5 In the circuit shown in Fig.5 switch K is closed at t=0, steady state having been attained in position 1. Find the values of I, first derivative and second derivative of I at t=0+. [10] CO3 L3

$i_L(0^-) = 0\ A = i_L(0^+)$

$V_c(0^-) = 0V = V_c(0^+)$

$\frac{di(0^+)}{dt} = 10\ A/s$

$\frac{d^2 i(0^+)}{dt^2} = -100\ A/sec^2$

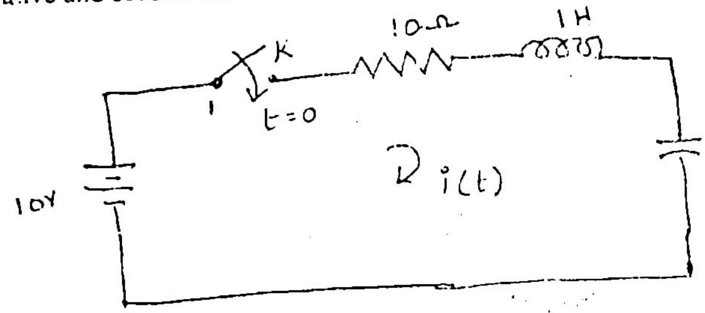


Fig.5

6 Obtain the transient response of series RL circuit. [10] CO3 L3

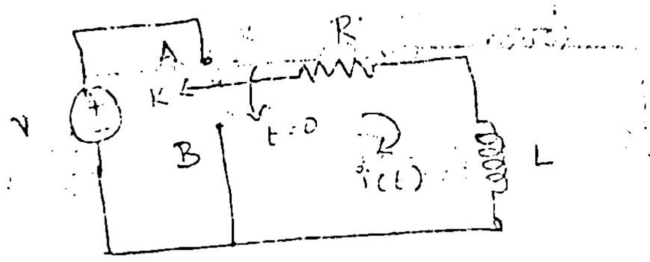


Fig.6

$i_L(0^-) = \frac{V}{R} = i(0^+)$

$i(t) = I_0 e^{-R/L\, t}$

$i(t) = \frac{V}{R} e^{-\frac{R}{L}t}$

7 Define 'h' and 'T' parameters. Derive' h' parameters interms of 'T' and 'T' parameters interms of h parameters. [10] CO5 L3

3 Define standard input function and obtain it's laplace transform. [10] CO4 L3

(i)Step (ii) Ramp (iii) Impulse

$1/s$ $1/s^2$ 1

h in terms of T:

$\frac{B}{D}$	$\frac{\Delta T}{D}$
$\frac{-1}{D}$	$\frac{C}{D}$

T in terms of h:

$-\frac{\Delta h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$
$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$

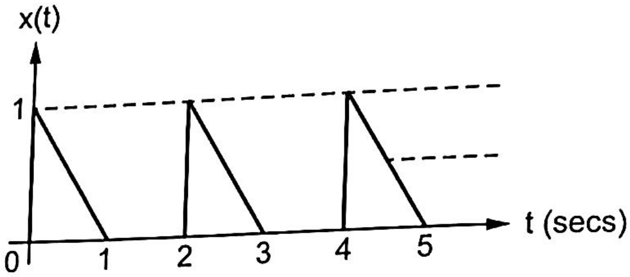
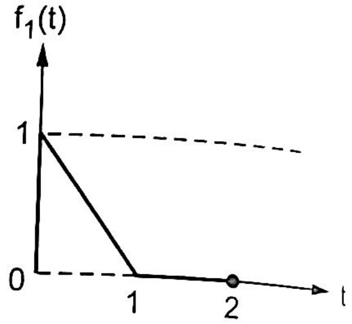


Fig. 4.7.3

Solution : As the signal is periodic, consider its first cycle as shown in the Fig. 4.7.3 (a).



(a)

It is made up of :

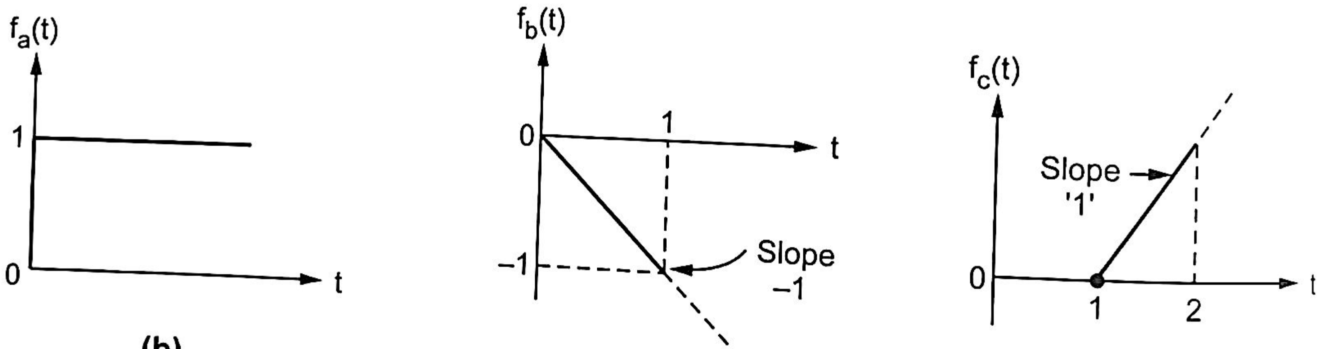
1) Step of '1' at $t = 0$ i.e. $f_a(t) = u(t)$.

2) Ramp at $t = 0$ with slope $\frac{0-1}{1-0} = -1$ i.e. $f_b(t) = -t\,u(t)$.

3) The ramp at $t = 0$ is stopped and converted to constant valued function at $t = 1$. So there exists a ramp at $t = 1$ of same slope as that of $f_b(t)$ but opposite sign.

$$\therefore \quad f_c(t) = +(t-1)\,u(t-1)$$

These signals are shown in the Fig. 4.7.3 (b), (c) and (d).



(b) (c) (d)

Fig. 4.7.3

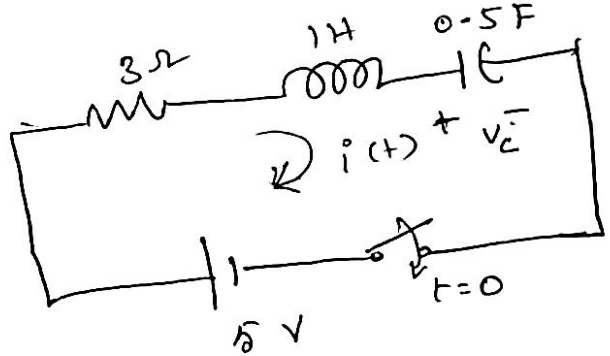
$$\therefore \quad f_1(t) = f_a(t) + f_b(t) + f_c(t) = u(t) - t\,u(t) + (t-1)\,u(t-1)$$

$$\therefore \quad F_1(s) = \frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2} e^{-s}$$

$$\therefore \quad F(s) = \frac{F_1(s)}{1-e^{-Ts}} = \boxed{\frac{\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s^2} e^{-s}}{1-e^{-2s}}} \qquad \ldots T = 2$$

$F(s) - \frac{}{s} \quad 3s$

eg: For series RLC ckt, the initial conditions are $i_L(0_-) = 2A$, $V_c(0_-) = 2V$. Find $i(t)$ at $t = 0_+$. (2011)



Soln: at $t = 0$, switch is closed.

apply KVL,

$$3\,i(t) + 1\frac{d\,i(t)}{dt} + \frac{1}{0.5}\int_{-\infty}^{t} i(t)\,dt = 5$$

$$3\,i(t) + \frac{d\,i(t)}{dt} + \frac{1}{0.5}\int_{-\infty}^{0_-} i(t)\,dt + \frac{1}{0.5}\int_{0^-}^{t} i(t)\,dt = 5$$

$V_c(0_-) = 2V$

$$3\,i(t) + \frac{d\,i(t)}{dt} + 2\int_{0}^{t} i(t)\,dt = 5 - 2$$

take LT

$$3I(s) + sI(s) - i(0_-) + 2\frac{I(s)}{s} = 3/s$$

$$I(s)\left\{3+s+\frac{2}{s}\right\}-2=\frac{3}{s}$$

$$I(s)\left\{3+s+\frac{2}{s}\right\}=\frac{3}{s}+2$$

$$I(s)\left[\frac{3s+s^2+2}{s}\right]=\frac{3+2s}{s}$$

$$I(s)=\frac{3+2s}{3s+s^2+2}=\frac{3+2s}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$$

$s=-1$ $s=-2$

$A=1$, $B=1$

$$I(s)=\frac{1}{s+1}+\frac{1}{s+2}$$

$$i(t)=e^{-t}+e^{-2t}\ A$$

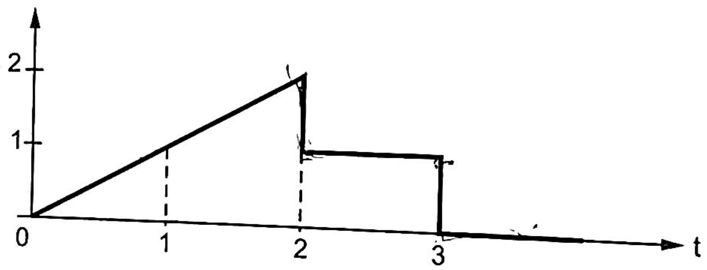


Fig. 4.6.3

Solution : Given function is not periodic. Identify the various step and ramp waveforms of which given waveform is made up of.

The first function in given waveform is ramp as shown in the Fig. 4.6.3 (a). Consider two points A (0, 0) and B(2, 2) so that slope of the ramp can be obtained as,

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{2-0} = 1$$

$$f_1(t) = 1.t\ u(t) = t\ u(t)$$

$\therefore$

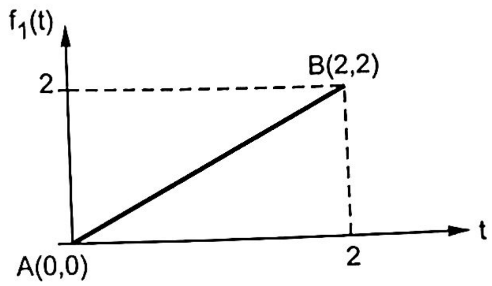


Fig. 4.6.3 (a)

To cancel this increasing ramp at t = 2, another ramp of same slope with opposite sign, starting at t = 2 must be added.

$$\therefore \quad f_2(t) = -(t-2)\ u(t-2)$$

So addition of these two is a constant value of 2 at t = 2. It is decreased to 1 at t = 2 so there exists a step function of magnitude – 1 at t = 2.

$$\therefore \quad f_3(t) = -u(t-2)$$

And the value 1 is further decreased to zero at t = 3 and continued to be zero at t = 3. So a step of magnitude –1 is added at t = 3.

$$\therefore \quad f_4(t) = -u(t-3)$$

So given waveform can be written as,

$$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t)$$

$$= t\ u(t) - (t-2)\ u(t-2) - u(t-2) - u(t-3)$$

$$\therefore \quad F(s) = \frac{1}{s^2} - \frac{1}{s^2}e^{-2s} - \frac{1}{s}e^{-2s} - \frac{1}{s}e^{-3s}$$

... of the function shown in Fig. 4.6.4.

Example 3.8 In a circuit as shown in *Fig. 3.8, the switch K is changed from position 1 to 2 at t = 0. The steady state having been reached before switching. Find values of* $i(t)$, $\frac{di(t)}{dt}$ *and* $\frac{d^2(t)}{dt^2}$ *at* $t = 0^+$.

VTU : Dec.-10, Marks 8

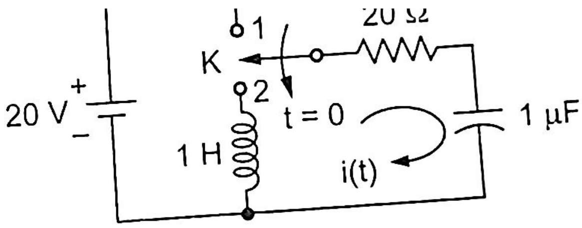


Fig. 3.8

Solution : At $t = 0^-$**, switch K is at position 1.** The network remains in steady state. Hence capacitor acts as open circuit. As inductor is connected to terminal 2 and at $t = 0^-$ it is not connected in circuit, current through it be zero.

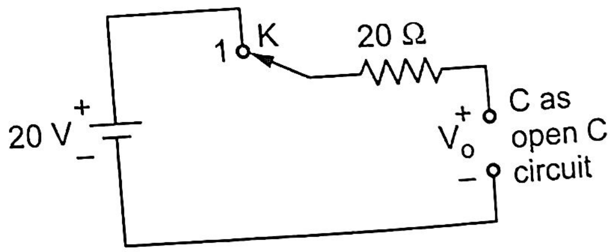


Fig. 3.8 (a)

$\therefore \quad v_C(0^-) = 20\text{ V} = v_C(0^+)$ ...(1)

And $i_L(0^-) = 0 = i_L(0^+)$ ...(2)

For all $t \geq 0^+$, switch K is moved to position 2 as shown in the Fig. 3.8 (b).

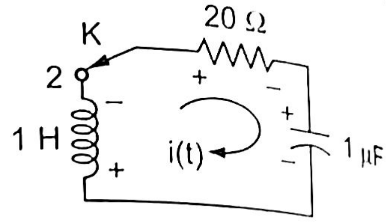


Fig. 3.8 (b)

Immediately after switching, inductor acts as open circuit, so current in loop will be zero.

Hence $i(0^+) = 0A$ ...(3)

Applying KVL to closed path, we get

$$20\,i(t) + \frac{1}{1\times10^{-6}}\int_{0^-}^{t} i(t)dt + \frac{di(t)}{dt} = 0 \quad ...(4)$$

At $t = 0^+$, equation (4) becomes,

$$20\,i(0^+) + \frac{1}{1\times10^{-6}}\int_{0^-}^{0^+} i(0^+)dt + \frac{di}{dt}(0^+) = 0$$

Second term represents initial voltage across capacitor. The value of such voltage is 20 V. And $i(0^+) = 0$ from equation (3) so we can write,

$$20\,(0) + 20 + \frac{di}{dt}(0^+) = 0$$

$$\therefore \frac{di}{dt}(0^+) = -\,20\text{ A/sec}$$

Differentiating equation (4) w.r.t. t, we get,

$$20\frac{di(t)}{dt} + \frac{i(t)}{1\times10^{-6}} + \frac{d^2i(t)}{dt^2} = 0 \quad ...(5)$$

At $t = 0^+$, equation (5) becomes,

$$20\frac{di(t)}{dt}(0^+) + \frac{i(0^+)}{1\times10^{-6}} + \frac{d^2i}{dt^2}(0^+) = 0$$

$$\therefore (20)\,(-\,20) + \frac{(0)}{1\times10^{-6}} + \frac{d^2i}{dt^2}(0^+) = 0$$

$$\therefore \quad \frac{d^2i}{dt^2}(0^+) = 400\text{ A/sec}^2$$

Example 3.1 *In the network shown in Fig. 3.1 the switch is closed at $t = 0$. Determine i, di/dt and $\frac{d^2i}{dt^2}$ at $t = 0^+$.* **VTU : Jan.-14, Marks 10**

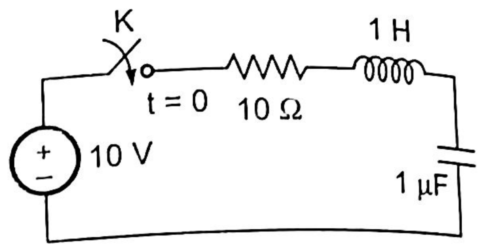


Fig. 3.1

Solution : At $t = 0^-$, switch K is open.

$$\therefore \quad i(0^-) = 0 = i(0^+) \quad \text{... (1)}$$

Current through inductor cannot change instantaneously.

$$\therefore \quad i(0^+) = 0 \text{ A}$$

$$\text{Also} \quad v_C(0^-) = 0 = v_C(0^+) \quad \text{... (2)}$$

Voltage across capacitor cannot change instantaneously.

For all $t \geq 0^+$, switch K is closed. Applying KVL, we get,

$$10\, i(t) + \frac{di(t)}{dt} + \frac{1}{1\times10^{-6}} \int_{-\infty}^{t} i(t)\, dt = 10$$

$$\therefore \quad \frac{di(t)}{dt} + 10\, i(t) + \left[\frac{1}{1\times10^{-6}} \int_{-\infty}^{0^-} i(t)\, dt + \frac{1}{1\times10^{-6}} \int_{0^-}^{t} i(t)\, dt\right] = 10$$

Now the first integral term on L.H.S. is the initial voltage across capacitor which is zero from equation (2). So we can write,

$$\frac{di(t)}{dt} + 10\, i(t) + \frac{1}{1\times10^{-6}} \int_{0^-}^{t} i(t)\, dt = 10$$

At $t = 0^+$, equation (3) becomes,

$$\frac{di}{dt}(0^+) + 10\, i(0^+) + \frac{1}{1\times10^{-6}} \int_{0^-}^{0^+} i(t)\, dt = 10$$

$$\therefore \quad \frac{di}{dt}(0^+) = \mathbf{10\ A/s}$$

Differentiating equation (3) with respect to t, we get,

$$\frac{d^2i(t)}{dt^2} + 10\frac{di(t)}{dt} + \frac{i(t)}{1\times10^{-6}} = 0$$

At t = 0^+, equation (4) becomes,

$$\frac{d^2i}{dt^2}(0^+) + 10\frac{di}{dt}(0^+) + \frac{i(0^+)}{1\times10^{-6}} = 0$$

$$\therefore \quad \frac{d^2i}{dt^2}(0^+) = -10\frac{di}{dt}(0^+) = -10\,(10) = \mathbf{-100\ A/s^2} \quad \ldots (\because\ i\,(0$$

[C] Interms of Transmission Parameters

The equations for transmission parameters are as follows,

$$V_1 = A V_2 + B(-I_2)$$

$$I_1 = C V_2 + D(-I_2)$$

We can rewrite equation (2) as follows,

$$-D I_2 = I_1 - C V_2$$

$$\therefore \quad I_2 = \left[\frac{-1}{D}\right] I_1 + \left[+\frac{C}{D}\right] V_2$$

Substituting value of I_2 in equation (1), we have,

$$V_1 = A V_2 + B\left\{-\left(\left[-\frac{1}{D}\right] I_1 + \left[\frac{C}{D} V_2\right]\right)\right\}$$

$$\therefore \quad V_1 = A V_2 + \left[\frac{B}{D}\right] I_1 - \frac{BC}{D} V_2$$

$$\therefore \quad V_1 = \left[\frac{B}{D}\right] I_1 + \left[\frac{AD-BC}{D}\right] V_2$$

Comparing equations (4) and (3) with equations (A) and (B) respectively

$$h_{11} = \frac{B}{D}, \quad h_{12} = \frac{AD-BC}{D}$$

$$h_{21} = -\frac{1}{D}, \quad h_{22} = \frac{C}{D}$$

In the matrix form, h-parameters can be written as,

$$[h] = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$