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Internal Assesment Test – III

Sub:	Information Theory and Coding		Sec	ECE & TCE				Code:	17EC54
Date:	18 / 11 /19	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE

			OBE	
	ANSWER ANY FIVE FULL QUESTIONS	MARKS	CO	RBT
1	For a systematic linear block code, the parity matrix is given by $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Draw the encoding and decoding circuits.	[10]	C504.4	L2
2	Construct a standard array for (6, 3) codes namely, (000000), (001110), (010011), (011101), (100101), (101011), (110110) and (111000). Let the received codeword be (000101). Decode this codeword using this standard array and obtain the correct sequence.	[10]	C504.4	L2
3	For the (7, 4) single error correcting cyclic code, $D(x) = d_0 + d_1x + d_2x^2 + d_3x^3$. Using generator polynomial $g(x) = 1 + x + x^3$, find all possible cyclic codes in non-systematic and systematic form.	[10]	C504.4	L2
4	Design an encoder for the (7, 4) binary cyclic code generated by $g(x) = 1 + x + x^3$ and verify its operation using the message (1011).	[10]	C504.4	L2
5	Write a note on Golay codes and BCH codes.	[10]	C504.5	L1
6	Consider the binary convolution encoder shown in Fig. Q6. Encode the message D=101 using code tree.	[10]	C504.5	L3

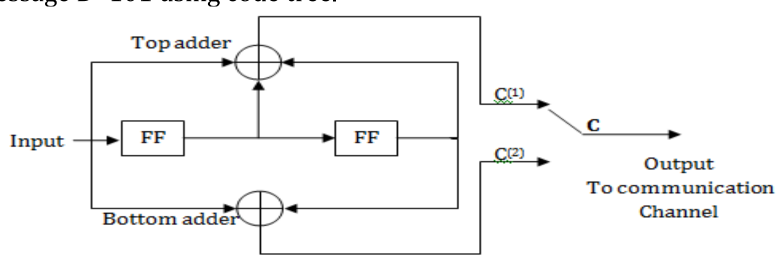

Fig. Q6. Convolutional Encoder

7	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)\ and\ g^{(3)} = (101)$. Draw the encoder circuit. Obtain the generator Matrix.	[10]	C504.5	L2
8	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)$ and $g^{(3)} = (101)$, Encode the message $d = 1011$ using, Time domain approach and Transform domain approach.	[10]	C504.5	L2

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Internal Assesment Test – III Scheme of Evaluation

Sub:	Information Theory and Coding		Sec	ECE & TCE				Code:	17EC54
Date:	18 / 11 /19	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE

	ANSWER ANY FIVE FULL QUESTIONS	MARKS	OBE CO	OBE RBT
1	For a systematic linear block code, the parity matrix is given by $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Draw the encoding and decoding circuits.	[10]	C504.4	L2
	Encoding circuit	05		
	Decoding circuit	05		
2	Construct a standard array for (6, 3) codes namely, (000000), (001110), (010011), (011101), (100101), (101011), (110110) and (111000). Let the received codeword be (000101). Decode this codeword using this standard array and obtain the correct sequence.	[10]	C504.4	L2
	Standard array Look up table	08		
	Writing the correct vector	02		
3	For the (7, 4) single error correcting cyclic code, $D(x) = d_0 + d_1x + d_2x^2 + d_3x^3$. Using generator polynomial $g(x) = 1 + x + x^3$, find all possible cyclic codes in non-systematic and systematic form.	[10]	C504.4	L2
	Cyclic codes in non-systematic form	05		
	Cyclic codes in systematic form	05		
4	Design an encoder for the (7, 4) binary cyclic code generated by $g(x) = 1 + x + x^3$ and verify its operation using the message (1011).	[10]	C504.4	L2
	Encoding circuit	05		
	Encoding the message	05		
5	Write a note on Golay codes and BCH codes.	[10]	C504.5	L1
	Note on Golay codes	05		
	Note on BCH codes	05		
6	Consider the binary convolution encoder shown in Fig. Q6. Encode the message D=101 using code tree.	[10]	C504.5	L3

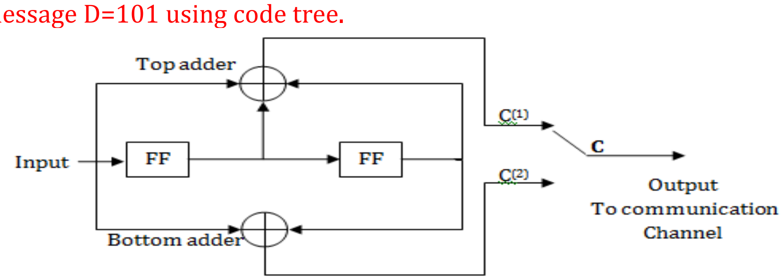


Fig. Q6. Convolutional Encoder

	Drawing the code tree using either state transition table or state diagram	*5*		
	Obtaining $C = [\mathbf{11\ 10\ 00\ 10\ 11}]$	*5*		
7	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)$ and $g^{(3)} = (101)$. Draw the encoder circuit. Obtain the generator Matrix.	[10]	C504.5	L2
	Encoder circuit	*5*		
	Generator matrix $[\boldsymbol{G}]_{\boldsymbol{LXn(L+m)}}$	*5*		
8	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)$ and $g^{(3)} = (101)$, Encode the message $d = 1011$ using, Time domain approach and Transform domain approach.	[10]	C504.5	L2
	Obtaining $C = [\mathbf{111\ 110\ 100\ 001\ 101\ 011}]$	*5*		
	Obtaining $\boldsymbol{C(x) = 1 + x + x^2 + x^3 + x^4 + x^6 + x^{11} + x^{12} + x^{14} + x^{16} + x^{17}}$	*5*		

1) For a systematic linear block code, the parity matrix is given by $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. draw the encoding & decoding circuits.

Ans

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\Longrightarrow [P]_{K \times (n-K)} = [P]_{3\times 3}$$

$$\therefore K = 3 \text{ \& } n = 6$$

Valid i/p combination $= 2^3 = 8$

$$[D]_{1\times K} = [D]_{1\times 3} = [d_1 \; d_2 \; d_3]_{1\times 3}$$

$$[C]_{1\times n} = [C]_{1\times 6} = [C_1 \; C_2 \; C_3 \; C_4 \; C_5 \; C_6]_{1\times 6}$$

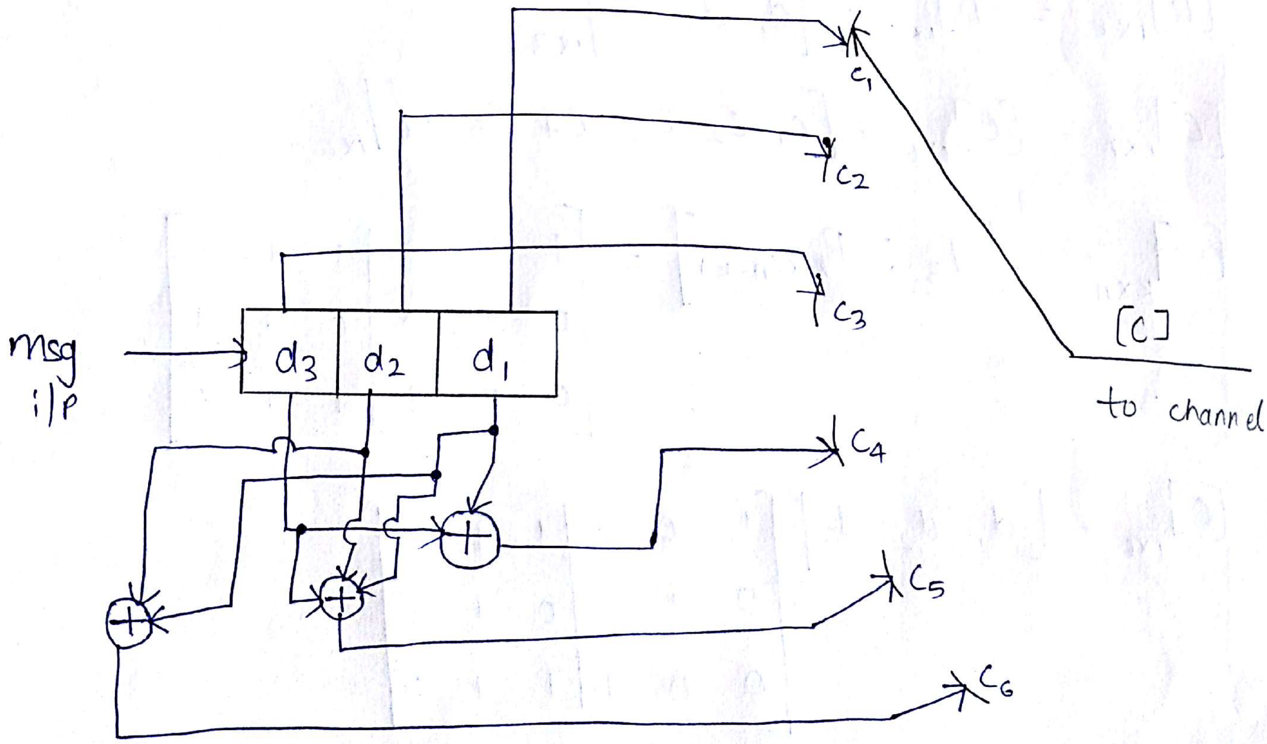
$$[G]_{K\times n} = [I_3 : P_{K\times(n-K)}] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$[C]_{1\times 6} = [d_1 \; d_2 \; d_3] \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$[C] = [d_1 \; d_2 \; d_3 \; (d_1 + d_3) \; (d_1 + d_2 + d_3) \; (d_1 + d_2)]$$

msg			CW					
d_1	d_2	d_3	c_1 d_1	c_2 d_2	c_3 d_3	c_4 (d_1+d_3)	c_5 $(d_1+d_2+d_3)$	c_6 (d_1+d_2)
0	0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	1	0
0	1	0	0	1	0	0	1	1
0	1	1	0	1	1	1	0	1
1	0	0	1	0	0	1	1	1
1	0	1	1	0	1	0	0	0
1	1	0	1	1	0	1	0	0
1	1	1	1	1	1	0	1	1

Encoding ckt :-



Decoding ckt :-

Syndrome	Co-set leader
000	000000
111	100000
011	010000
110	001000
100	000100
010	000010
001	000001
↑ ↑ ↑	↑ ↑ ↑ ↑ ↑ ↑
s_1 s_2 s_3	e_1 e_2 e_3 e_4 e_5 e_6

$$
\begin{array}{rl}
 & R = r_1 \; r_2 \; r_3 \; r_4 \; r_5 \; r_6 \\
+ & E = e_1 \; e_2 \; e_3 \; e_4 \; e_5 \; e_6 \\
\hline
 & C = c_1 \; c_2 \; c_3 \; c_4 \; c_5 \; c_6
\end{array}
$$

$$S = RH^T = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \Big[\underbrace{(r_1 + r_3 + r_4)}^{s_1} \; \overbrace{(r_1 + r_2 + r_3 + r_5)}^{s_2} \; \overbrace{(r_1 + r_2 + r_6)}^{s_3} \Big]$$

$e_1 = s_1 s_2 s_3$

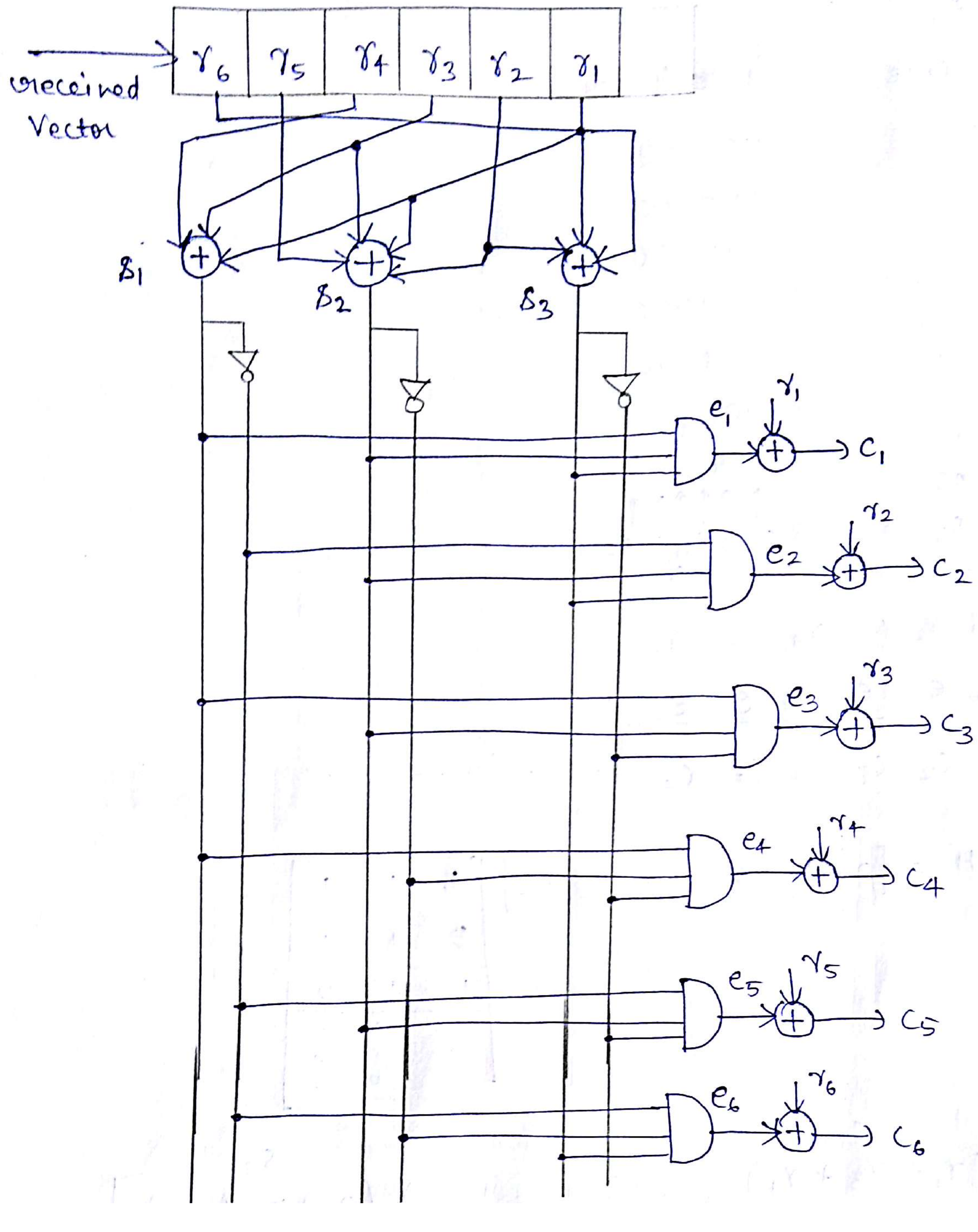
$e_2 = \bar{s}_1 s_2 s_3$

$e_3 = s_1 s_2 \bar{s}_3$

$e_4 = s_1 \bar{s}_2 \bar{s}_3$

$e_5 = \bar{s}_1 s_2 \bar{s}_3$

$e_6 = \bar{s}_1 \bar{s}_2 s_3$



received Vector
r6
r5
r4
r3
r2
r1
s1
s2
s3
e1
e2
e3
e4
e5
e6
c1
c2
c3
c4
c5
c6

2) Construct a Standard Array for (6,3) codes namely, (000000), (001110), (010011), (011101), (100101), (101011), (110110) & (111000). Let the received codeword be (000101). decode this code word using this std. array & obtain the correct sequence.

Syndrome	Co-set leader							
000	000000	001110	010011	011101	100101	101011	110110	111000
111	100000	101110	110011	111101	$\boxed{000101}$	001011	010110	011000
011	010000	011110	000011	001101	110101	111011	100110	101000
110	001000	000110	011011	010101	101101	100011	111110	110000
100	000100	001010	010111	011001	100001	101111	110010	111100
010	000010	001100	010001	01111[illegible]	100111	101001	110100	111010
001	000001	001111	010010	011100	100100	101010	110111	111001
101	110000	111110	100011	101101	010101	011011	000110	001000

$R \rightarrow 000101$

$E \rightarrow 100000$

$C \rightarrow 100101$

$$S = [0\,0\,0\,1\,0\,1]\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ \hline 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Error is in first bit

3) For the (7,4) single Error Correcting Cyclic Code, $D(x) = d_0 + d_1 x + d_2 x^2 + d_3 x^3$. using generator polynomial $g(x) = 1 + x + x^3$, find all ~~possible~~ possible Cyclic Codes in Non-~~Symeto~~ systematic & Systematic form.

Ans Non-~~Sym~~ Systematic form

$(n, k) = (7, 4)$

$g(x) = 1 + x + x^3$

WKT $g(x) = g_0 + g_1 x^1 + g_2 x^2 + g_3 x^3$

$g_0 = 1 ;\ g_1 = 1 ;\ g_2 = 0 ;\ g_3 = 1$

$V(x) = D(x)\, g(x)$

$= (d_0 + d_1 x + d_2 x^2 + d_3 x^3)(1 + x + x^3)$

$= d_0 + d_1 x + d_2 x^2 + d_3 x^2$

$\quad + d_0 x + d_1 x^2 + d_2 x^3 + d_3 x^4$

$\quad\quad + d_0 x^3 + d_1 x^4 + d_2 x^5 + d_3 x^6$

$d_0 + (d_0 + d_1)x + (d_1 + d_2)x^2 + (d_0 + d_2 + d_3)x^3 + (d_1 + d_3)x^4$

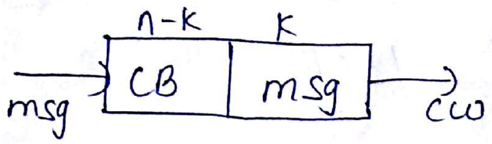
$+ d_2 x^5 + d_3 x^6$

Since $K = 4$

Valid i/p Comb^n $= 2^k = 2^4 = \underline{\underline{16}}$

msg d_0	d_1	d_2	d_3	Codeword (cw) V_0	V_1	V_2	V_3	V_4	V_5	V_6
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1	1	0	1
0	0	1	0	0	0	1	1	0	1	0
0	0	1	1	0	0	1	0	1	1	1
0	1	0	0	0	1	1	0	1	0	0
0	1	0	1	0	1	1	1	0	0	1
0	1	1	0	0	1	0	1	1	1	0
0	1	1	1	0	1	0	0	0	1	1
1	0	0	0	1	1	0	1	0	0	0
1	0	0	1	1	1	0	1	1	0	1
1	0	1	0	1	1	1	0	0	1	0
1	0	1	1	1	1	1	1	1	1	1
1	1	0	0	1	0	1	1	1	0	0
1	1	0	1	1	0	1	0	0	0	1
1	1	1	0	1	0	0	0	1	1	0
1	1	1	1	1	0	0	1	0	1	1

Systematic form :



$$V(x) = D(x)\,g(x)$$

$$[V]_{1\times n} = [D]_{1\times k}\,[G]_{k\times n}$$

$$g(x) = 1 + x + x^3$$

$$g(x) = 1 + x + 0x^2 + x^3 + 0x^4 + 0x^5 + 0x^6 \Rightarrow 1101000$$

$$x\,g(x) = 0x^0 + 1x + 1x^2 + 0x^3 + 1x^4 + 0x^5 + 0x^6 \Rightarrow 0110100$$

$$x^2 g(x) = 0x^0 + 0x + 1x^2 + 1x^3 + 0x^4 + 1x^5 + 0x^6 \Rightarrow 0011010$$

$$x^3 g(x) = 0x^0 + 0x + 0x^2 + 1x^3 + 1x^4 + 0x^5 + 1x^6 \Rightarrow 0001101$$

$$D = [d_0\ d_1\ d_2\ d_3]$$

$$[G] = [P_{4\times 3} : I_4]_{k\times n} = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$R_3 \rightarrow R_3 + R_1$

$R_4 \rightarrow R_4 + R_1 + R_2$

$$[G] = \left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$[V] = [D][G]$$

$$= [d_0\ d_1\ d_2\ d_3]\left[\begin{array}{ccc|cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$= [(d_0+d_2+d_3)\ (d_0+d_1+d_2)\ (d_1+d_2+d_3)\ d_0\ d_1\ d_2\ d_3]$$

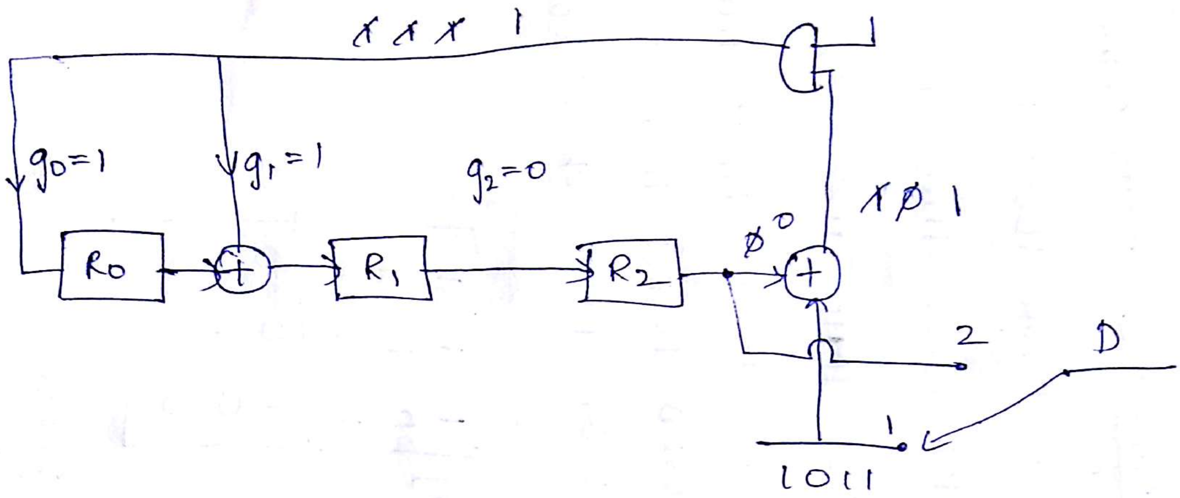
msg				cw						
d_0	d_1	d_2	d_3	V_0	V_1	V_2	V_3	V_4	V_5	V_6
0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	0	1	0	0	0	1
0	0	1	0	1	1	1	0	0	1	0
0	0	1	1	0	1	0	0	0	1	1
0	1	0	0	0	1	1	0	1	0	0
0	1	0	1	1	1	0	0	1	0	1
0	1	1	0	1	0	0	0	1	1	0
0	1	1	1	0	0	1	0	1	1	1
1	0	0	0	1	1	1	1	0	0	0
1	0	0	1	0	1	1	1	0	0	1
1	0	1	0	0	0	1	1	0	1	0
1	0	1	1	1	0	0	1	0	1	1
1	1	0	0	1	0	1	1	1	0	0
1	1	0	1	0	0	0	1	1	0	1
1	1	1	0	0	1	0	1	1	1	0
1	1	1	1	1	1	1	1	1	1	1

4) Design an encoder for the (7,4) BCC generated by $g(x) = 1 + x + x^3$ & Verify its operation using the msg (1011)

Ans

$g(x) = 1 + x + x^3$

$g_0 = 1 \quad g_1 = 1 \quad g_2 = 0 \quad g_3 = 1$



Number of shifts	Input D	Shift Register Contents R_0	R_1	R_2	Remainder bits → R
Initialisation → switch S is in position-1 and gate is turned ON		0	0	0	–
1	1	1	1	0	–
2	1	1	0	1	–
3	0	1	0	0	–
4	1	1	0	0	–
Switch S moves to position-2 and gate is turned OFF					
5	X	0	1	0	0
6	X	0	0	1	0
7	X	0	0	0	1

5) Write a Note on Golay codes & BCH codes.

Ans Golay Code :- It is a (23, 12) binary code capable of correcting upto 3 Errors in a block of 23 bits.

It is a perfect Binary Code because it satisfies the Hamming bound with Equality Sign for t=3

$$2^{n-k} \geq \sum_{i=0}^{t} {}^{n}C_{i}$$

It can be characterised as

1) No of code words (n) = 23
2) no of msg/ data bits (k) = 12
3) No of parity bits (n-k) = 11
4) Minimum distance (dmin) = 7
5) ECC (t) = 3

The generated polynomial is obtained from

$$x^n+1 = x^{23}+1$$
$$= (x+1)\,g_1(x)\,g_2(x)$$

Where

$$g_1(x) = 1+x^2+x^4+x^5+x^6+x^{10}+x^{11}$$

$$g_2(x) = 1+x+x^5+x^6+x^7+x^9+x^{11}$$

BCH codes :-

It is a sub-set of CC capable of correcting 3t errors. hence it is also called as 't' error correcting BCH codes.

for any +ve integer $m \geq 3$, there exists a B code with

block length $n = 2^m - 1$

parity bits $(n-k) \leq mt$

$d_{min} \geq 2t+1$

wher

t = Error correcting capability

m = no of flipflop's or registers

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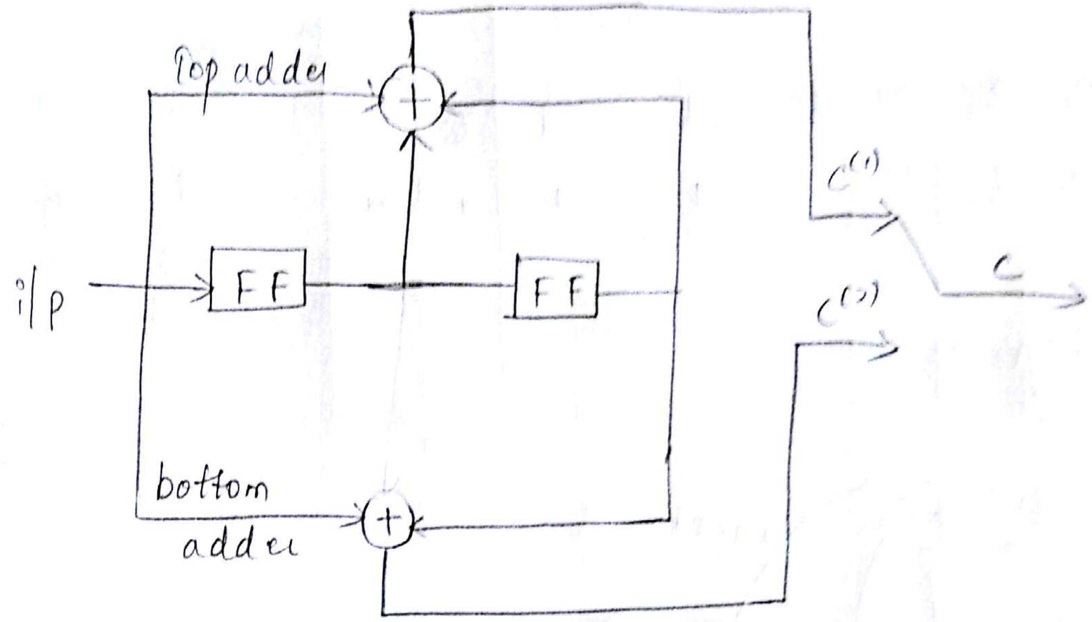
6) Consider a binary Convolution Encoder shown in fig. Q6. Encode the msg d = 101 using code tree.



Ans $d = 101$

$g^{(1)} = 111 \quad g^{(2)} = 101$

State table

No of State $= 2^m = 2^2 = 4$

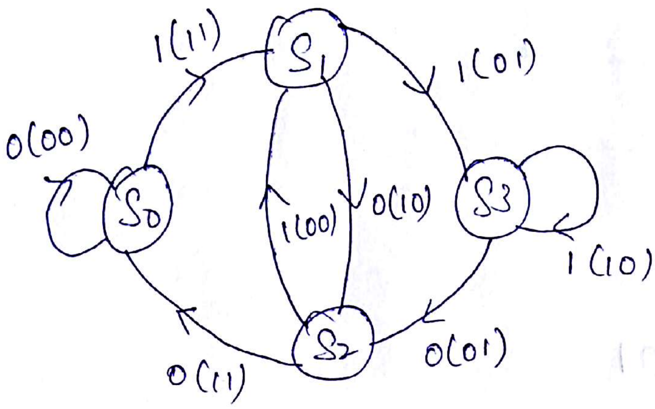
State	S_0	S_1	S_2	S_3
BD	00	10	01	11

State transition table is

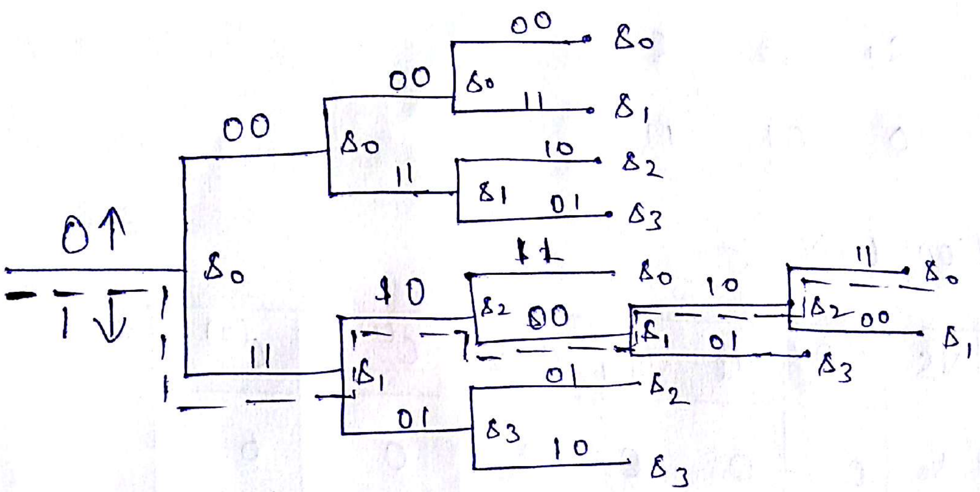
PS	BD	i/p	NS	BD	d_l	d_{l-1}	d_{l-2}	$c^{(1)}$	$c^{(2)}$
S_0	00	0	S_0	00	0	0	0	0	0
		1	S_1	10	1	0	0	1	1
S_1	10	0	S_2	01	0	1	0	1	0
		1	S_3	11	1	1	0	0	1

S_2	01	0	S_0	00	0	0	1	~~1~~	1
		1	S_1	11	1	0	1	0	0
S_3	11	0	S_2	01	0	1	1	0	1
		1	S_3	11	1	1	1	1	0

State diagram :-



Code tree



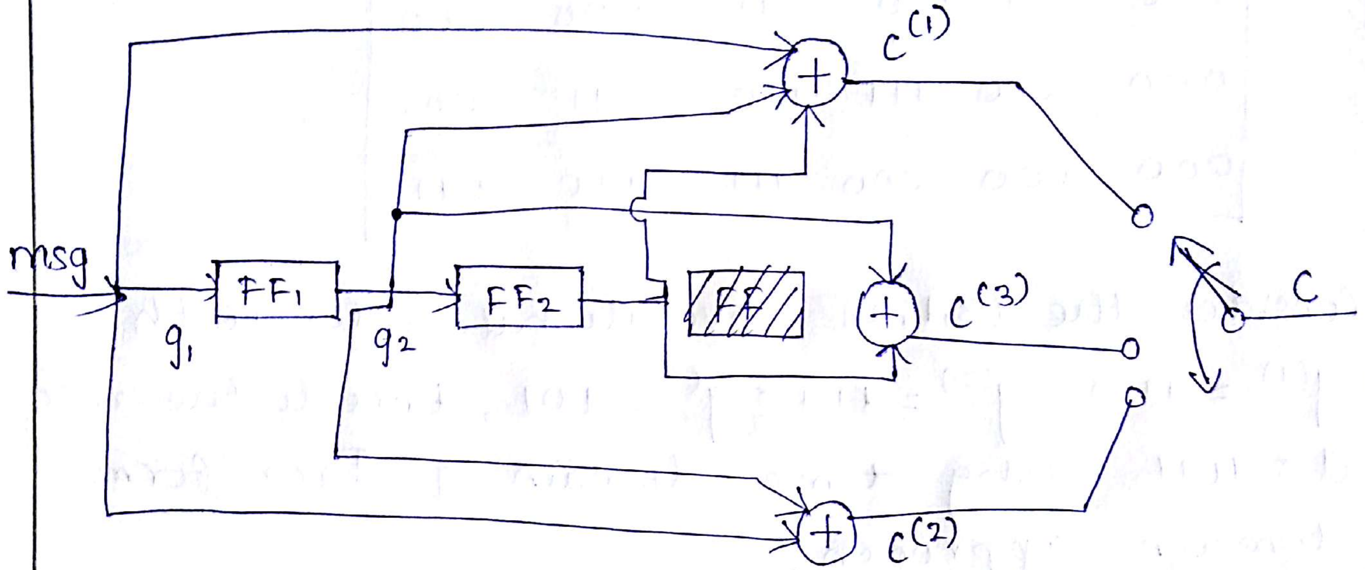
i/p	1	0	1	–	–
o/p	11	10	00	10	11

Consider a (3,1,2) Convolutional code with $g^{(1)} = (110)$
$g^{(2)} = (111)$ & $g^{(3)} = (101)$.
draw the encoder ckt
Obtain the generator matrix

Ans given/. $(n,k,m) = (3,1,2)$ ~~# i/p = 2^k~~

$g^{(1)} = 110$ $g^{(2)} = 111$ $g^{(3)} = 101$

Encoder ckt :- Let $d = 1011 \Rightarrow L = 4$



$[C]_{\cancel{1\times (4)}} =$

$$[C]_{1\times n(L+m)} = [d]_{1\times L} * [G]_{L\times n(L+m)}$$

$$[C]_{1\times 3(4+2)} = [d]_{1\times 4} * [G]_{4\times 18}$$

$$[C]_{1\times 18} = [d]_{1\times 4} * [G]_{4\times 18}$$

$$[G] = \begin{bmatrix} g_1^{(1)} g_1^{(2)} g_1^{(3)} & g_2^{(1)} g_2^{(2)} g_2^{(3)} & g_3^{(1)} g_3^{(2)} g_3^{(3)} & 000 & 000 & 000 \\ 000 & g_1^{(1)} g_1^{(2)} g_1^{(3)} & g_2^{(1)} g_2^{(2)} g_2^{(3)} & g_3^{(1)} g_3^{(2)} g_3^{(3)} & 000 & 000 \\ 000 & 000 & g_1^{(1)} g_1^{(2)} g_1^{(3)} & g_2^{(1)} g_2^{(2)} g_2^{(3)} & g_3^{(1)} g_3^{(2)} g_3^{(3)} & 000 \\ 000 & 000 & 000 & g_1^{(1)} g_1^{(2)} g_1^{(3)} & g_2^{(1)} g_2^{(2)} g_2^{(3)} & g_3^{(1)} g_3^{(2)} g_3^{(3)} \end{bmatrix}$$

$$[G] = \begin{bmatrix} 111 & 110 & 011 & 000 & 000 & 000 \\ 000 & 111 & 110 & 011 & 000 & 000 \\ 000 & 000 & 111 & 110 & 011 & 000 \\ 000 & 000 & 000 & 111 & 110 & 011 \end{bmatrix}$$

8) Consider the (3,1,2) Convolutional Codes with $g^{(1)} = 110$ $g^{(2)} = 111$ & $g^{(3)} = 101$, Encode the msg $d = 1011$ using time domain & Transform domain approach.

Ans <u>Time Domain :-</u>

Given $(n, K, m) = (3, 1, 2)$

$g^{(1)} = 110$ $g^{(2)} = 111$ $g^{(3)} = 101$

$d = 1011 \Longrightarrow \underline{L = 4}$

$$[C]_{1 \times 3(4+2)} = [d]_{1\times 4} * [G]_{4 \times 3(4+2)}$$

$$[C]_{1\times 18} = [d]_{1\times 4} * [G]_{4\times 18}$$

$$[G] = \begin{bmatrix} 111 & 110 & 011 & 000 & 000 & 000 \\ 000 & 111 & 110 & 011 & 000 & 000 \\ 000 & 000 & 111 & 110 & 011 & 000 \\ 000 & 000 & 000 & 111 & 110 & 011 \end{bmatrix}$$

$$C = [1\ 0\ 1\ 1] \begin{bmatrix} 111 & 110 & 011 & 000 & 000 & 000 \\ 000 & 111 & 110 & 011 & 000 & 000 \\ 000 & 000 & 111 & 110 & 011 & 000 \\ 000 & 000 & 000 & 111 & 110 & 011 \end{bmatrix}$$

$$C = [111 \quad 110 \quad 100 \quad 001 \quad 101 \quad 011]$$

Transform domain Approach :-

Given $(n, k, m) = (3, 1, 2)$

$d = 1011 \Rightarrow d(x) = 1 + x^2 + x^3$

$g^{(1)} = 1 + x$

$g^{(2)} = 1 + x + x^2$

$g^{(3)} = 1 + x^2$

output from top adder is $c^{(1)}(x)$

$$c^{(1)}x = d(x) * g^{(1)}(x)$$

$$= (1 + x^2 + x^3)(1 + x)$$

$$= 1 + \quad + x^2 + x^3$$
$$\quad x + \quad + x^3 + x^4$$

$$1 + x + x^2 + x^4$$

$$c^{(1)}(x^3) = 1 + x^3 + x^6 + x^{12}$$

o/p from middle adder $= c^{(2)}(x^3)$

$$c^{(2)}(x) = d(x) * g^{(2)}(x)$$

$$= (1+x^2+x^3)(1+x+x^2)$$

$$\begin{aligned} = \; & 1 + + \cancel{x^2} + \cancel{x^3} \\ & x + + \cancel{x^3} + \cancel{x^4} + \cancel{x^5} \\ & \cancel{x^2} + + \cancel{x^4} + x^5 \end{aligned}$$

$$\underline{\underline{1 + x + x^5}}$$

$$x\,c^{(2)}(x^3) = x[1 + x^3 + x^{15}] = \underline{\underline{x + x^4 + x^{16}}}$$

o/p from bottom adder $= c^{(3)}(x)$

$$c^{(3)}(x) = d(x) * g^{(3)}(x)$$

$$= (1+x^2+x^3)(1+x^2)$$

$$\begin{aligned} = \; & 1 + \cancel{x^2} + x^3 \\ & \cancel{x^2} + + x^4 + x^5 \end{aligned}$$

$$1 + x^3 + x^4 + x^5$$

$$x^2 c^{(3)}(x^3) = x^2[1 + x^9 + x^{12} + x^{15}] = x^2 + x^{11} + x^{14} + x^{17}$$

$$c(x) = c^{(1)}(x^3) + x\,c^{(2)}(x^3) + x^2 c^{(3)}(x^3)$$

$$= 1 + x^3 + x^6 + x^{12} + x + x^4 + x^{16} + x^2 + x^{11} + x^{14} + x^{17}$$

$$\underline{\underline{c = [111\ 110\ 100\ 001\ 101\ 011]}}$$