

Modified

# CBCS SCHEME

USN

--	--	--	--	--	--	--	--	--	--

17EC54

## Fifth Semester B.E. Degree Examination, Dec.2019/Jan.2020 Information Theory and Coding

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

### Module-1

- 1 a. Suppose you are planning a trip to Miami, Florida from Minneapolis in the winter time. You are receiving the following information from Miami Weather bureau:  
 (i) Mild and Sunny day      (ii) Cold day      (iii) Possible snow flurries  
 Explain the amount of information content in each statement. (06 Marks)
- b. The output of an information source consists of 128 symbols, 16 of which occurs with probability of  $\frac{1}{32}$  and the remaining 112 occurs with probability of  $\frac{1}{224}$ . The source emits 1000 symbols/sec. Assuming that the symbols are chosen independently. Find the Average Information Rate of this source. (06 Marks)
- c. The state diagram of a stationary Mark off Source is shown in Fig.Q1(c):  
 (i) Find the entropy of each state  
 (ii) Find the entropy of the source  
 (iii) Find  $G_1$  and  $G_2$  and verify that  $G_1 \geq G_2 \geq H$ .

Assume  $P(1) = P(2) = P(3) = \frac{1}{3}$

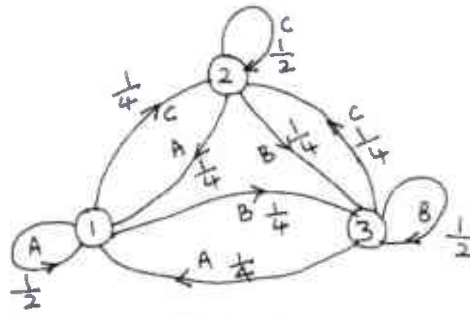


Fig.Q1(c)

(08 Marks)

OR

- 2 a. What is self information? Mentions its various measuring units and also mentions the reasons for choosing logarithmic function. (06 Marks)
- b. A binary source is emitting an independent sequence of 0's 1's with probabilities of  $P$  and  $1 - P$  respectively. Plot the entropy of this source versus probability. (06 Marks)
- c. For the first order Markov statistical model as shown in Fig.Q2(c).  
 (i) Find the probability of each state      (ii) Find  $H(s)$  and  $H(s^{-2})$

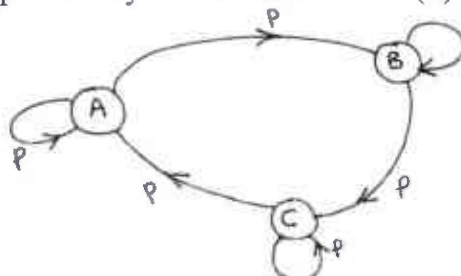


Fig.Q2(c)

where A, B, and C are the states.

(08 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.  
 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

**Module-2**

- 3 a. Identify whether the codes shown in Table.Q3(a) are instantaneous. Justify your answer.

Symbols	Code A	Code B	Code C
S <sub>1</sub>	00	1	0
S <sub>2</sub>	01	01	100
S <sub>3</sub>	10	001	101
S <sub>4</sub>	11	00	111

Table.Q3(a)

(06 Marks)

- b. Consider a Discrete Memory Source (DMS) with  $S = \{X, Y, Z\}$  with  $P = \{0.6, 0.2, 0.2\}$ . Find the code word for the message "YXZXY" using Arithmetic code. (06 Marks)
- c. An information source produces a sequence of independent symbols having the following probabilities. More composite symbol as slow as possible.

Symbol	A	B	C	D	E	F	G
Probabilities	$\frac{1}{3}$	$\frac{1}{27}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{27}$

Construct Binary Huffman encoding and find its efficiency.

(08 Marks)

**OR**

- 4 a. Write the Shannon's Encoding Algorithms. (06 Marks)
- b. Consider the following source with probabilities:  
 $S = \{A, B, C, D, E, F\}$        $P = \{0.4, 0.2, 0.2, 0.1, 0.08, 0.02\}$   
 Find the code words using Shannon-Fano algorithm and also find its efficiency. (06 Marks)
- c. Consider the following discrete memoryless source:  
 $S = \{S_0, S_1, S_2, S_3, S_4\}$        $P = \{0.55, 0.15, 0.15, 0.1, 0.05\}$   
 Compute Huffman code by placing composite symbol as high as possible. Also find average code word length and variance of the code word. (08 Marks)

**Module-3**

- 5 a. What is Joint Probability Matrix? How it is obtained from Channel Matrix and also mention properties of JPM. (06 Marks)
- b. For the communication channel shown in Fig.Q5(b), determine Mutual Information and Information Rate if  $r_s = 1000$  symbols/sec. Assume  $P(X_1) = 0.6$  and  $P(X_2) = 0.4$ .

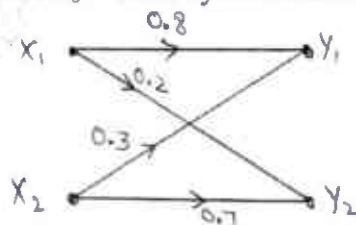


Fig.Q5(b)

(06 Marks)

- c. Discuss the Binary Erasure Channel and also prove that the capacity a Binary Erasure Channel is  $C = \bar{P} \cdot r_s$  bits/sec. (08 Marks)

**OR**

- 6 a. What is Mutual Information? Mention its properties. (06 Marks)
- b. The noise characteristics of a channel shown in Fig.Q6(b). Find the capacity of a channel if  $r_s = 2000$  symbols/sec using Muroga's method.

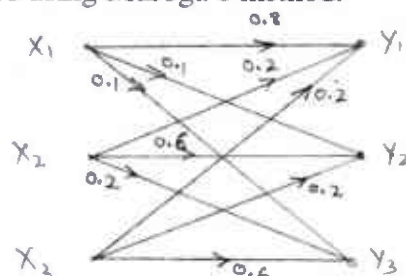


Fig.Q6(b)

(06 Marks)

- c. State and prove the Shannon-Hartley Law. (08 Marks)

**Module-4**

- 7 a. What are the advantages and disadvantages of Error Control Coding? Discuss the methods of controlling Errors. (06 Marks)
- b. The parity check bits of a (7, 4) Hamming code are generated by  
 $C_5 = d_1 + d_3 + d_4$   
 $C_6 = d_1 + d_2 + d_3$   
 $C_7 = d_2 + d_3 + d_4$   
 where  $d_1, d_2, d_3$  and  $d_4$  are the message bits.  
 (i) Find G and H for this code. (06 Marks)  
 (ii) Prove that  $GH^T = 0$ . (06 Marks)
- c. Design a syndrome calculating circuit for a (7, 4) cyclic code with  $g(X) = 1 + X + X^3$  and also calculate the syndrome of the received vector  $R = 1110101$ . (08 Marks)

**OR**

- 8 a. For a systematic (6, 3) linear block code, the Parity Matrix P is given by

$$[P] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

- (i) Find all possible code words. (06 Marks)  
 (ii) Find error detecting and correcting capability. (06 Marks)
- b. A (7, 4) cyclic code has the generator polynomial  $g(X) = 1 + X + X^3$ . Find the code vector both in systematic and non-systematic form for the message bits (1101). (06 Marks)
- c. Draw the Encoder circuit of a cyclic code using  $(n - K)$  bit shift Registers and explain it. (08 Marks)

**Module-5**

- 9 a. Consider (3, 1, 2) Convolution Encoder with  $g^{(1)} = 110$ ,  $g^{(2)} = 101$  and  $g^{(3)} = 111$ .  
 (i) Draw the encoder diagram. (16 Marks)  
 (ii) Find the code word for the message sequence (11101) using generator Matrix and Transform domain approach. (04 Marks)
- b. Discuss the BCH codes. (04 Marks)

**OR**

- 10 a. Consider the convolution encoder shown in Fig.Q10(a).  
 (i) Write the impulse response and its polynomial. (16 Marks)  
 (ii) Find the output corresponding to input message (10111) using time and transform domain approach. (04 Marks)

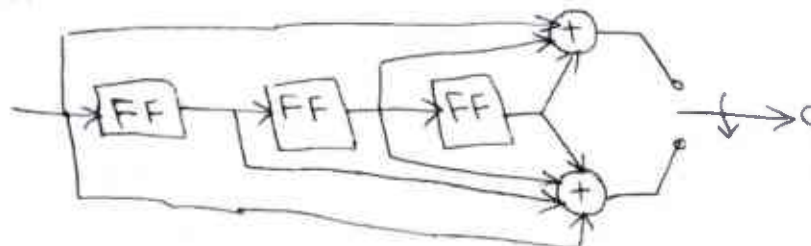


Fig.Q10(a)

- b. Write a note on Golay codes. (16 Marks)

(04 Marks)

\*\*\*\*\*

Question Number	Solution	P-1/7	Marks Allocated
1 a)	<p>(i) Contains very little information, since the weather in Miami is mild and sunny most of the time</p> <p>(ii) cold day contains more information, since it is not an event that occurs often.</p> <p>(iii) snow blizzards convey even more information, since the occurrence of snow in Miami is a rare event</p>		2M 2M 2M
b)	$H(S) = \sum_{i=1}^n p_i \log \frac{1}{p_i} = 16 \times \frac{1}{32} \log_2 32 + 112 \times \frac{1}{924} \log_2 224$ $H(S) = 6.4036 \text{ bits/sym}$		3M
c)	$R_s = H(S) r_s = 6.4036 \times 1000 = 6403.67 \text{ bits/sec}$		3M
	$H_i = \sum_{j=1}^3 p_{ij} \log \frac{1}{p_{ij}}$ $H_1 = H_2 = H_3 = 1.5 \text{ bits/message symbol}$		2M
	<p>(ii) <math>H = p_1 H_1 + p_2 H_2 + p_3 H_3 = (\frac{1}{3} \times 1.5) \times 3 = 1.5 \text{ bits/ms}</math></p>		2M
	<p>(iii) Construct tree for state 1, 2 &amp; 3</p> <p>Find <math>G_1</math>: <math>P(A) = \frac{1}{3}, P(B) = \frac{1}{3}, P(C) = \frac{1}{3}</math></p>		2M
	$G_1, H(S) = 3 \times \frac{1}{3} \log_2 3 = 1.585 \text{ bits/ms}$		1M
	<p>Find <math>G_2</math>:</p> <p><math>P(AA) = \frac{1}{6}, P(AC) = \frac{1}{12}, P(AB) = \frac{1}{12}, P(CA) = \frac{1}{12}</math></p> <p><math>P(CC) = \frac{1}{6}, P(CB) = \frac{1}{12}, P(BA) = \frac{1}{12}, P(BC) = \frac{1}{12}, P(BB) = \frac{1}{6}</math></p>		2M
	$G_2 = \frac{1}{2} [3 \times \frac{1}{6} \log_2 6 + 6 \times \frac{1}{12} \log_2 12] = 1.5425 \text{ bits/ms}$		2M
	$\therefore G_1 \geq G_2 \geq H$		2M
2 a)	$E_k = \log \frac{1}{p_k}$		2M
	<p>where <math>p_k</math> - probability occurrence of <math>k^{\text{th}}</math> symbol</p> <p><math>\log_2 \rightarrow</math> Bits, <math>\log_{10} \rightarrow</math> Hartley, <math>\log_e \rightarrow</math> NATS</p>		2M
	<p>Reasons (1) <math>E_k</math> - Non-negative</p>		2M
	<p>(2) Lowest possible <math>E_k</math> is zero, which occurs for sure events</p>		2M
	<p>(3) More information is carried by</p>		2M



Question Number	Solution	P-2/7	Marks Allocated
-----------------	----------	-------	-----------------

b)  $H = -\sum_{i=1}^n P_i \log_2 \frac{1}{P_i} = -P \log_2 \frac{1}{P} + (1-P) \log_2 \frac{1}{1-P}$

P	0	0.2	0.4	0.5	0.6	0.8	1
H	0	0.721	0.917	1	0.917	0.721	0

c) (1)  $P(A) = P P(A) + P P(C)$  — (1)  
 $P(B) = P P(B) + P P(A)$  — (2)  
 $P(C) = P P(C) + P P(B)$  — (3)

Using (3) & solving  $P = \frac{1}{2}$

$\therefore P(A) = P(B) = P(C) = \frac{1}{2}$

(ii)  $H(A) = H(B) = H(C) = \text{bits/m.s}$   
 $H(S) = P(A)H(A) + P(B)H(B) + P(C)H(C) = 1 \text{ bit/m.s}$   
 $H(S^2) = 2H(S) = 2 \text{ bits/m.s}$

3 a) Code A: [s] instantaneous  
 $\sum_{i=1}^n r^{-li} = 2^{-2} + 2^{-2} + 2^{-2} + 2^{-2} \leq 1 \Rightarrow$  Satisfy Kraft/McMillan inequality

Code B: Not instantaneous  
 $\sum_{i=1}^n r^{-li} = 2^{-1} + 2^{-2} + 2^{-3} + 2^{-2} > 1 \Rightarrow$  NOT Satisfy KMT

Code C: [n] instantaneous  
 $\sum_{i=1}^n r^{-li} = 2^{-1} + 2^{-3} + 2^{-3} + 2^{-3} < 1 \Rightarrow$  Satisfy KMT

b) Given  $S = \{X, Y, Z\}$  &  $P = \{0.6, 0.2, 0.2\}$

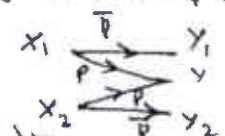
Code for "YXZXY" lies bet<sup>n</sup> 0.70464 to 0.70752

c) Code sym prob

1	A	1/3	---	1/3	---	1/3	---	1/3	---	1/3
00	C	1/3	---	1/3	---	1/3	---	1/3	---	1/3
011	D	1/9	---	1/9	---	1/9	---	1/9	---	1/9
0100	E	1/9	---	1/9	---	1/9	---	1/9	---	1/9
01011	B	1/27	---	1/27	---	1/27	---	1/27	---	1/27
010100	F	1/27	---	1/27	---	1/27	---	1/27	---	1/27
010101	G	1/27	---	1/27	---	1/27	---	1/27	---	1/27

$L = 2.4074 \text{ bit/m.s}$   
 $H(S) = 2.2895 \text{ bit/m.s}$   
 $\eta_c = \frac{H(S)}{L} \times 100 = 95.09\%$

Question Number	Solution	Marks Allocated																					
4 a)	<p style="text-align: right;"><math>p = 3/7</math></p> <ul style="list-style-type: none"> <li>List the source symbols in the decreasing order of prob</li> <li>Compute the sequence <math>\alpha_i</math></li> <li>Determine the smallest integer value <math>l_i</math> so <math>\frac{\alpha_i}{2^{l_i}} \geq \frac{1}{2}</math></li> <li>Expand <math>\alpha_i</math> in binary form upto <math>l_i</math> places</li> <li>Remove the binary point to get the desired code</li> </ul> <p>b)</p> <table border="0" style="width: 100%;"> <thead> <tr> <th>Sym</th> <th>P</th> <th>code</th> </tr> </thead> <tbody> <tr> <td>A</td> <td>0.4</td> <td>00</td> </tr> <tr> <td>B</td> <td>0.2</td> <td>01</td> </tr> <tr> <td>C</td> <td>0.2</td> <td>10</td> </tr> <tr> <td>D</td> <td>0.1</td> <td>110</td> </tr> <tr> <td>E</td> <td>0.08</td> <td>1110</td> </tr> <tr> <td>F</td> <td>0.02</td> <td>1111</td> </tr> </tbody> </table> <p><math>L = 2.3 \text{ bits/ms}, H(S) = 2.194 \text{ bit/ms}</math></p> <p><math>\eta_c = \frac{H(S)}{L} \times 100 = 95.39\%</math></p>	Sym	P	code	A	0.4	00	B	0.2	01	C	0.2	10	D	0.1	110	E	0.08	1110	F	0.02	1111	<p>6M</p> <p>3M</p> <p>1M</p> <p>2M</p>
Sym	P	code																					
A	0.4	00																					
B	0.2	01																					
C	0.2	10																					
D	0.1	110																					
E	0.08	1110																					
F	0.02	1111																					
5 a)	<p>c)</p> <table border="0" style="width: 100%;"> <thead> <tr> <th>code</th> <th>sym</th> <th>prob</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>S<sub>0</sub></td> <td>0.55</td> </tr> <tr> <td>100</td> <td>S<sub>1</sub></td> <td>0.15</td> </tr> <tr> <td>101</td> <td>S<sub>2</sub></td> <td>0.15</td> </tr> <tr> <td>110</td> <td>S<sub>3</sub></td> <td>0.1</td> </tr> <tr> <td>111</td> <td>S<sub>4</sub></td> <td>0.05</td> </tr> </tbody> </table> <p><math>L = 1.9 \text{ bit/ms}, \text{Var}(l_i) = \sum_{i=1}^n (l_i - L)^2 = 0.99</math></p> <p><math>P(x_i, y_j) = P(y_i x_i) P(x_i) = P(x_i y_j) P(y_j)</math></p> <p>Which is obtained by multiply all the elements of 1<sup>st</sup> row of Channel Matrix by <math>P(x_1)</math>, 2<sup>nd</sup> row by <math>P(x_2)</math> ... and n<sup>th</sup> row by <math>P(x_n)</math></p> <p>ie</p> $P(y_i x_i) = \begin{bmatrix} P(y_1 x_1) & P(y_2 x_1) & \dots & P(y_m x_1) \\ P(y_1 x_2) & P(y_2 x_2) & \dots & P(y_m x_2) \\ \vdots & \vdots & \ddots & \vdots \\ P(y_1 x_n) & P(y_2 x_n) & \dots & P(y_m x_n) \end{bmatrix}$	code	sym	prob	0	S <sub>0</sub>	0.55	100	S <sub>1</sub>	0.15	101	S <sub>2</sub>	0.15	110	S <sub>3</sub>	0.1	111	S <sub>4</sub>	0.05	<p>5M</p> <p>3M</p> <p>2M</p>			
code	sym	prob																					
0	S <sub>0</sub>	0.55																					
100	S <sub>1</sub>	0.15																					
101	S <sub>2</sub>	0.15																					
110	S <sub>3</sub>	0.1																					
111	S <sub>4</sub>	0.05																					

Question Number	Solution	Marks Allocated
	<p style="text-align: center;">P-4/7</p> $P(Y_i/x_i) P(x_i) = \begin{bmatrix} P(y_1/x_1)P(x_1) & P(y_2/x_1)P(x_1) & \dots & P(y_m/x_1)P(x_1) \\ P(y_1/x_2)P(x_2) & P(y_2/x_2)P(x_2) & \dots & P(y_m/x_2)P(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ P(y_1/x_n)P(x_n) & P(y_2/x_n)P(x_n) & \dots & P(y_m/x_n)P(x_n) \end{bmatrix}$ <p>Property ① Add column wise of JPM to get the probability of all symbols                  ② Add row wise of JPM to get the prob. of P(x<sub>i</sub>)                  ③ Sum of all elements of JPM = 1</p> <p>b) <math>P(Y/x) = \begin{bmatrix} 0.8 &amp; 0.2 \\ 0.3 &amp; 0.7 \end{bmatrix} \Rightarrow P(X, Y) = \begin{bmatrix} 0.48 &amp; 0.12 \\ 0.12 &amp; 0.28 \end{bmatrix}</math>  <math>P(X) = [0.6, 0.4] \quad P(Y) = [0.6, 0.4]</math>  <math>H(Y) = 0.6 \log_2 \frac{1}{0.6} + 0.4 \log_2 \frac{1}{0.4} = 0.9709 \text{ bit/sym}</math>  <math>H(Y/x) = \sum \sum P(x_i, y_j) \log \frac{1}{P(y_j/x_i)} = 0.7855 \text{ bit/sym}</math>  <math>C(X, Y) = H(Y) - H(Y/x) = 0.1854 \text{ bit/sym}</math>  <math>R_c = \{C(X, Y)\} r_s = 0.1854 \times 1000 = 185.4 \text{ bit/sec}</math></p>	<p>2M</p> <p>2M</p> <p>1M</p> <p>1M</p> <p>2M</p> <p>2M</p>
	<p>c) When ever error occur, sym x<sub>1</sub> will be received and no decision will be made about the information, but an immediate request will be made (i.e. ARQ) through a reverse channel for retransmission of the transmitted signal till a correct signal is received. This ensure 100% correct data recovery.</p> <p></p> <p><math>P(Y/x) = \begin{bmatrix} \bar{p} &amp; p &amp; 0 \\ 0 &amp; p &amp; \bar{p} \end{bmatrix} \Rightarrow P(X, Y) = \begin{bmatrix} \bar{p}w &amp; pw &amp; 0 \\ 0 &amp; p\bar{w} &amp; p\bar{w} \end{bmatrix}</math>  <math>P(x_1=0) = w \text{ \&amp; } P(x_2=1) = \bar{w}, \text{ but } w + \bar{w} = 1</math>  <math>P(Y) = [\bar{p}w \quad p \quad p\bar{w}]</math>  <math>P(X Y) = \begin{bmatrix} 1 &amp; w &amp; 0 \\ 0 &amp; \bar{w} &amp; 1 \end{bmatrix}</math>  <math>H(X Y) = \sum \sum P(x_i, y_j) \log \frac{1}{P(x_i y_j)}</math>  <math>H(X Y) = P H(X)</math>  <math>C(X, Y) = H(X) - H(X Y) = (1-P) H(X) = \bar{p} H(X)</math>  <math>C = \text{Max} \{C(X, Y)\} r_s = \{\bar{p} H(X)_{\text{max}}\} r_s = \bar{p} \log_2^2 \cdot r_s</math>  <math>C = \bar{p} r_s \text{ bits/sec}</math></p>	<p>2M</p> <p>6M</p>



Question Number	Solution	Marks Allocated
6 a)	$I(X, Y) = H(X) - H(X Y) \text{ with Expn}$ <p>Property 4: - <math>I(X, Y) = I(Y, X)</math>  <math>- I(X, Y) \geq 0</math>  <math>I(X, Y) = H(X) - H(X Y)</math>  <math>I(X, Y) = H(Y) - H(Y X)</math>  <math>I(X, Y) = H(X) + H(Y) - H(X, Y)</math></p>	<p>3M 3M 6M</p>
b)	$P(Y X) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$ $\begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0.8 \log_2 0.8 + 0.1 \log_2 0.1 + 0.1 \log_2 0.1 \\ 0.2 \log_2 0.2 + 0.6 \log_2 0.6 + 0.2 \log_2 0.2 \\ 0.2 \log_2 0.2 + 0.2 \log_2 0.2 + 0.6 \log_2 0.6 \end{bmatrix}$ $= \begin{bmatrix} -0.922 \\ -1.271 \\ -1.371 \end{bmatrix}$ <p>Solving above matrix, we get  <math>Q_1 = -0.7723, Q_2 = -1.5207, Q_3 = -1.5207</math></p> $C = \log_2 [2^{Q_1} + 2^{Q_2} + 2^{Q_3}] \times R_s = 718 \text{ bit/sec}$	<p>2M 2M 2M 6M</p>
c)	$C = B \log_2 \left(1 + \frac{S}{N}\right) \text{ bit/sec}$ <p><math>B</math> = Channel BW (Hz), <math>N</math> = Noise power (W) = <math>\eta B</math>  <math>S</math> = Signal power (W)</p>	<p>2M 2M 6M</p>
7 a)	<p>Adv: Improve Data Quality, Reduction in <math>E_b/N_0</math>  Reduction in Transmitted Power</p>	<p>3M</p>
	<p>Disadv: Increased BW, System becomes more Complex</p>	<p>6M</p>
	<p>Method of controlling Error:</p> <ul style="list-style-type: none"> <li>- Forward acting error correction } with Expn</li> <li>- Error detection Method</li> </ul>	<p>3M</p>
b)	$C = [d_1, d_2, d_3, d_4, \quad e_5, e_6, e_7]$ $C = D G = [d_1, d_2, d_3, d_4] \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$ $H = [P^T   I_{n-k}] = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ $G H^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$	<p>2M 2M 2M 6M</p>



Question Number	Solution	Marks Allocated
-----------------	----------	-----------------

$R = 1110101$   
 No of Shift | Cp  
 Shift | Cp  
 1 | 1  
 2 | 0  
 3 | 1  
 4 | 0  
 5 | 1  
 6 | 1  
 7 | 1

Shift	Cp	s0	s1	s2	Commentary
1	1	0	0	0	← Clear the content of Register
2	0	1	0	0	
3	1	1	0	1	
4	0	1	0	0	
5	1	1	1	0	
6	1	1	1	1	
7	1	0	0	1	← Indicate the error

3M  
5M

8 a)

$S = 001 \Rightarrow S(x) = x^2$   
 $C = DG = [d_1 d_2 d_3] \begin{bmatrix} 1 & 0 & 0 & | & 1 & 0 & 1 \\ 0 & 1 & 0 & | & 0 & 1 & 1 \\ 0 & 0 & 1 & | & 1 & 1 & 0 \end{bmatrix}$   
 $d_{min} = 3$

code	wt
0 - 000 000	-3
1 001 110	-3
2 010 011	-3
3 011 101	-4
4 100 101	-3
5 101 011	-4
6 110 110	-4
7 111 000	-3

Error detection =  $d_{min} - 1 = 2$   
 Error Correction =  $\frac{d_{min} - 1}{2} = 1$

3M

6M

b)

$D = 1101 \Rightarrow D(x) = 1 + x + x^3$   
 Non-systematic:  
 $V(x) = D(x)g(x)$   
 $V(x) = 1 + x^2 + x^4$   
 $V = 1010001$

Systematic:  
 $\frac{x^{n-k}}{D(x)} = \frac{x^3 + x^4 + x^6}{1 + x + x^3}$   
 $x^3 + x + 1 \overline{) x^6 + x^4 + x^3}$   
 $\underline{x^6 + x^4 + x^3}$   
 $\quad \quad \quad 0$   
 $\therefore V(x) = x^{n-k} \frac{0}{D(x)} + R(x)$

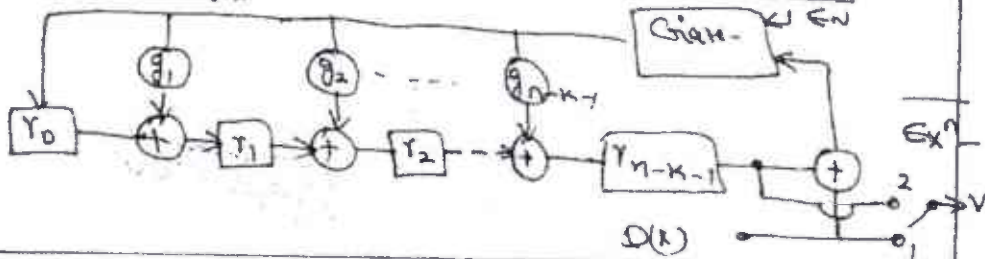
$V(x) = x^3 + x^4 + x^6$   
 $V = 0001101$

$V(x) = x^3 + x^4 + x^6$   
 $V = 0001101$

4M  
2+4=6M

c)

$R(x) = \frac{x^{n-k}}{g(x)} + Q(x)$



3M  
5M

8M

Question Number	Solution	Marks Allocated
-----------------	----------	-----------------

9 a)



(ii) Generator Matrix

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$C = D G = [11101] [G] = [111, 010, 001, 110, 100, 101, 011]$$

Transform domain:

$$c^1(x) = 1 + x^3 + x^4 + x^5 \Rightarrow 1001110$$

$$c^2(x) = 1 + x + x^3 + x^6 \Rightarrow 1101001$$

$$c^3(x) = 1 + x^2 + x^5 + x^6 \Rightarrow 1010011$$

$$\therefore C = [111, 010, 001, 110, 100, 101, 011]$$

BCH codes

10 a)

b) (i)  $g^1 = 1011$   $g^2 = 1111$

$$g^1(x) = 1 + x^2 + x^3, \quad g^2(x) = 1 + x + x^2 + x^3$$

(ii) Time domain,  $L = 5, l = 8$

$$\text{Using } c_i^d = \sum_{k=0}^{L-1} d_{k-i} g_{k+1}^d$$

$$c^1 = 10000001, \quad c^2 = 11011101$$

$$C = [11, 01, 00, 01, 01, 01, 00, 11]$$

Transform domain:

$$c^1(x) = D(x) g^1(x) \Rightarrow 10000001$$

$$c^2(x) = Q(x) g^2(x) = 11011101$$

$$C = [11, 01, 00, 01, 01, 01, 00, 11]$$

b) Golay codes