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Internal Assessment Test 1 – Sept. 2019

Sub:	Energy Engineering				Sub Code:	15ME71	Branch:	Mechanical
Date:	07/09/2019	Duration:	90 mins	Max Marks: 50	Sem / Sec:	7 A & B		OBE

Answer all Questions

		MARKS	CO	RBT
1	Derive an expression for power coefficient for a horizontal axis wind turbine.	[10]	CO4	L2
2	An aero-generator, installed at sea shore generates an output of 1200W at wind speed of 6m/s at one atmosphere pressure and a temperature of 27ºC. What will be the output, if the same aero-generator is installed on the top of a hill where the temperature is 15ºC, pressure is 0.85 atmosphere and wind speed is 8 m/s.	[10]	CO4	L3
3	Following data were measured for a HAWT: Speed of wind = 8 m/s at 1 atmosphere and 20ºC, Diameter of rotor = 120m, Speed of rotor = 40 rpm. Calculate the maximum possible torque produced at the shaft.	[10]	CO4	L3
4	Wind blow with a velocity of 15 m/s at 15ºC and 1 standard atm. pressure. The turbine diameter is 120 m with operating speed 40 rpm at maximum efficiency. Propeller type wind turbine is considered. Calculate the following: (i) Total power density in the wind stream, (ii) The maximum obtainable power density, (iii) A reasonably obtainable power density, (iv) Total power, (v) Maximum axial thrust, (vi) Torque at maximum efficiency Assume R = 0.287 kJ/kgK, η = 35%.	[20]	CO4	L3

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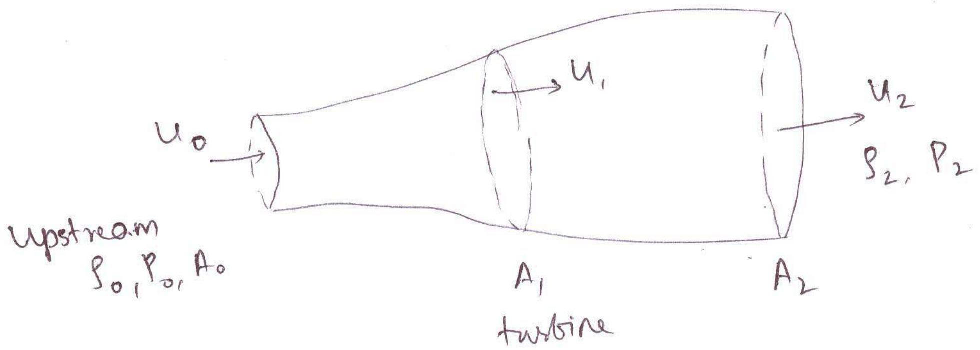
1) <u>Expression for power coefficient</u>



The rotor of the turbine is replaced as disk (actuator disk theory). The control volume for analysis is shown in figure. The control volume here is a streamtube, and hence all the mass entering the control volume upstream convects downstream. ie

$$\dot{m} = \rho A_0 u_0 = \rho A_1 u_1 = \rho A_2 u_2$$

The reduction in momentum of the fluid results in thrust on the turbine. ie

$$F = \dot{m}(u_0 - u_2)$$

The power extracted by the turbine

$$P_T = F u_1 = \dot{m}(u_0 - u_2) u_1 \quad \rightarrow (1)$$

The power extracted is also equal to loss in KE

$$P_W = \frac{1}{2}\dot{m}(u_0^2 - u_2^2) \quad \rightarrow (2)$$

Equation (1) and (2)

$$\dot{m}(u_0 - u_2)u_1 = \frac{1}{2}\dot{m}(u_0^2 - u_2^2)$$

$$\Rightarrow u_1 = \left(\frac{u_0 + u_2}{2}\right) \quad \rightarrow (3)$$

The ~~sta~~ interference factor 'a' is defined as

$$a = \frac{\Delta u}{u_0} = \left(\frac{u_0 - u_1}{u_0}\right)$$

This can be rewritten as

$$a u_0 = u_0 - u_1 \Rightarrow u_1 = u_0(1 - a) \quad \rightarrow (4)$$

Equation (3) and (4)

$$u_1 = \left(\frac{u_0 + u_2}{2}\right) = u_0(1-a)$$

$$u_0 + u_2 = 2u_0(1-a)$$

$$u_0 + u_2 = 2u_0 - 2au_0$$

$$\Rightarrow 2au_0 = (u_0 - u_2)$$

$$\Rightarrow a = \left(\frac{u_0 - u_2}{2u_0}\right) \rightarrow (5)$$

The expression for power can be recast as

$$P_T = \frac{1}{2}\dot{m}(u_0^2 - u_2^2)$$

$$= \frac{1}{2}\rho A_1 u_1 (u_0 - u_2)(u_0 + u_2)$$

$$= \frac{1}{2}\rho A_1 u_0(1-a)(2au_0)(2u_1)$$

$$= \frac{1}{2}\rho A_1 u_0 (1-a) 2au_0 (2u_0)(1-a)$$

$$P_T = 4a(1-a)^2\left(\frac{1}{2}\rho A_1 u_0^3\right)$$

The power in wind is $P_0 = \frac{1}{2}\dot{m}u_0^2$

$$P_0 = \frac{1}{2}(\rho A_1 u_0)u_0^2$$

$$P_0 = \frac{1}{2}\rho A_1 u_0^3$$

power coefficient $C_p = \left(\frac{P_T}{P_0}\right) = \dfrac{4a(1-a)^2\left(\frac{1}{2}\rho A_1 u_0^3\right)}{\left(\frac{1}{2}\rho A_1 u_0^3\right)}$

$$\boxed{C_p = 4a(1-a)^2}$$

2) Given for case 1 : $P = 1200W$ $U_0 = 6\ m/s$

$p = 1atm = 1 \times 101.325 \times 10^3\ N/m^2$

$T = 27°C = 300K$

$P = C_p P_0$ $\rho = \frac{p}{RT}$

$1200 = C_p\left(\frac{1}{2}\rho A U_0^3\right)$

$= (C_p A)\left(\frac{1}{2} \times \frac{101.325 \times 10^3}{R \times 300} \times 6^3\right) \rightarrow ①$

For case 2: $P = ?$ $U_0 = 8m/s$

$p = 0.85\ atm = 0.85 \times 101.325 \times 10^3\ N/m^2$

$T = 15°C = 288K$

$P = C_p P_0$

$? = C_p\left(\frac{1}{2} \times \frac{0.85 \times 101.325 \times 10^3 \times A}{R \times 288} \times 8^3\right) \rightarrow ②$

② / ① gives

$\frac{?}{1200} = \frac{1 \times 6^3}{300} \times \frac{288}{0.85 \times 8^3} = \underline{\underline{2518\ W}}$

It is ~~assumped~~ assumed that the turbine operates at the same power coefficient under both conditions.

3) Given. $U_0 = 8\,m/s$

$D = 120\,m$

$N = 40\,rpm$

Density $\rho = \frac{P}{RT}$ $\quad P = 101.325 \times 10^3\,N/m^2$

$R = 287\ J/kg\,K$

$T = 20^\circ C = 293K$

$= \frac{101.325 \times 10^3}{287 \times 288}$

$= \underline{1.204\,kg/m^3}$

max Power produced by turbine

$P_T = C_p P_0$

at max power output $C_p = \left(\frac{16}{27}\right)$

$P_T = \left(\frac{16}{27}\right) P_0$

$= \left(\frac{16}{27}\right)\left(\frac{1}{2}\rho A U_0^3\right)$

$= \left(\frac{16}{27}\right)\left(\frac{1}{2} \times 1.202 \times \frac{\pi \times 120^2}{4} \times 8^3\right)$

$= 2.065 \times 10^6\,W$

$= \underline{2.065\,MW}$

$P = \frac{2\pi N T}{60}$, where T is the torque

$T = \frac{60\,P}{2\pi N}$

$= \frac{60 \times 2.065 \times 10^6}{2 \times \pi \times 40}$

$= 0.492 \times 10^6\,Nm$

$T = \underline{492\,kNm}$

4) Given $u_0 = 15\, m/s$

$T = 15°C = 288K$

$p = 1 atm = 101.325 \times 10^3\, N/m^2$

$D = 120 m$

$N = 40 rpm$ $\qquad \rho = \frac{p}{RT}$

Power density in wind stream

$$\frac{P_0}{A} = \left(\frac{1}{2}\rho u_0^3\right)$$

$$\rho = \frac{p}{RT} = \frac{101.325 \times 10^3}{287 \times 288} = \underline{\underline{1.226\, kg/m^3}}$$

(i) Power density in wind stream

$$\left(\frac{P_0}{A}\right) = \frac{1}{2}\rho u_0^3$$
$$= \frac{1}{2} \times 1.226 \times 15^3$$
$$= \underline{\underline{2068\, W/m^2}}$$

(ii) Max obtainable power density

$$= \left(\frac{P_{max}}{A}\right) = (C_p)_{max}\left(\frac{P_0}{A}\right)$$
$$= \left(\frac{16}{27}\right)\left(\frac{P_0}{A}\right)$$
$$= \frac{16}{27} \times 2068 = \underline{\underline{1225\, W/m^2}}$$

(iii) Reasonably obtainable power density

$$= (C_p)_{reasonable} \times \left(\frac{P_0}{A}\right)$$
$$= 0.35 \times 2068$$
$$= \underline{\underline{723.8\, W/m^2}}$$

(iv) Total power

Assumin C_p = ~~Optimum~~ 0.35

$$P_{max} = A \times \left(\frac{P_{max}}{A}\right)$$

$$= \pi \times \frac{120^2}{4} \times \cancel{1225}\ 723.8$$

$$= 8.185 \times 10^6\ W$$

$$= \underline{\underline{8.185\ MW}}$$

(v) Max axial thrust

$$F = \frac{1}{2} \rho A U_0^2$$

$$= \frac{1}{2} \times 1.226 \times \pi \times \frac{120^2}{4} \times 15^2$$

$$= \underline{\underline{1.559 \times 10^6\ N}}$$

(vi) power at max efficiency

$$P_{max} = A\left(\frac{P_{max}}{A}\right) = 1225 \times \pi \times \frac{120^2}{4}$$

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{60\ P}{2\pi N}$$

$$= \frac{60 \times 1225 \times \pi \times \frac{120^2}{4}}{2\pi \times 40}$$

$$= \underline{\underline{3.307\ kNm}}$$