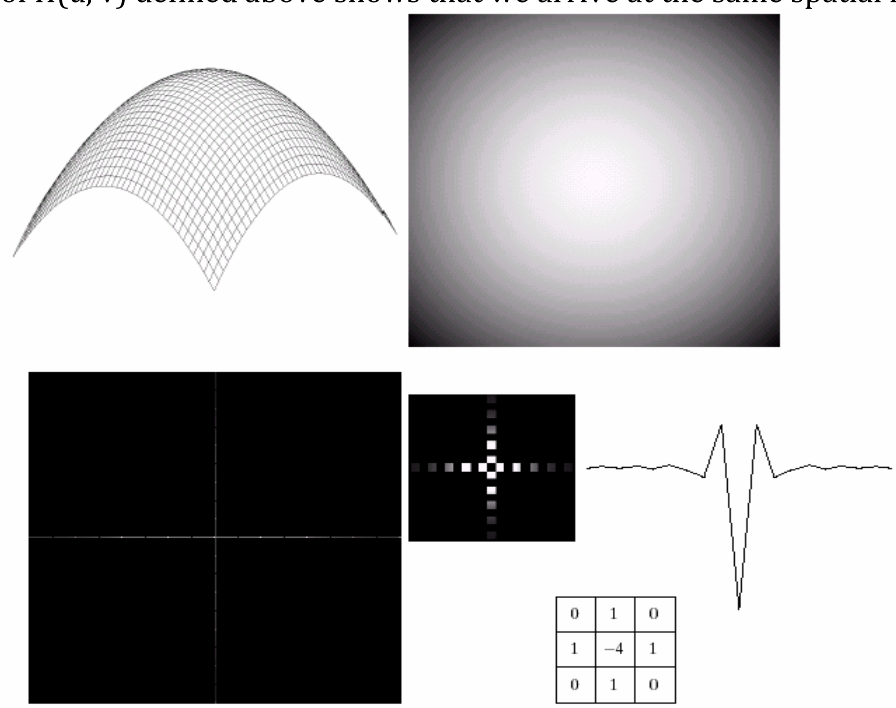
Laplacian filtered image in spatial domain is obtained by computing the inverse Fourier transform of $H(u,v)F(u,v)$ as

$$\nabla^2 f(x,y) = \Im^{-1}(-((u-M/2)^2 + (v-N/2)^2)F(u,v))$$

Conversely, we can apply convolution theorem to get the Fourier transform pair notation

$$\nabla^2 f(x,y) \Leftrightarrow -((u-M/2)^2 + (v-N/2)^2)F(u,v)$$

Interesting properties of spatial domain Laplacian filter by taking the inverse Fourier transform of H(u, v) defined above shows that we arrive at the same spatial mask:



0	1	0
1	–4	1
0	1	0

a b
c d e
f

FIGURE 4.27 (a) 3-D plot of Laplacian in the frequency domain. (b) Image representation of (a). (c) Laplacian in the spatial domain obtained from the inverse DFT of (b). (d) Zoomed section of the origin of (c). (e) Gray-level profile through the center of (d). (f) Laplacian mask used in Section 3.7.

Subtracting the Laplacian from the original image gives you the enhanced image

$$g(x,y) = f(x,y) - \nabla^2 f(x,y)$$

The filter can be combined into one operation as

$$H(u,v) = 1 + ((u - M/2)^2 + (v - N/2)^2)$$

The enhanced image is obtained with a single transform by

$$g(x,y) = \Im^{-1}((1 + ((u - M/2)^2 + (v - N/2)^2))F(u,v))$$

UNSHARP MASKING, HIGH-BOOST FILTERING AND HIGH-FREQUENCY EMPHASIS FILTERING

- The common thing in highpass filtered images is that it has dark background. This is because HPFs eliminate zero frequency from the image's Fourier Transform.
- The solution to this is adding a portion of image back to the filtered image.
- Enhancement using Laplacian does precisely this.
- But, sometimes it is required to increase the contribution of original image to the overall filtered image. This approach, called as the high-boost filtering; is a generalization of unsharp masking.
- Unsharp masking consists simply of generating a sharp image by subtracting a blurred version of an image with the image. That means, a high pass filtered image is obtained by subtracting an image with its lowpass filtered image as shown:

  $$f_{\text{hp}}(x,y) = f(x,y) - f_{\text{lp}}(x,y)$$

- High-boost filtering generalizes this by multiplying *f(x,y)* by a constant **A ≥ 1:**

You can also write it as

$$\begin{aligned} f_{\text{hb}}(x,y) &= (A-1)f(x,y) + f(x,y) - f_{\text{lp}}(x,y) \\ &= (A-1)f(x,y) + f_{\text{hp}}(x,y) \end{aligned}$$

For $A = 1$, high-boost filtering reduces to highpass filtering
For $A >> 1$, image contribution becomes more dominant
Also make sure that you normalize the result back by dividing the coefficients by A

In frequency domain, it can be implemented directly using composite filter as follows:

$F_{\text{hp}}(u,v) = F(u,v) - F_{\text{lp}}(u,v)$
$F_{\text{lp}}(u,v) = H_{\text{lp}}(u,v)F(u,v)$
The composite filter in frequency domain is

$$H_{\text{hp}}(u,v) = 1 - H_{\text{lp}}(u,v)$$

High-boost filtering in frequency domain is

$$H_{\text{hb}}(u,v) = (A-1) + H_{\text{hp}}(u,v)$$

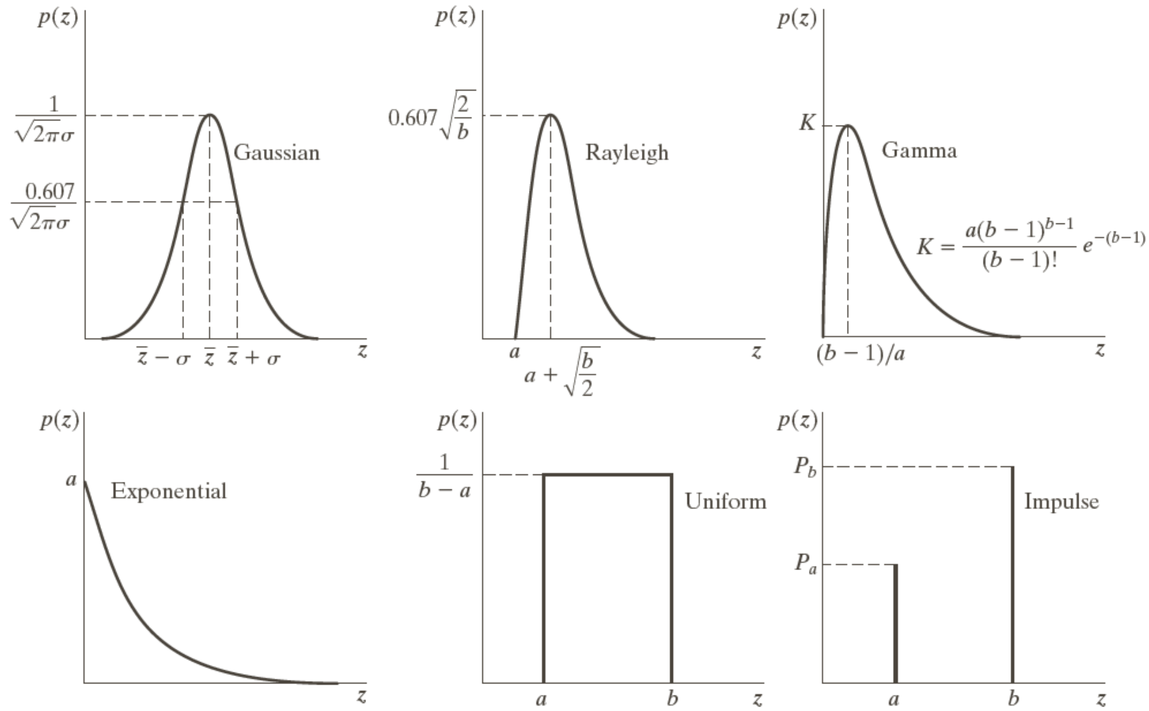
4. The different types of noise are characterized by their probability distributions. Each probability density distribution has its unique shape. In general it is not possible to identify the type of noise which corrupts an image simply by visually inspecting the image.

It is important to emphasize here that noise is additive to the image intensity at a pixel location. Unless otherwise stated, the noise is considered spatially independent, i.e. the amount of noise corruption at a given pixel does not depend on the spatial coordinates of the pixel in the image.
The Gaussian distribution is often used to describe, at least approximately, any variable that tends to cluster around the mean $\bar{z}$ Gaussian distribution can be completely characterized by its mean $\bar{z}$ and the standard deviation σ.
The Gaussian function has certain very useful mathematical properties. It is symmetric around the point $z = \bar{z}$. The PDF of a gausian random variable, z , is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$

Where z represents the intensity, $\bar{z}$ is the mean(average) value of z, σ is its standard deviation. The standard deviation squared, σ^2 is called the variance of z. approximately 70% of p(z) values will be in the range $[(\bar{z} - \sigma), (\bar{z} + \sigma)]$ and 95% will be in the range $[(\bar{z} - 2\sigma), (\bar{z} + 2\sigma)]$.

The Gaussian model is suitable to model the electronic circuitry noise in image acquisition systems. It is also useful to characterize the sensor noise which can be due to factors like poor illumination or high temperature.



a b c
d e f

FIGURE 5.2 Some important probability density functions.

The Rayleigh Distribution is left skewed distribution with light tails. An attractive feature of the Rayleigh distribution is that the mode can be estimated from the mean. The range is determined by the scale parameter b. Its skewness is constant.

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-b)e^{-\frac{(z-a)^2}{b}} & for\ z \geq a \\ 0 & for\ z < a \end{cases}$$

The mean and variance of this density are given by

$\bar{z} = a + \sqrt{\pi b/4}$

And

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

The formula for Rayleigh distribution has 2 factors, the first one (z-a) is a linearly increasing term and the second one $e^{-\frac{(z-a)^2}{b}}$ is an exponentially decaying term like the one in Gaussian. The second term also indicates that the parameter b plays a role similar to the variance.

The Rayleigh distribution is useful for modeling skewed distributions. The Rayleigh distribution is suitable for characterizing noise in range imaging.

The Erlang distribution is also skewed like the Rayleigh distribution. Similar to the Rayleigh distribution, its formula shows two factors, the term z^{b-1} is responsible for the increase and the other term e^{-az} is responsible for the exponential decay. The exponential decay in Erlang distribution is slower compared to Rayleigh because Rayleigh has a quadratic decay term.

- The Erlang distribution is suitable for characterizing noise in range imaging.

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ z \geq 0 \\ 0\ for\ z < 0 \end{cases}$$

Where a>0, bis positive integer and "!" indicates factorial.
The mean and variance of this density are given by

$$\bar{z} = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$

Exponential noise is a special case of Erlang distribution with the parameter b = 1
The PDF is given by

$$p(z) = \begin{cases} ae^{-az} & for\ z \geq 0 \\ 0\ for\ z < 0 \end{cases}$$

Wher a>0. The mean and variance of this density function are given by

$$\bar{z} = \frac{1}{a}$$

And $\sigma^2 = \frac{1}{a^2}$

The uniform noise has a PDF which remains constant for specified bounds$a \leq z \leq b$ of the noise amplitude. The constant value of probability is pegged at $\frac{1}{b-1}$ because the total area under the pdf curve is 1. This noise is less practical and is used for random number generator.
The PDF is given by

$$p(z) = \begin{cases} \frac{1}{b-a} & if\ a \leq z \leq b \\ o & otherwise \end{cases}$$

The mean and variance of this density function are given by

$$\bar{z} = \frac{a+b}{2}$$

And $\sigma^2 = \frac{(b-a)^2}{12}$

Impulse noise generally corresponds to extreme values (intensity 0 for dark and 255 for bright) in the image. The noise has only two allowable values the negative impulse causing dark points a = 0 and the positive impulse causing the bright points b = 255. The probabilities of the two types of noise impulses can be either same or different. If one of the probabilities is zero, the noise will be either a salt noise or a pepper noise.
The PDF is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

- bipolar if $P_a \neq 0,\ P_b \neq 0$
- unipolar if one of P_a and P_b is 0

- noise looks like salt-and-pepper granules if $P_a \approx P_b$
- negative or positive; scaling is often necessary to form digital images
- extreme values occur (e.g. $a = 0$, $b = 255$)

- The Salt and Pepper noise is suitable for characterizing noise due to electrical or illumination transients during imaging or communication.

Periodic noise is a spatially dependent noise. This can be in the form of spatially sinusoidal noise corrupting the image. The Fourier transform of a pure sinusoid is a pair of conjugate impulses located at the conjugate frequencies of the sinusoid. Hence the Fourier spectrum of the noisy image would indicate a pair of impulse for each frequency in the periodic noise. The impulses will be more pronounced if the sinusoid amplitude is large enough.

5. There are 3 principal ways to estimate the degradation function: 1) Observation. 2) Experimentation 3) Mathematical Modeling. The process of restoring an image by using degradation function that has been estimated in some way is called blind deconvolution.

Estimation by Image Observation: This method of estimating the degradation function is used when we have absolutely no clue of what caused the image degradation. We just have the degraded image given to us. In order to restore the image we must have some idea of what the original image could be looking like. On the given degraded image we select a small patch which has relatively less noise and has good contrast. Following our guesswork, we attempt to restore this patch by applying image operations like sharpening, contrast or brightness adjustment, etc. Our objective here is to get the restored patch. It does not depend on what operations we apply and in what sequence. Let the Fourier transform of the degraded patch be $G_s(u,v)$ and that of the restored patch be $\hat{F}_s(u,v)$ Then the Fourier transform of the degradation function $H_s(u,v)$ can be estimated as:

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

Following our assumption that $H(u,v)$ is position invariant, the degradation function $H(u,v)$ will have the same basic shape as $H_s(u,v)$. However the scale of $H(u,v)$ will be larger compared to that of $H_s(u,v)$.

Estimation by Experimentation: If the image acquisition system which was used to acquire the degraded image is available to us, then we can tune the system settings so that we get an image (not necessarily of the same scene/object) of similar degradation. The idea is to recover the same system settings which were responsible for producing the degradation which we want to estimate. Once we are able to achieve those system settings we need to know the response of the system to an impulse signal. An impulse can be simulated using a small bright dot of light. We record the system's response for this impulse as G_δ in frequency domain. Since the Fourier transform of an impulse is a constant say A the frequency domain representation of the system transfer function, i.e. the degradation $H(u,v)$ is given as:

$$H(u,v) = \frac{G_\delta(u,v)}{A}$$

Estimation by Modeling: We can mathematically model the physical phenomena or the imaging conditions which lead to degraded images. This requires extensive research. For example, it has been possible to model the different type of blurring effects (low-pass filtering) due to various degrees of severity of atmospheric turbulence conditions. For simple cases of blurring due to image motion, it is possible to mathematically derive the degradation function

When we acquire the image of a moving object we generally get a blurred image because of the relative motion between the sensor and the object. In this section we consider how to mathematically model the blur due to image motion. To simplify the modeling we assume that the image moves along a plane and the time varying displacement in x and y directions for every pixel is given as $x_0(t)$ and $y_0(t)$ respectively. If T is the duration for which the camera shutter is open, the intensity at each pixel on the blurred image $g(x,y)$ is computed as an integration of the (true, unblurred) image . $f(x,y)$ intensities over the period T.

Relation between Fourier transform of $f(x,y)$ and $g(x,y)$

The Fourier transform of $g(x,y)$ can be written as:

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, e^{-j2\pi(ux+vy)}dxdy$$

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$$\begin{aligned} G(u,v) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\int_0^T f[x-x_0(t), y-y_0(t)]\,dt\right] e^{-j2\pi(ux+vy)}dxdy \\ &= \int_0^T \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x-x_0(t), y-y_0(t)]\, e^{-j2\pi(ux+vy)}dxdy\right] dt \end{aligned}$$

Using the Fourier transform shift property $\mathscr{F}[f(x-a,y-b)] = F(u,v)\, e^{-j2\pi(ua+vb)}$ we have

$$\begin{aligned} G(u,v) &= \int_0^T F(u,v) e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \\ &= F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \\ &= F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \\ &= F(u,v)\, H(u,v) \end{aligned}$$

since $F(u,v)$ does not depend on t

We find that the Fourier transform of degradation due to motion blurring can be formulated as:

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

As we notice from this formulation, the degradation function can be estimated only when the image motion is planar and the time-varying displacements $x_0(t)$ and $y_0(t)$ are known. For general objects which can have articulated motion, it is very difficult to estimate the values of $x_0(t)$ and $y_0(t)$ for the different parts of the objects.

Once we have estimated the degradation function $H(u,v)$ the next step is to use it to restore the image. This process is called as filtering the degraded image so as to get the restored image as the output

6. The Wiener filter solves the signal estimation problem for stationary signals. The filter was introduced by Norbert Wiener in the 1940's. A major contribution was the use of a statistical model for the estimated signal. The noise present in the signal is reduced by comparison with an estimation of the desired noiseless signal. The filter is optimal in the sense of the minimum mean square error. The Wiener filtering approach takes into account both the degradation function and the noise characteristics for estimating the undegraded image. The Wiener filter computes an optimal estimate of the undegraded image.

Assumptions:

1. The power spectrum $S_\eta(u,v)$ of the noise is available.

2. The power spectrum $S_f(u,v)$ of the original image is available.

3. The image signal and the noise signal are uncorrelated.
4. Either the image signal or the noise must have zero mean.
5. The intensity levels in the restored image are a linear function of the levels in the degraded image.

Optimality Criteria

The Wiener filter is a minimum mean square error filter. The optimality criteria is to minimize the expected value (i.e. the mean) of the square error between the original image f and the estimate of the un-degradedimage $\hat{f}$. $e^2 = E\{(f-\hat{f})^2\}$

Here the $E\{\circ\}$ denotes the expected value.

The Wiener Filter expression

$$
\begin{aligned}
\hat{F}(u,v) &= \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2+S_\eta(u,v)}\right]G(u,v) \\
&= \left[\frac{H^*(u,v)}{|H(u,v)|^2+\frac{S_\eta(u,v)}{S_f(u,v)}}\right]G(u,v) \\
&= \left[\frac{1}{H(u,v)}\,\frac{|H(u,v)|^2}{|H(u,v)|^2+\frac{S_\eta(u,v)}{S_f(u,v)}}\right]G(u,v)
\end{aligned}
$$

where $H(u,v)$ = degradation function

$H^*(u,v)$ = complex conjugate of $H(u,v)$

$|H(u,v)|^2$ = $H^*(u,v)H(u,v)$

$S_\eta(u,v)$ = $|N(u,v)|^2$ = power spectrum of noise

$S_f(u,v)$ = $|F(u,v)|^2$ = power spectrum of undegraded image

- If the $H(u,v)$ is zero, the denominator will remain non-zero unless the noise power spectrum is also zero. This is an advantage over the inverse filter.
- The problem with the Wiener filter is that it requires an estimate of $S_\eta(u,v)$ and $S_f(u,v)$. The latter quantity is difficult to guess because we don't have access to the original image $f(x,y)$.
- In a simplified expression of the Wiener filter we assume that the ratio $\frac{S_\eta(u,v)}{S_f(u,v)}$ is a constant K.

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

-
- When restoring a degraded image using the Wiener filter, we can interactively adjust the value of K as per our visual assessment and obtain the most satisfactory restored image.