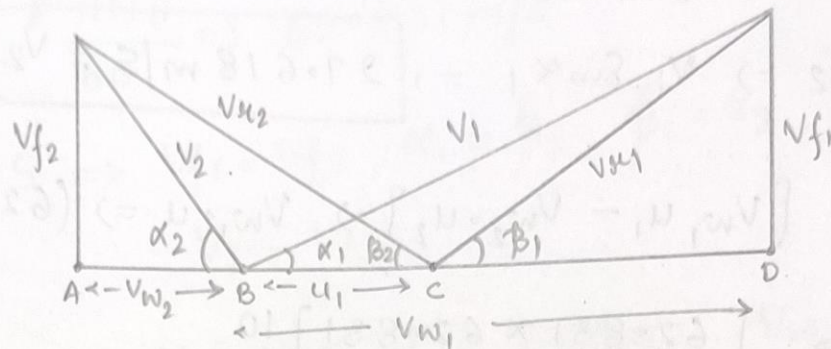


TURBOMACHINES

IAT 2-OCTOBER 2019 SOLUTIONS



We know Euler Equation

$$u(Vw_1 + Vw_2)$$

$$AD \Rightarrow Vw_1 + Vw_2 \Rightarrow AC + CD \quad \text{--- (1)}$$

$$\Rightarrow AC \Rightarrow \cos \beta_2 = \frac{AC}{Vw_2} \Rightarrow AC \Rightarrow Vw_2 \cos \beta_2 \quad \text{--- (2)}$$

$$\Rightarrow CD \Rightarrow \cos \beta_1 = \frac{CD}{Vw_1} \Rightarrow CD \Rightarrow Vw_1 \cos \beta_1 \quad \text{--- (3)}$$

Putting eq (2) and (3) in (1).

We have.

$$Vw_1 + Vw_2 \Rightarrow Vw_1 \cos \beta_1 + Vw_2 \cos \beta_2$$

$$\Rightarrow Vw_1 \cos \beta_1 \left[1 + \frac{Vw_2}{Vw_1} \cdot \frac{\cos \beta_2}{\cos \beta_1} \right]$$

$$\text{We know } \frac{Vw_2}{Vw_1} = C_b \quad \frac{\cos \beta_2}{\cos \beta_1} = K = \text{constant}$$

$$\Rightarrow Vw_1 \cos \beta_1 (1 + C_b \cdot K) \quad \text{--- (4)}$$

Now $\cos \alpha_1 \Rightarrow \frac{u + cD}{v_1} \Rightarrow C_D \Rightarrow v_1 \cos \alpha_1 - u$.

$v_{x1} \cos \beta_1 = v_1 \cos \alpha_1 - u$ — (5)

Putting eq (5) in (4)

we have.

$\Rightarrow (v_1 \cos \alpha_1 - u) \cdot (1 + C_b \cdot K)$

Putting the above eqn in Euler's eqn we have.

$\Rightarrow u (v_1 \cos \alpha_1 - u) (1 + C_b \cdot K)$

$\Rightarrow v_1 u (\cos \alpha_1 - u/v_1) (1 + C_b \cdot K)$ — (6)

Putting eq (6) in w.D eq we have.

w.D $\Rightarrow \frac{v_1 u (\cos \alpha_1 - \phi) (1 + C_b \cdot K)}{v_2 v_1^2}$ ($u/v_1 = \phi$)

w.D $\Rightarrow 2 \cdot \frac{u}{v} \cdot (\cos \alpha_1 - \phi) (1 + C_b \cdot K)$

w.D $\Rightarrow 2\phi \cos \alpha_1 - 2\phi^2 (\cos \alpha_1 - \phi) (1 + C_b \cdot K)$

for maximum efficiency $\Rightarrow \frac{dW_D}{d\phi} = 0$

$d [2\phi (\cos \alpha_1 - \phi) (1 + C_b \cdot K)] = 0$

$\frac{d}{d\phi} [2\phi (\cos \alpha_1 - \phi) (1 + C_b \cdot K)] = 0$

$\phi(-1) + (\cos \alpha_1 - \phi) = 0$
 $\phi = \cos \alpha_1 / 2$

Putting back in equation we have.

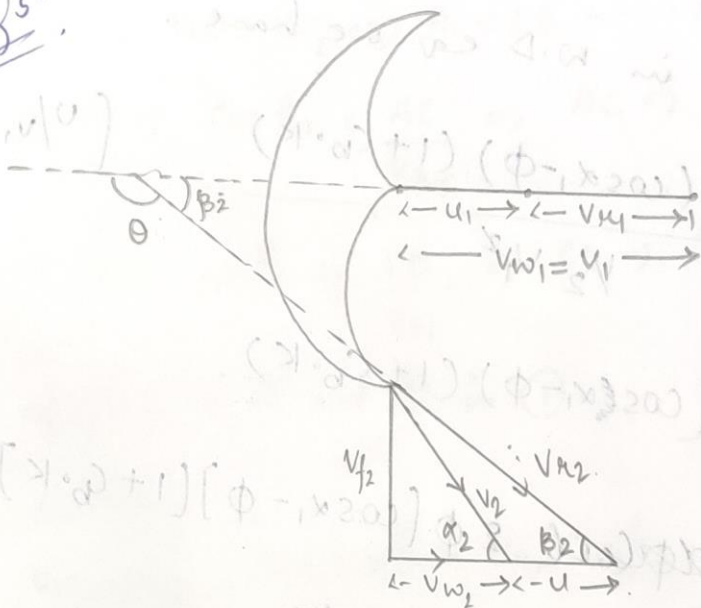
$$\frac{1}{2} (\cos \alpha_1 / \phi) (\cos \alpha_1 - \cos \alpha_1 / 2) (1 + C_b \cdot k)$$

$$\Rightarrow \frac{\cos \alpha_1^2}{2} (1 + C_b \cdot k)$$

Now if $C_b = 1$ and $k = 1$, then:

$$\eta_{\max} = \cos^2 \alpha_1$$

Q30



from the outlet velocity Δ , we have.

$$\cos \beta_2 \Rightarrow \frac{V_{w2} + u}{V_{x2}} \Rightarrow V_{x2} \cos \beta_2 - u = V_{w2}$$

$$\text{Euler equation } \Rightarrow E \Rightarrow u(V_{w1} \pm V_{w2})$$

$$\text{Assuming } V_{x1} = V_{x2}$$

$$V_{w_2} = V_{x_1} \cos \beta_2$$

$$\Rightarrow V_{x_1} = V_1 - u$$

$$\Rightarrow V_{w_2} = (V_1 - u) \cos \beta_2$$

$$\Rightarrow u [V_1 + (V_1 - u) \cos \beta_2]$$

Now we know bucket coefficient = $C_b = V_{x_2}/V_{x_1}$

Now to find out efficiency of man condition we have.

$$\eta \Rightarrow \frac{u [V_1 + (V_1 - u) \cos \beta_2]}{\frac{1}{2} V_1^2}$$

$$\frac{d\eta}{du} \Rightarrow 0$$

Solving the above equation we have.

$$\Rightarrow 2u = V \quad \text{or} \quad \frac{1}{2} V = u \Rightarrow \boxed{u = 0.5V}$$

Hence Bucket Speed is half of jet velocity.

Q. $D = 1.2 \text{ m}$ $N = 3000 \text{ rpm}$ $\alpha = 18^\circ$ $\phi = 0.42$.

$\frac{V_{x_2}}{V_{x_1}} = 0.9$ $\beta_2 = \beta_1 - 3^\circ$ $\dot{m} = 5 \text{ kg/s}$.

$$u = \frac{\pi D N}{60} \Rightarrow \frac{\pi \times 1.2 \times 3000}{60} \Rightarrow 188.49 \text{ m/s}$$

$$u/V_1 = \phi \quad V_1 = u/\phi \Rightarrow 188.49/0.42 = 448.79 \text{ m/s}$$

$$V_{x2} = 0.9 \times V_{x1} \Rightarrow V_{x2} = 0.9 \times 300 \Rightarrow 270 \text{ m/s}$$

① Velocity of whirl $\Rightarrow \Delta V_w \Rightarrow [V_{w1} \pm V_{w2}]$
 $\Rightarrow 400 \text{ m/s}, 395 \text{ m/s}$

② Axial thrust $\Rightarrow \dot{m} [V_{f1} - V_{f2}]$
 $\Rightarrow 5 [215 - 170] = [5] [25]$
 $\Rightarrow 125 \text{ N}$

③ Blade angles $\Rightarrow \beta_1$

④ Axial thrust $\Rightarrow \dot{m} (V_{f1} - V_{f2})$
 $\Rightarrow 5 [215 - 175] \Rightarrow \underline{\underline{200 \text{ N}}}$

⑤ Blade angles $\beta_1 = 46^\circ$
 $\beta_2 = \beta_1 - 3^\circ$
 $\beta_2 = 46 - 3 = 43^\circ$

⑥ Power Developed

$$\Rightarrow \dot{m} u (\Delta V_w)$$

$$\Rightarrow \dot{m} \times u \times \Delta V_w \Rightarrow 5 \times 188.49 \times 395$$

$$\Rightarrow 372\,267.75 \text{ W}$$

$$\Rightarrow 372.267 \text{ kW}$$

Q6. $H = 70 \text{ m}$, $N = 600 \text{ RPM}$, $S.P = 370 \text{ kW}$, $O.E = 0.80$.
 $H.E = 0.95$, $\psi = 0.25$, $B_1/D_1 = 0.1$, $D_2 = 2D_1$

$t = 0.1 \text{ PD}$. $V_{f1} = V_{f2} = V_f$ $\alpha_2 = 90^\circ$ $V_{w2} = 0$.

$P(H) \eta_o = \frac{P}{\rho \cdot g \cdot Q \cdot H} \Rightarrow 0.8 = \frac{370 \times 10^3}{1000 \times 9.81 \times Q \times 70}$

$\Rightarrow Q \Rightarrow 0.6735 \text{ m}^3/\text{s}$

$\eta_h = \frac{\rho g Q H (V_{w1} u_1 \pm V_{w2} u_2)}{\rho g Q H}$ $0.95 \times 9.81 \times 70 = V_{w1} u_1$

$\Rightarrow V_{w1} u_1 = 652.365$

from inlet Δ we have.

$\tan \alpha_1 = \frac{V_{f1}}{V_{w1}}$

$V_{f1} = \psi \sqrt{2gh}$

$V_{f1} = 0.25 \sqrt{2 \times 9.81 \times 70}$

$V_{f1} = 9.26 = V_{f2}$

$Q = A_{f1} \times V_{f1}$

$Q = \frac{\pi}{4} (D_1)^2 \pi C \cdot D_1 \cdot B_1 \times V_{f1}$

$Q \Rightarrow \pi (1) D_1 (0.1 D_1) 9.26$

$0.6735 \Rightarrow \pi (0.1 D_1^2) (9.26)$

$D_1 = 0.481 \text{ m}$

$D_2 = 0.9623 \text{ m}$

$u_1 \Rightarrow \frac{\pi D_1 N}{60}$

$u_1 \Rightarrow 15.11 \text{ m/s}$

$u_2 = \frac{\pi D_2 N}{60}$

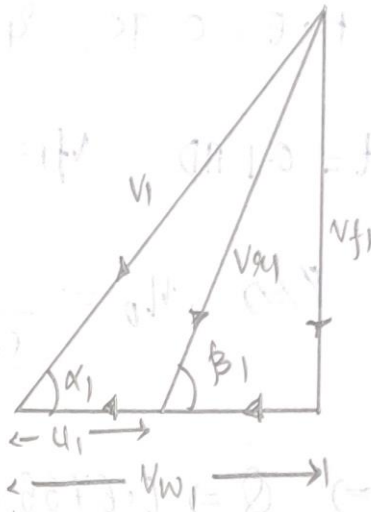
$u_2 \Rightarrow 30.23 \text{ m/s}$

$$V_{w1} = \frac{652 \cdot 365}{43.1743} \Rightarrow 43.1743$$

$$V_{w1} > u_1$$

$$\tan \alpha_1 = \frac{V_{f1}}{V_{w1}}$$

$$\alpha_1 = \tan^{-1} \frac{9.26}{43.174}$$

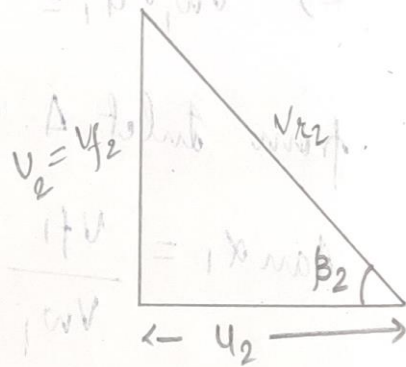


$$\alpha_1 = 12.105^\circ$$

$$\tan \beta_1 \Rightarrow \frac{V_{f1}}{V_{w1} - u_1} \Rightarrow \frac{9.26}{43.174 - 15.11} = 18.26^\circ$$

$$\tan \beta_2 \Rightarrow \frac{V_{f2}}{u_2}$$

$$\beta_2 \Rightarrow \tan^{-1} \frac{9.26}{30.23}$$

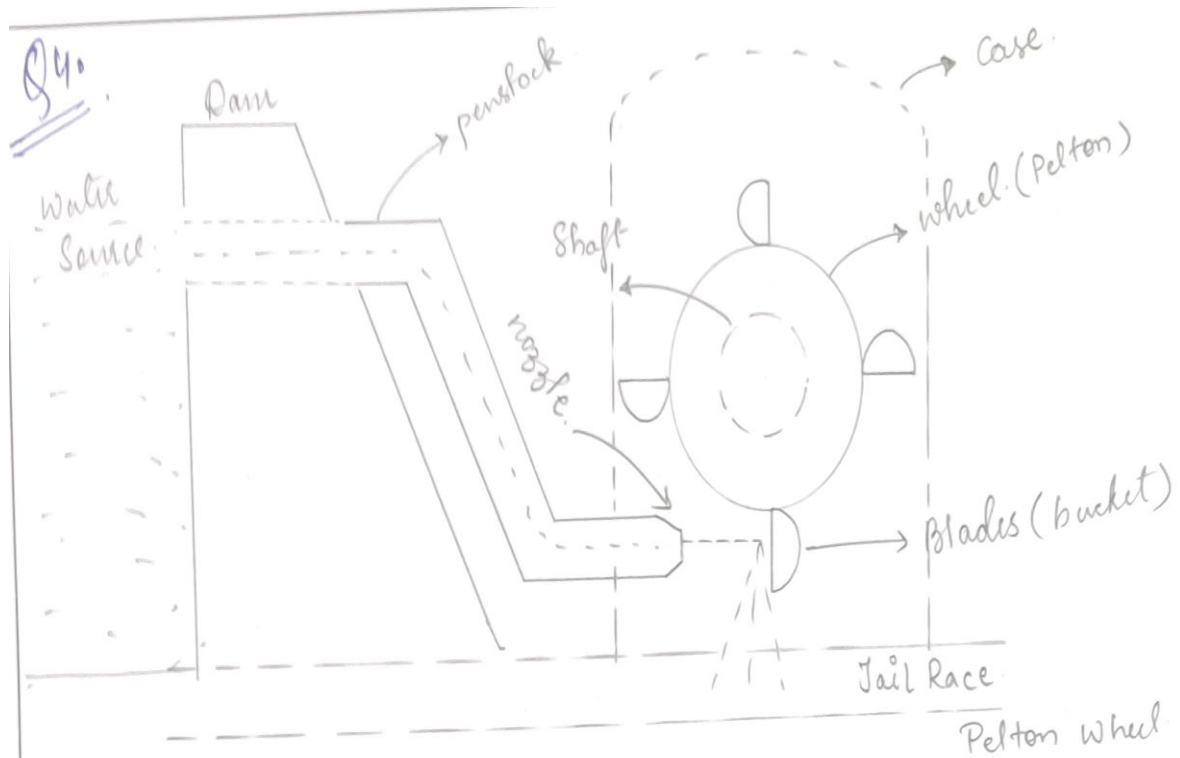


$$\beta_2 = 17.03^\circ$$

width of the wheel $\Rightarrow t \Rightarrow 0.171$

$$t \Rightarrow 0.1(17 \times 0.481)$$

$$t \Rightarrow 0.15 \text{ m}$$



In the pelton wheel mechanism the water is provide from a water source near the dam from a certain height. The water is transported with the help of penstock. This penstock carries the water at high velocity and low pressure.

This high velocity low pressure water hits the blades of the pelton wheel (buckets) which cause the wheel to rotate. Thus the rotation of the wheel causes the movement of generator shaft which in turn produces electricity.