CMR
INSTITUTE OF
TECHNOLOGY

USN | | | | | | | | | | |

Internal Assesment Test – III

Sub: Digital Signal Processing   Code: 17EC52
Date: 16/ 11/ 2019   Duration: 90 mins   Max Marks: 50   Sem: V   Branch: ECE: A,B & C

Answer any 5 questions.

		Marks	OBE CO	OBE RBT
1	Derive an expression for frequency response of frequency response of a symmetric FIR filter for N odd.	[10]	Co4	L3
2	Design a linear phase FIR filter using Hanning window for the following desired frequency response $$H_d(e^{jw}) = \begin{cases} e^{-j3w}, \frac{\pi}{6} \leq \lvert w \rvert \leq \pi \\ 0, \ \lvert w \rvert \leq \frac{\pi}{6} \end{cases}$$	[10]	Co4	L3
3	Obtain the Direct form II, Cascade and parallel realization of $$H(z) = \frac{(1-\frac{1}{2}z^{-1})}{(1+2z^{-1})(1+\frac{1}{2}z^{-1})}$$	[10]	Co4	L3
4	Explain the design of IIR filter by impulse invariance technique.	[10]	Co4	L3
5	a) Compare FIR and IIR Filters b) Compare Butterworth and Chebyshev Filters.	[10]	Co4	L3

—

CMR
INSTITUTE OF
TECHNOLOGY

USN | | | | | | | | | | |

Internal Assesment Test – III

Sub: Digital Signal processing   Code: 17EC52
Date: 16/ 11 / 2019   Duration: 90 mins   Max Marks: 50   Sem: V   Branch: ECE: A,B &C

Answer any 5 questions.

		Marks	OBE CO	OBE RBT
1	Derive an expression for frequency response of frequency response of a symmetric FIR filter for N odd.	[10]	Co4	L3
2	Design a linear phase FIR filter using Hanning window for the following desired frequency response $$H_d(e^{jw}) = \begin{cases} e^{-j3w}, \frac{\pi}{6} \leq \lvert w \rvert \leq \pi \\ 0, \ \lvert w \rvert \leq \frac{\pi}{6} \end{cases}$$	[10]	Co4	L3
3	Obtain the Direct form II, Cascade and parallel realization of $$H(z) = \frac{(1-\frac{1}{2}z^{-1})}{(1+2z^{-1})(1+\frac{1}{2}z^{-1})}$$	[10]	Co4	L3
4	Explain the design of IIR filter by impulse invariance technique.	[10]	Co4	L3
5	a) Compare FIR and IIR Filters	[10]	Co4	L3

b) Compare Butterworth and Chebyshev Filters.

6 Design a Digital Chebyshev filter with maximum passband attenuation of 2.5 dB at 20 rad/sec and stopband attenuation of 30 dB at 50 rad/sec using Bilinear Transformation. Sampling rate is 1 hz. [10] Co4 L3

7 Design an IIR filter H(z) that will satisfy the following specifications [10] Co4 L3

i) Passband ripple of 1dB at $100\pi\ rad/sec$

ii) Stopband attenuation of 30dB or greater at $1000\ \pi\ rad/sec$

iii) Monotonic pass and stop band

iv) Sampling rate 2000 samples/sec

Use Bilinear Transformation

8 a) Transform the analog filter $H(S) = \frac{S+1}{S^2+5S+6}$ into H(z) using Impulse Invariant Transformation. [10] Co4 L3

b) Let $H(s) = \frac{b}{(S+a)^2+b^2}$.Obtain H(z) using Impulse Invariant Transformation method.

6 Design a Digital Chebyshev filter with maximum passband attenuation of 2.5 dB at 20 rad/sec and stopband attenuation of 30 dB at 50 rad/sec using Bilinear Transformation. Sampling rate is 1 hz. [10] Co4 L3

7 Design an IIR filter H(z) that will satisfy the following specifications [10] Co4 L3

I. Passband ripple of 1dB at $100\pi\ rad/sec$

II. Stopband attenuation of 30dB or greater at $1000\ \pi\ rad/sec$

III. Monotonic pass and stop band

IV. Sampling rate 2000 samples/sec

Use Bilinear Transformation

8 a) Transform the analog filter $H(S) = \frac{S+1}{S^2+5S+6}$ into H(z) using Impulse Invariant Transformation. [10] Co4 L3

b) Let $H(s) = \frac{b}{(S+a)^2+b^2}$.Obtain H(z) using Impulse Invariant Transformation method.

Scheme Of Evaluation

Internal Assessment Test 1 – Mar.2019

Sub:	Digital Signal Processing							**Code:**	17EC52
Date:	06/09/2019	Duration:	90mins	Max Marks:	50	**Sem:**	V	**Branch:**	ECE A,B &C

Note: Answer Any Five Questions

Question #	**Description**	**Marks Distribution**	**Max Marks**
1	Derive an expression for frequency response of frequency response of a symmetric FIR filter for N odd. • Derivation	- 10	10
2	Design a linear phase FIR filter using Hanning window for the following desired frequency response $$H_d(e^{jw}) = \begin{cases} e^{-j3w}, \frac{\pi}{6} \le \lvert w \rvert \le \pi \\ 0, \ \lvert w \rvert \le \frac{\pi}{6} \end{cases}$$ • Computation of order and cut-off frequency • computing impulse response and window function • Frequency response	- 2 6 2	10
3	Obtain the Direct form II, Cascade and parallel realization of $$H(z) = \frac{(1 - \frac{1}{2}z^{-1})}{(1 + 2z^{-1})(1 + \frac{1}{2}z^{-1})}$$ • Direct form II • Cascade Realization • Parallel realization	- 5 5	10
4	Explain the design of IIR filter by impulse invariance technique. • Derivation	- 10	10
5	a) Compare FIR and IIR Filters b) Compare Butterworth and Chebyshev Filters. • Compare FIR and IIR Filters • Compare Butterworth and Chebyshev Filters	- 5 5	10
6	Design a Digital Chebyshev filter with maximum passband attenuation of 2.5 dB at 20 rad/sec and stopband attenuation of 30 dB at 50 rad/sec using Bilinear Transformation. Sampling rate is 1 hz. • Order N, epsilon and beta • Obtaining H(S)	- 4 6	10
7	Design an IIR filter H(z) that will satisfy the following specifications I. Passband ripple of 1dB at $100\pi\ rad/sec$ II. Stopband attenuation of 30dB or greater at $1000\ \pi\ rad/sec$ III. Monotonic pass and stop band IV. Sampling rate 2000 samples/sec Use Bilinear Transformation. • Obtaining H(s) • Obtaining H(z)	7 3	2

Question #	Description	Marks Distribution	Max Marks
8	a) Transform the analog filter $H(S) = \frac{S+1}{S^2+5S+6}$ into H(z) using Impulse Invariant Transformation. b) Let $H(s) = \frac{b}{(S+a)^2+b^2}$.Obtain H(z) using Impulse Invariant Transformation method. • a) H(s) to H(z) • b) H(s) to H(z) •	 5 5	10

Solution

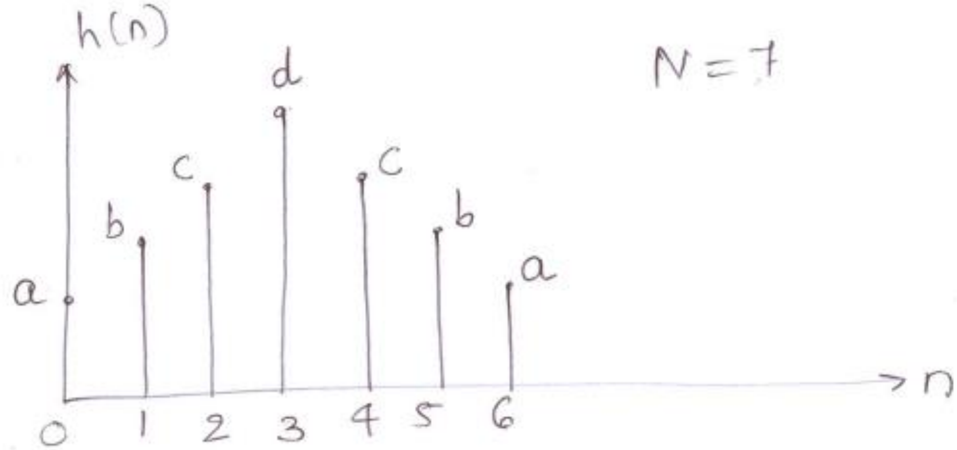
1.

Type-I FIR Filter

N- odd

h(n)- symmetric.

Example:



z- transform of $h(n)$,

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + \cdots + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \cdots \cdots + h(N-2) z^{-(N-2)} + h(N-1) z^{-(N-1)}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + \cdots + h\left(\frac{N-1}{2}\right) z^{-\left(\frac{N-1}{2}\right)} + \cdots + h(1) z^{-(N-2)} + h(0) z^{-(N-1)}$$

$$= z^{-\left(\frac{N-1}{2}\right)} \left[h(0) z^{\left(\frac{N-1}{2}\right)} + h(1) z^{\left(\frac{N-3}{2}\right)} + h(2) z^{\left(\frac{N-5}{2}\right)} + \cdots + h\left(\frac{N-1}{2}\right) + \cdots + h(1) z^{-\left(\frac{N-3}{2}\right)} + h(0) z^{-\left(\frac{N-1}{2}\right)} \right]$$

$$= z^{-\left(\frac{N-1}{2}\right)} \left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) \left\{ z^{\frac{(N-1-2n)}{2}} + z^{-\frac{(N-1-2n)}{2}} \right\} \right] \quad \cdots (1)$$

To obtain the frequency response,

we substitute $z = e^{j\omega}$.

$$\therefore H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n).\, 2\cos\left(\frac{\omega(N-1-2n)}{2}\right)\right] \quad \text{---} (2)$$

Let $$H_R(\omega) = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}}\left[h(n)\cos\left(\frac{\omega(N-1-2n)}{2}\right)\right] \quad \text{---} (3)$$

The phase response of type-I filter is given by

$$\theta(\omega) = \begin{cases} -\omega\left(\frac{N-1}{2}\right), & H_R(\omega) > 0 \\ -\omega\left(\frac{N-1}{2}\right) + \pi, & H_R(\omega) < 0 \end{cases}$$

2.

$$H_d(e^{jw}) = \begin{cases} e^{-j3w}, \frac{\pi}{6} \le |w| \le \pi \\ 0, \quad |w| \le \frac{\pi}{6} \end{cases}$$

$$\alpha = 3$$

$$\alpha = \frac{N-1}{2}$$

$$\therefore N = 7$$

$$w_c = \frac{\pi}{6}$$

$$h_1(n) = \begin{cases} -\frac{\sin(w_c\,(n-\alpha))}{\pi(n-\alpha)}, n \ne \alpha \\ 1 - \frac{w_C}{\pi}, n = \alpha \end{cases}$$

n	$h_1(n)$	w(n)	$h(n) = h(n) * w(n)$

0	-0.10610	0	0
1	-0.13783	0.25	-0.03446
2	-0.15915	0.75	-0.11937
3	0.83333	1	0.83333
4	-0.15915	0.75	-0.11937
5	-0.13783	0.25	-0.03446
6	-0.10610	0	0

$$H(e^{jw}) = h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} 2h(n)\cos(w(n-(\frac{N-1}{2})))$$

$$H(e^{jw}) = h(3) + \sum_{n=0}^{2} 2h(n)\cos(w(n-(3)))$$

$$H(e^{jw}) = h(3) + 2h(0)\cos(w(-3)) + 2h(1)\cos(w(1-3)) + 2h(2)\cos(w(2-3))$$

$$H(e^{jw}) = h(3) + 2h(0)\cos(3w) + 2h(1)\cos(2w) + 2h(2)\cos(w)$$

$$H(e^{jw}) = 0.8333 - 0.0689\cos(2w) - 0.2387\cos(w)$$

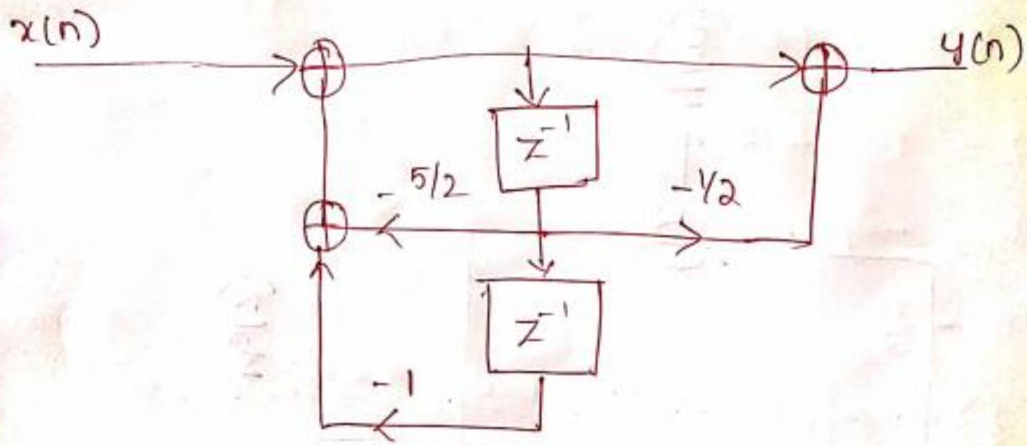
3.

3)

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{(1+2z^{-1})(1+\frac{1}{2}z^{-1})}$$

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}+2z^{-1}+z^{-2}} = \frac{1-\frac{1}{2}z^{-1}}{1+\frac{5}{2}z^{-1}+z^{-2}}$$

$$y(n) = x(n) - \frac{1}{2}x(n-1) - \frac{5}{2}y(n-1) - y(n-2)$$

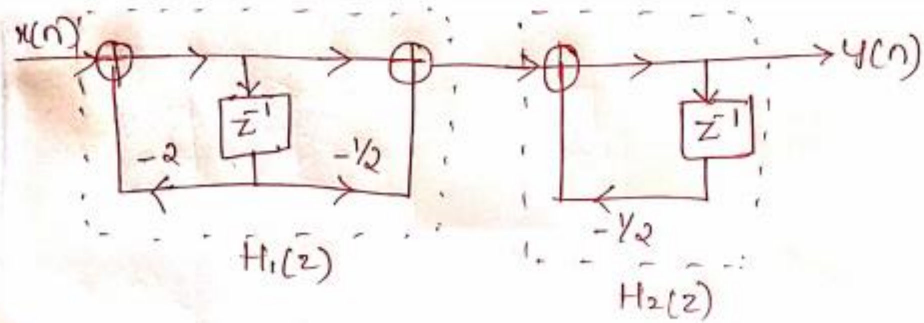


DF−II Structure.

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{(1+2z^{-1})(1+\frac{1}{2}z^{-1})}$$

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{(1+2z^{-1})} \cdot \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$H(z) = H_1(z) \cdot H_2(z)$$



Cascade realization.

3. Parallel realization.

$$H(z) = \frac{1-\frac{1}{2}z^{-1}}{(1+2z^{-1})(1+\frac{1}{2}z^{-1})} = \frac{A}{1+2z^{-1}} + \frac{B}{1+\frac{1}{2}z^{-1}}$$

$$1-\frac{1}{2}z^{-1} = A(1+\tfrac{1}{2}z^{-1}) + B(1+2z^{-1}) \quad \text{— (1)}$$

put $\cancel{z^{-1} = 1/2}$ $z^{-1} = -2$

$\cancel{1-\frac{1}{2}(\frac{1}{2}) = 2A}$

$$1-\frac{1}{2}(-2) = A(1+\tfrac{1}{2}(-2)) + B(1+2(-2))$$

$$2 = B(1-4)$$

$$B = -3/2.$$

put $z^{-1} = -\frac{1}{2}$

$$1-\frac{1}{2}(-\tfrac{1}{2}) = A(1+\tfrac{1}{2}(-1/2)) + B(1+2(-1/2))$$

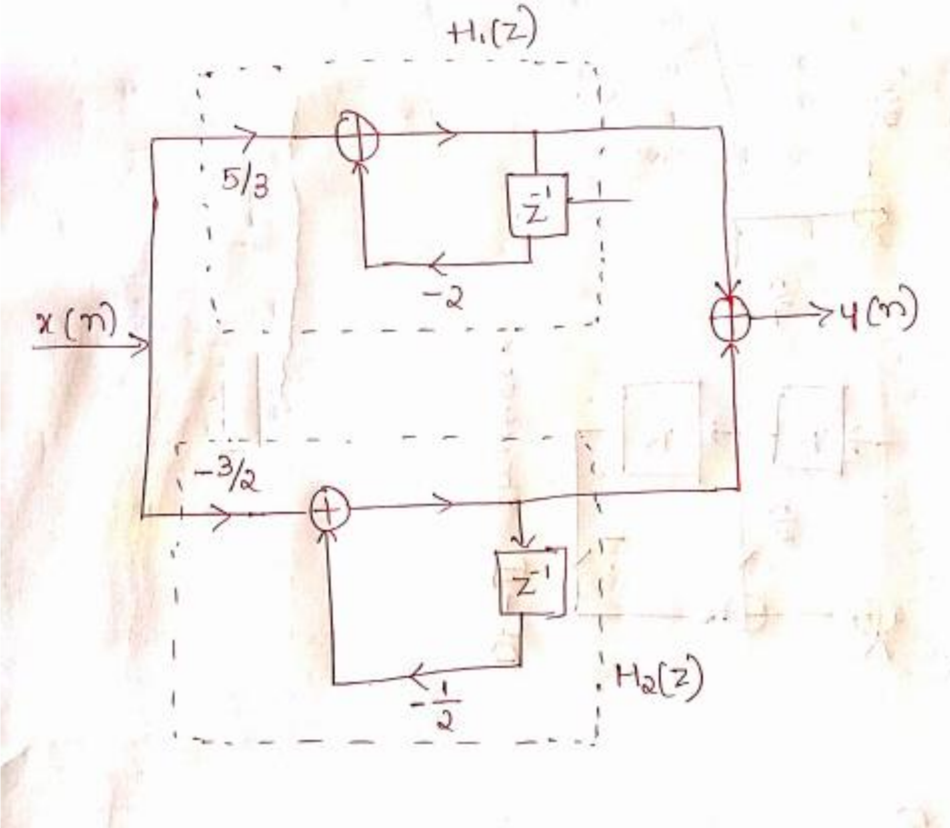
$$1+\frac{1}{4} = A(1-\tfrac{1}{4}) + 0 \qquad 1\tfrac{1}{4}$$

$$\frac{5}{4} = \frac{3}{4}A$$

$$A = 5/3.$$

$$H(z) = \frac{5/3}{1+2z^{-1}} - \frac{3/2}{1+\frac{1}{2}z^{-1}}$$

$$H(z) = H_1(z) + H_2(z)$$



H₁(z)
5/3
z⁻¹
-2
x(n)
y(n)
-3/2
z⁻¹
-1/2
H₂(z)

4.

Let $H(s)$ be the transfer function of an analog filter.

$$H(s) = \sum_{k=1}^{N} \frac{c_k}{1 - P_k} \quad \text{—— (1)}$$

where P_k are the poles of the analog filter and C_k are the co-efficients in the partial fraction expansion.

Take inverse Laplace Transform of (1)

$$h(t) = \sum_{k=1}^{N} c_k e^{P_k t} \quad \text{—— (2)}$$

Sample $h(t)$ periodically at $t = nT_s$

$$h(n) = h(t)\Big|_{t=nT_s}$$

$$h(n) = \sum_{k=1}^{N} c_k e^{P_k n T_s} \quad \text{—— (3)}$$

Taking Z-Transform on both the sides of (3)

$$H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$H(z) = \sum_{n=0}^{\infty} \sum_{k=1}^{N} c_k e^{P_k n T_s} z^{-n}$$

$$= \sum_{k=1}^{N} c_k \left[\sum_{n=0}^{\infty} (e^{P_k T_s} z^{-1})^n \right]$$

$$H(Z) = \sum_{K=1}^{N} C_K \frac{1}{1 - e^{P_K T_S} Z^{-1}} \quad \text{——} \quad (4)$$

$$\left[\because \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \right]$$

Compare (1) & (4)

$$\frac{1}{S - P_K} \longrightarrow \frac{1}{1 - e^{P_K T_S} Z^{-1}}$$

From (1) the poles of the analog filter are at $S = P_K$ —— (5)

From (4) the poles of the digital filter are at $Z = e^{P_K T_S}$ —— (6)

put (5) in (6)

$$Z = e^{S T_S} \quad \text{——} \quad (7)$$

$$r e^{j\omega} = e^{\sigma T_S} \cdot e^{j \Omega T_S}$$

$$\therefore r = e^{\sigma T_S} \quad \text{——} \quad (8)$$

$$\omega = \Omega T_S \quad \text{——} \quad (9)$$

From (8),

i) If $\sigma < 0 \Rightarrow r < 1$

i.e., left half of the S-plane is mapped inside the unit circle in the Z-plane.

ii) If $\sigma = 0 \Rightarrow r = 1$

i.e., imaginary axis of the s-plane is mapped onto the unit circle in the z-plane.

iii) If $\sigma > 0 \Rightarrow r > 1$

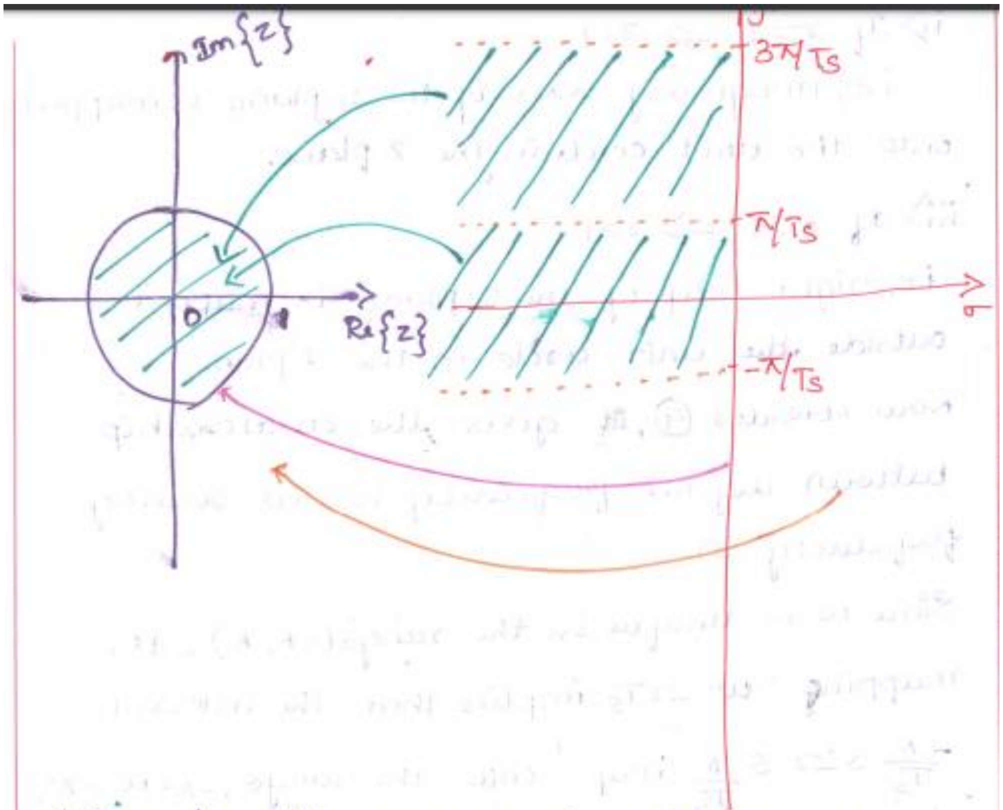
i.e., right half of the s-plane is mapped outside the unit circle in the z-plane.

Now consider (9), it gives the relationship between digital frequency ω and analog frequency Ω.

Since ω is unique in the range$(-\pi, \pi)$, the mapping $\omega = \Omega T_s$ implies that the interval $-\frac{\pi}{T_s} \le \Omega \le \frac{\pi}{T_s}$ maps onto the range, $-\pi \le \omega \le \pi$.

Further, $\frac{\pi}{T_s} \le \Omega \le \frac{3\pi}{T_s}$ also maps into the interval $-\pi \le \omega \le \pi$. In general, the interval $(2k-1)\frac{\pi}{T_s} \le \Omega \le (2k+1)\frac{\pi}{T_s}$ maps into the interval $-\pi \le \omega \le \pi$.

Thus the mapping from analog frequency 'Ω' to digital frequency 'ω' is many to one, which reflects the effect of ~~sampling~~ aliasing due to sampling.



* Due to the presence of aliasing, the impulse invariant method is appropriate for the design of lowpass & bandpass filters only. It is not suitable for high pass filters.

5a)

1. All the poles of a causal FIR filters lie at $Z=0$, in the Z-plane. So FIR filters are always stable.
Poles of IIR filters may lie anywhere in the Z-plane. Therefore IIR filters are not always stable.

2. Linear phase can be achieved in FIR filters. Causal & Stable IIR filters cannot achive linear phase.

3. FIR filters do not have feedback. They are non recursive.
IIR filters have feedback. They are recursive filters.

6. For the given transition width, order of FIR filter will be greater than the order of the IIR filter.

7. IIR filter ~~are~~ used where sharp cut off characteristics with minimum order are required.
FIR filters are used where linear phase characteristic is essential.

b)

1. Poles of a Butterworth filter lie on a circle

Poles of a Chebyshev filter lie on an ellipse.

2. For the given specifications the order of the Butterworth filter is higher than chebyshev.

3. Transition band of a Butterworth filter is broader than chebyshev for given order.

4. The frequency response of a Butterworth filter is monotonically decreasing function.

5. The frequency response of Chebyshev filter is

Type I : Ripple in passband

Monotonic in stopband

Type II : Ripple in stopband

Monotonic in Passband.

7.

Given that Monotonic pass and stop band is required. So it's a Butterworth filter.

$A_{PB} = -1$ dB or $K_P = 1$

$\Omega''_p = 100\pi$

$A_{SB} = -30$ or $K_S = 30$

$\Omega''_s = 1000\pi$

$F_s = 2000 \Rightarrow T_s = \frac{1}{2000} = 5\times10^{-4}$ sec.

$$\Omega'_p = \frac{2}{T_s}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{T_s}\tan\left(\frac{\Omega''_p}{F_s \cdot 2}\right)$$

$$= \frac{2}{1/2000}\tan\left(\frac{100\pi}{2000 \cdot 2}\right)$$

$\Omega'_p = 314.80$ rad/sec.

III^{ly} $\Omega'_s = 4000$ rad/sec.

Convert specifications to PLPT

$\Omega_p = 1$ rad/sec

$\Omega_s = \frac{\Omega_s'}{\Omega_p'} = 12.7$ rad/sec.

$$N = \frac{\log_{10}\left(\frac{10^{APB/10}-1}{10^{ASB/10}-1}\right)}{2\log\left(\frac{\Omega_p}{\Omega_s}\right)} = \frac{\log_{10}\left(\frac{10^{1/10}-1}{10^{30/10}-1}\right)}{2\log\left(\frac{1}{12.7}\right)}$$

$N = 1.62$ (Round off to next integer)

$N \simeq 2$

$$\Omega_c' = \frac{\Omega_p}{\left(10^{APB/10}-1\right)^{1/2N}} = \frac{1}{\left(10^{1/10}-1\right)^{1/4}}$$

$\Omega_c' = 1.4$ rad/sec.

For $N=2$,

$$H'(s) = \frac{1}{s^2+\sqrt{2}\,s+1}$$

Replace $s \rightarrow \frac{s}{\Omega_c'} = \frac{s}{1.4}$

$$H''(s) = \frac{1}{\left(\frac{s}{1.4}\right)^2 + \sqrt{2}.\frac{s}{1.4}+1} = \frac{(1.4)^2}{s^2+\sqrt{2}\times 1.4\,s+(1.4)^2}$$

$$H''(s) = \frac{1.96}{s^2+1.97s+1.96}$$

$s \rightarrow \Omega_p . \frac{s}{\Omega_p'} = 1 . \frac{s}{314.8}$

$$H(S) = \frac{1.96}{\left(\frac{S}{314.8}\right)^2 + 1.97\frac{S}{314.8} + 1.96}$$

$$= \frac{1.96\,(314.8)^2}{S^2 + 1.97 \times 314.8 S + (314.8)^2 \times 1.96}$$

$$H(S) = \frac{194234.118}{S^2 + 620.156 S + 194234.118}$$

convert $H(s)$ to $H(z)$ using Bilinear Transformation

$$S \rightarrow \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$$

$$= \frac{2}{1/2000}\frac{1-z^{-1}}{1+z^{-1}} = 4000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$$

$$H(z) = \frac{194234.18}{\left(4000\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right)^2 + 620.156\left(4000\frac{(1-z^{-1})}{1+z^{-1}}\right) + 194234.18}$$

8a)

$$H(S) = \frac{S+1}{S^2+5S+6}$$
$$H(s) = -\frac{1}{S+2} + \frac{2}{S+3}$$
$$\frac{1}{S-p_k} \rightarrow \frac{1}{1-e^{p_k T_s}Z^{-1}}$$

$$H(z) = -\frac{1}{1-e^{-2}Z^{-1}} + \frac{2}{1-e^{-3}Z^{-1}}$$

$$H(z) = \frac{z^2 - 0.221z}{z^2 - 0.18z + 6.6 * 10^{-3}}$$

8b)

$$H(s) = \frac{b}{(s+a)^2 + b^2}$$

Inverse laplace Transform of H(s) is h(t)

$$h(t) = e^{-at} \sin(bt) u(t)$$

$$h(n) = h(t)\Big|_{t = nT_s}$$

$$h(n) = e^{-anT_s} \sin(bnT_s)\, u(nT_s)$$

$$\therefore H(z) = \sum_{n=0}^{\infty} h(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} e^{-anT_s} \left[\frac{e^{jbnT_s} - e^{-jbnT_s}}{2j} \right] z^{-n}$$

$$H(z) = \frac{1}{2j}\left[\sum_{n=0}^{\infty} e^{-(anT_s - jbnT_s)} z^{-1} - \sum_{n=0}^{\infty} e^{-(anT_s + jbnT_s)} z^{-n}\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1 - e^{-(aT_s - jbT_s)} z^{-1}} - \frac{1}{1 - e^{-(aT_s + jbT_s)} z^{-1}}\right]$$

$$= \frac{1}{2j}\left[\frac{1 - e^{-(aT_s + jbT_s)} z^{-1} - 1 + e^{-(aT_s - jbT_s)} z^{-1}}{1 - e^{-(aT_s + jbT_s)} z^{-1} - e^{-(aT_s - jbT_s)} z^{-1} + e^{-2aT_s} z^{-2}}\right]$$

$$H(z) = \frac{1}{2j}\left[\frac{\left(-e^{-jbT_s} + e^{jbT_s}\right) z^{-1} e^{-aT_s}}{1 - z^{-1} e^{-aT_s}\, 2\cos(bT_s) + z^{-2} e^{-2aT_s}}\right]$$

$$= \frac{1}{2j}\left[\frac{2j \sin(bT_s)\, z^{-1} e^{-aT_s}}{1 - 2e^{-aT_s}\cos(bT_s) z^{-1} + e^{-2aT_s} z^{-2}}\right]$$

$$H(z) = \frac{\sin(bT_s) z^{-1} e^{-aT_s}}{1 - 2e^{-aT_s}\cos(bT_s) z^{-1} + e^{-2aT_s} z^{-2}}$$