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Internal Assesment Test – 2

Sub: Mechanics of Materials				Code: 18ME32
Date: 15/10/2019	Duration: 90 mins	Max Marks: 50	Sem: 3	Branch (sections): ME (A,B)
Answer any two questions from **part A** and one question from **part B**. Good luck!				

PART A	Marks	OBE CO	OBE RBT
1 Draw the shear force diagram and bending moment diagram for the beam shown in fig. 1.	[20]	CO4	L3
2 Draw the shear force diagram and bending moment diagram for the beam shown in fig. 2 indicating the principal values.	[20]	CO4	L3
3 An element is subjected to stress conditions as shown in fig. 3. Find: i) Normal stress, tangential stress and resultant stress acting on the inclined plane. ii) Angle of obliquity. iii) Maximum and minimum normal stress and their locations. iv) Maximum and minimum shear stress and their locations. v) Verify above answers by Mohr's circle method.	[10]	CO2	L3

1 Draw the shear force diagram and bending moment diagram for the beam shown in fig. 1. [20]

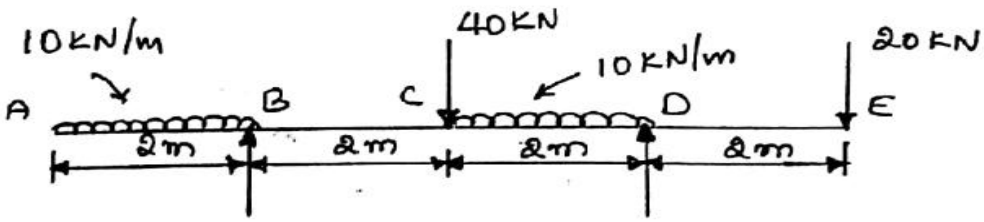


Fig 1

2 Draw the shear force diagram and bending moment diagram for the beam shown in fig. 2 indicating the principal values. [20]

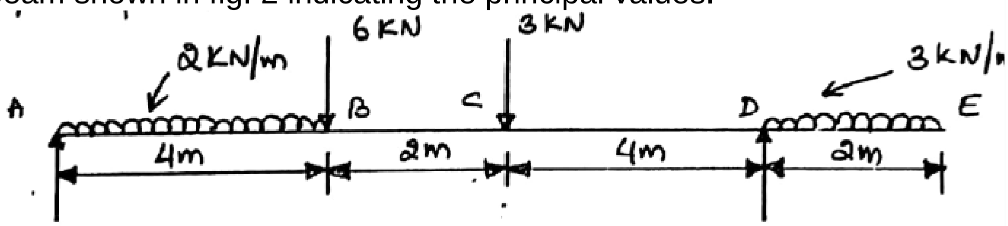


Fig 2

3 An element is subjected to stress conditions as shown in fig. 3. Find: [10]

i) Normal stress, tangential stress and resultant stress acting on the inclined plane.
ii) Angle of obliquity.
iii) Maximum and minimum normal stress and their locations.
iv) Maximum and minimum shear stress and their locations.
v) Verify above answers by Mohr's circle method.

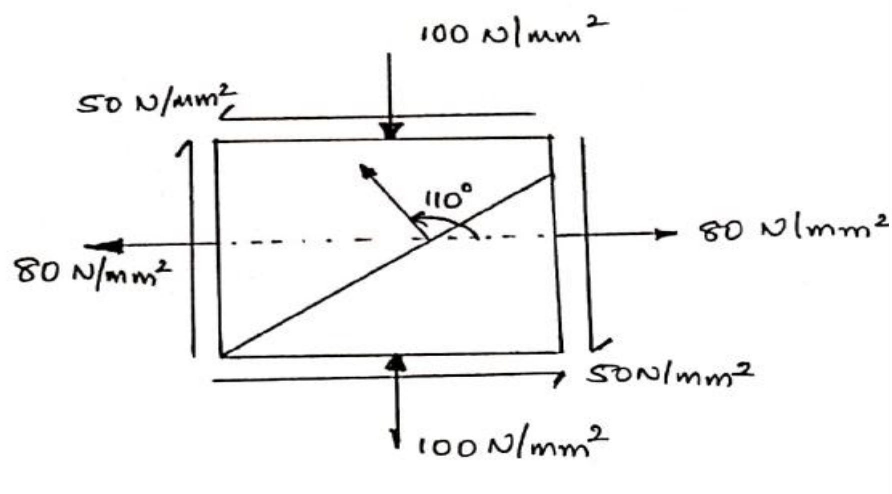


Fig 3

PART B

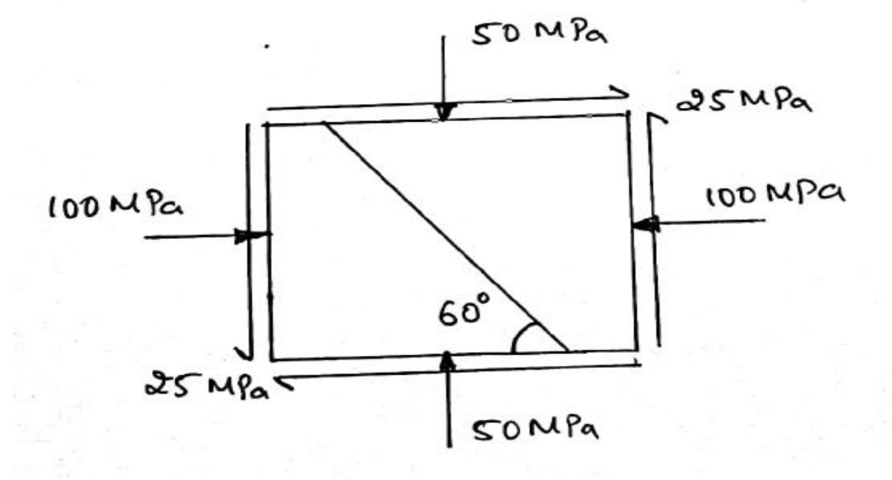
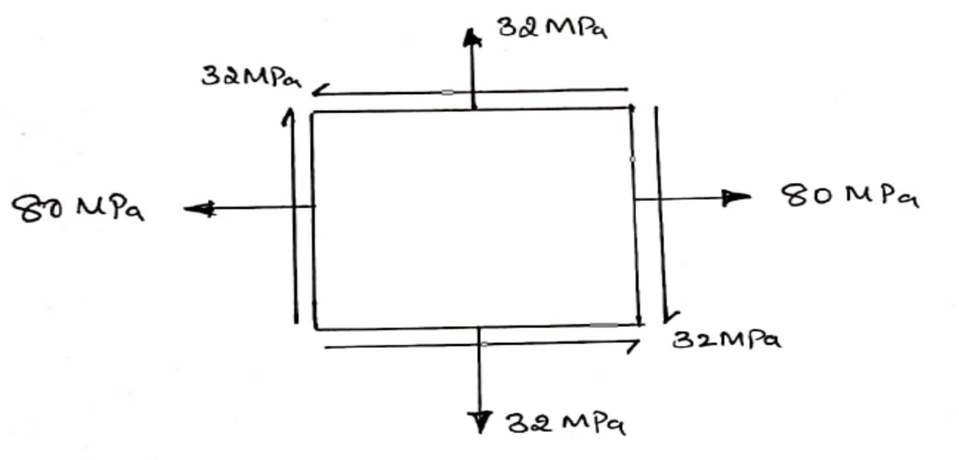
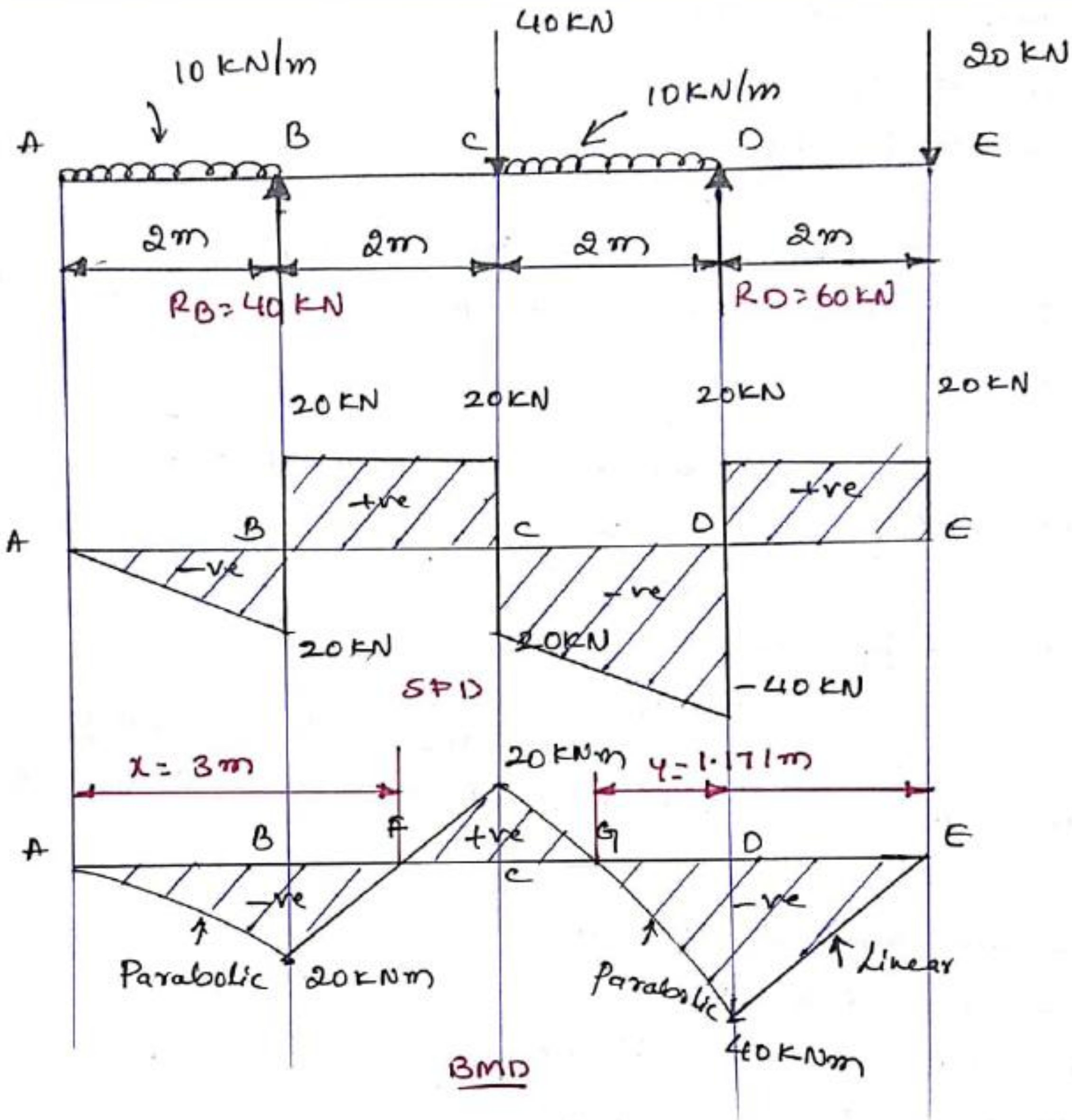
4	An element is subjected to stress conditions as shown in fig. 4. Using Mohr's circle method find: i) Normal stress, tangential stress and resultant stress acting on the inclined plane. ii) Maximum and minimum principal stress and their locations. iii) Maximum and minimum shear stress and their locations.  **Fig 4**	[10]	CO2	L3
5	An element is subjected to stress conditions as shown in fig. 5. Find analytically: i) Maximum and minimum normal stress and their locations. ii) Maximum and minimum shear stress and their locations. iii) Normal stress acting on shear stress plane. 	[10]	CO2	L3

Fig 5

CI **CCI** **HOD**

Q. NO 1



To find reaction at supports (R_B & R_D)

Take moments about B

$$R_D \times 4 - 40 \times 2 - (10 \times 2)\left(\tfrac{2}{2} + 2\right) - 20 \times 6 + (10 \times 2)(2/2) = 0$$

$$R_D \times 4 + (10 \times 2)(2/2) = 40 \times 2 + (10 \times 2)(2/2 + 2) + 20 \times 6$$

$$R_D \times 4 = 80 + 60 + 120 - 20$$

$$\therefore R_D = \underline{60\,KN}$$

Total load = $R_B + R_D$

$$R_B + 60 = (10 \times 2) + 40 + (10 \times 2) + (20)$$

$$\therefore R_B = \underline{40\,KN}$$

Shear force diagram

SF at A, $F_A = 0$

SF at B, $F_B = -10 \times 2 = \underline{-20 KN}$

SF at B, $F_B = -20 + 40 = \underline{+20 KN}$

[Sudden variation due to R_B]

SF at C, $F_C = +20 KN$ (Shear force remains constant between B and C)

SF at C $F_C = +20 - \underline{40}$

$= \underline{-20 KN}$

[Sudden variation due to point load]

SF at D, $F_D = -20 - \underline{10 \times 2}$

$= \underline{-40 KN}$

SF at D, $F_D = -40 + 60$

$= \underline{+20 KN}$

[Sudden variation due to Point Load]

Shear force remains constant between D and C

SF at C, $F_C = +20 KN$

Bending moment diagram

Bm at A, $M_A = 0$

Bm at B, $M_B = -(10 \times 2)(2/2) = \underline{-20 KNm}$

Bm at C, $M_C = -(10 \times 2)(2/2 + 2) + R_B \times 2 = -60 + 80$

$= \underline{+20 KNm}$

Bm at D, $M_D = -(10 \times 2)(2/2 + 4) + R_B \times 4 - 40 \times 2 - (10 \times 2)(2/2)$

$= -100 + 160 - 80 - 20$

$= \underline{-40 KNm}$

Bm at E, $M_E = -(10 \times 2)(2/2 + 6) + R_B \times 6 - 40 \times 4 - (10 \times 2)(2/2 + 2) + R_E \times 2$

$= -140 + 240 - 160 - 60 + 120$

$= 0$

Point of contraflexure

Bm at G, $M_G = -20(y+2) + R_D y - (10y)(y/2)$

$$0 = -5y^2 + 40y - 40$$

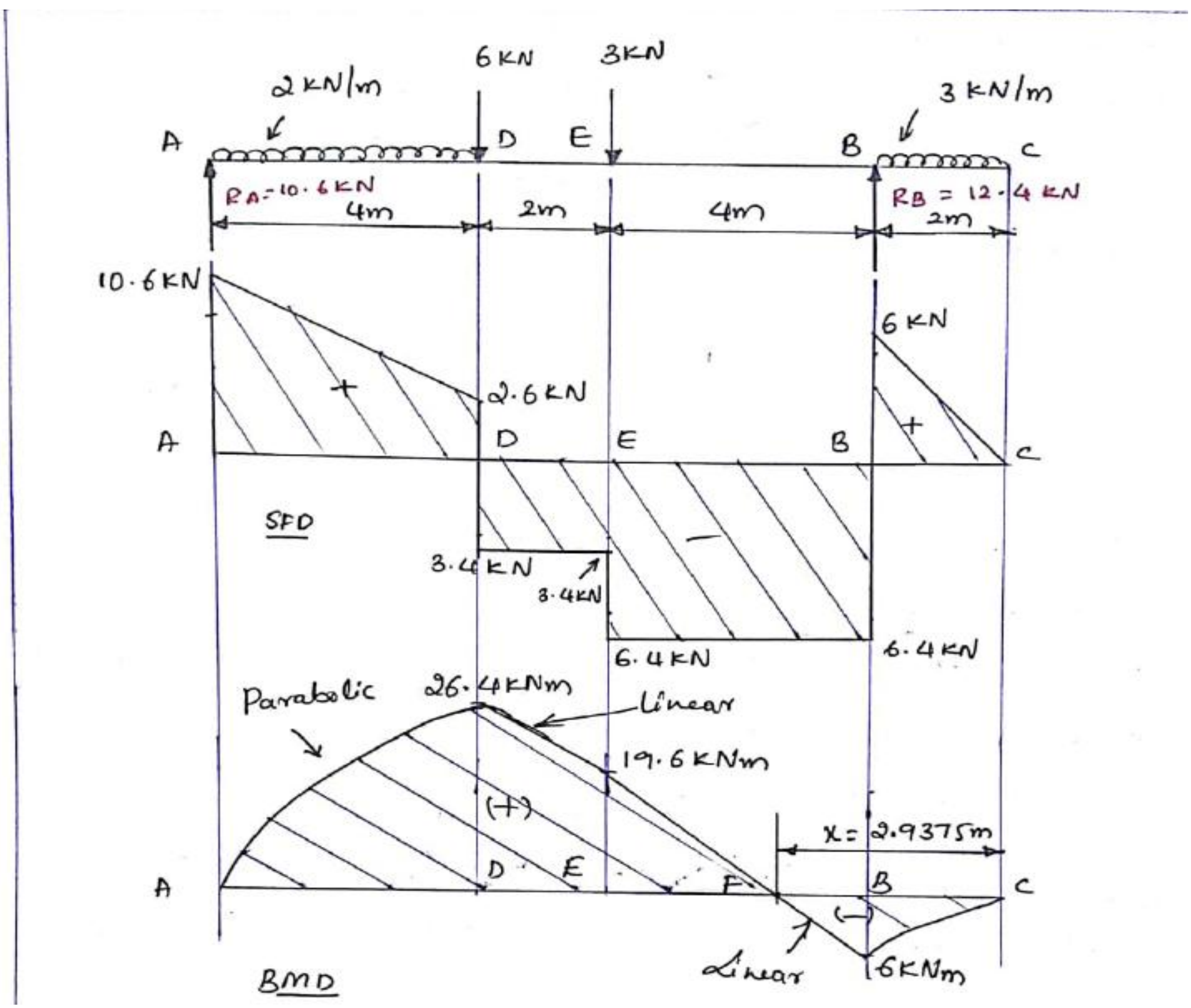
$$5y^2 - 40y + 40 = 0$$

(or) $y^2 - 8y + 8 = 0$

$$= \frac{+8 \pm \sqrt{8^2 - 4 \times 8}}{2 \times 1}$$

$\therefore\ y = \underline{1.171m}$

Q. NO 2:



To find Reactions at supports (R_A & R_B).

Take moments about A

$$R_B \times 10 = (2\times4)(4/2) + 6\times4 + 3\times6 + (3\times2)(2/2+10)$$

$$\therefore R_B = 12.4\,KN \text{ (Ans)}$$

Total load = $R_A + R_B$

$$R_A + 12.4 = (2\times4) + 6 + 3 + (3\times2)$$

$$\therefore R_A = 10.6\,KN \text{ (Ans)}$$

Shear force diagram (SFD)

S.F at A, $F_A = +R_A = +10.6\,KN$

S.F at D, $F_B = R_A - 2\times4 = 2.6\,KN$

S.F at D, $F_D = 2.6 - 6 = -3.4\,KN$

[Sudden variation due to Point load at D]

Shear force remains constant between D and E

S.F at E, $F_E = -3.4\,KN$

S.F at E, $F_E = -3.4 - 3 = -6.4\,KN$

[Sudden variation due to Point load at E]

Shear force remains constant between E and B

S.F at B, $F_B = -6.4\,KN$

S.F at B, $F_B = -6.4 + 12.4 = 6\,KN$

[Sudden variation due to R_B]

S.F at C, $F_C = 6 - 3\times2 = 0$

Bending moment diagram (BMD)

B.m at A, $M_A = 0$

B.m at B, $M_D = R_A \times 4 - (2\times4)(4/2) = +26.4\ kNm$

B.m at E, $M_E = R_A\times6 - (2\times4)(4/2+2) - 6\times2 = +19.6\ kNm$

Bm at B, $M_B = R_A\times10 - (2\times4)(\frac{4}{2}+6) - 6\times6 - 3\times4 = -6\ kNm$

B.M at c, $M_c = 0$

$R_A\times12 - (2\times4)(4/2+8) - 6\times8 - 3\times6 + R_B\times2 - (3\times2)(2/2) = 0$

Point of contraflexure:

Bending moment at F, $M_F = -(3\times2)(x-2/2) + R_B(x-2)$

$$0 = -6x + 6 + 12.4x - 24.8$$

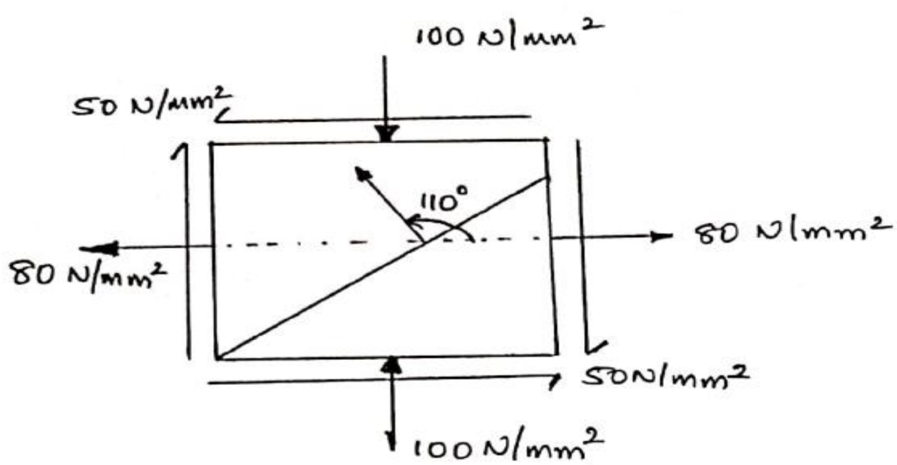
$$0 = +6.4x - 18.8$$

$$6.4x = 18.8$$

$$\therefore x = \frac{18.8}{6.4}$$

$$\therefore x = 2.9375\ m$$

Q. NO 3



Q.3 Data: $\sigma_x = 80\ N/mm^2$; $\sigma_y = -100\ N/mm^2$; $\tau_{xy} = -50\ N/mm^2$

$\theta = 110°$ or $-70°$ (CW)

(1)

Normal stress acting on inclined plane

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{80-100}{2} + \left(\frac{80+100}{2}\right)\cos[2(-70°)] + (-50)\sin[2(-70°)]$$

$$\therefore \sigma_n = -46.8\ N/mm^2 \text{ (Compressive)}$$

Tangential or shear stress acting on the inclined plane

$$\sigma_t = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{80+100}{2}\right)\sin[2(-70°)] + (-50)\cos[2(-70°)]$$

$$\therefore \sigma_t = +96.15\ N/mm^2$$

Resultant stress: $\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$

$$= \sqrt{(-46.8)^2 + (96.15)^2}$$

$$\therefore \sigma_R = 106.935\ N/mm^2$$

(2) Angle of obliquity

$$\phi = \tan^{-1}\left(\frac{\sigma_t}{\sigma_n}\right) = \tan^{-1}$$

$$\therefore \phi = \tan^{-1}\left(\frac{96.15}{-46.8}\right)$$

$$\therefore \phi = -64.046° \text{ (CW)}$$

(3) Max. normal stress or Major principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80-100}{2} + \sqrt{\left(\frac{80+100}{2}\right)^2 + (-50)^2}$$

$$= -10 + 102.956$$

$$\therefore \sigma_1 = \underline{92.956 \text{ N/mm}^2} \quad \text{(Tensile)}$$

Min. normal stress or Minor principal stress

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80-100}{2} - \sqrt{\left(\frac{80+100}{2}\right)^2 + (-50)^2}$$

$$= -10 - 102.956$$

$$\therefore \sigma_2 = \underline{-112.956 \text{ N/mm}^2} \quad \text{(Compressive)}$$

Location of principal planes:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \; ; \quad \theta = \frac{1}{2}\tan^{-1}\frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{1}{2}\tan^{-1}\frac{2(-50)}{80+100}$$

$$\therefore \theta = \underline{-14.5273^\circ} = \theta_1$$

$$\theta_2 = \theta_1 + 90^\circ = \underline{-14.5273^\circ} + 90^\circ = \underline{75.4727^\circ}$$

(4) Max and min shear stress

$$\tau_{max} \text{ or } \tau_{min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm \sqrt{\left(\frac{80+100}{2}\right)^2 + (-50)^2}$$

$$= \pm 102.956 \text{ N/mm}^2$$

(or) $$\tau_{max} \text{ or } \tau_{min} = \pm \frac{\sigma_1 - \sigma_2}{2} = \frac{92.956 + 112.956}{2}$$

$$= \pm \underline{102.956 \text{ N/mm}^2}$$

Location of shear stress planes

$\theta_1' = \theta_1 + 45° = -14.5273 + 45° = \underline{30.4727}$

$\theta_2' = \theta_1 + 135° = -14.5273 + 135° = \underline{120.4727°}$

From Mohr's circle:

Principal stresses: $\sigma_1 = OP \times \text{Scale} = 4.7 \times 20 = \underline{94 \text{ N/mm}^2}$

$\sigma_2 = OQ \times \text{Scale} = 5.7 \times 20 = \underline{114 \text{ N/mm}^2}$

Location of principal planes: $2\theta_1 = -29°$; $\theta_1 = \underline{-14.5°}$

$2\theta_2 = 2\theta_1 + 180°$; $\theta_2 = \underline{75.5°}$

Shear stresses: $\tau_{max} = GC \times \text{Scale} = 5.2 \times 20 = +104 \text{ N/mm}^2$

$\tau_{min} = CH \times \text{Scale} = 5.2 \times 20 = \underline{-104 \text{ N/mm}^2}$

Location of shear stress planes: $2\theta_1' = 2\theta_1 + 90°$; $\theta_1' = \underline{30.5°}$

$2\theta_2' = 2\theta_1 + 270°$; $\theta_2' = \underline{120.5°}$

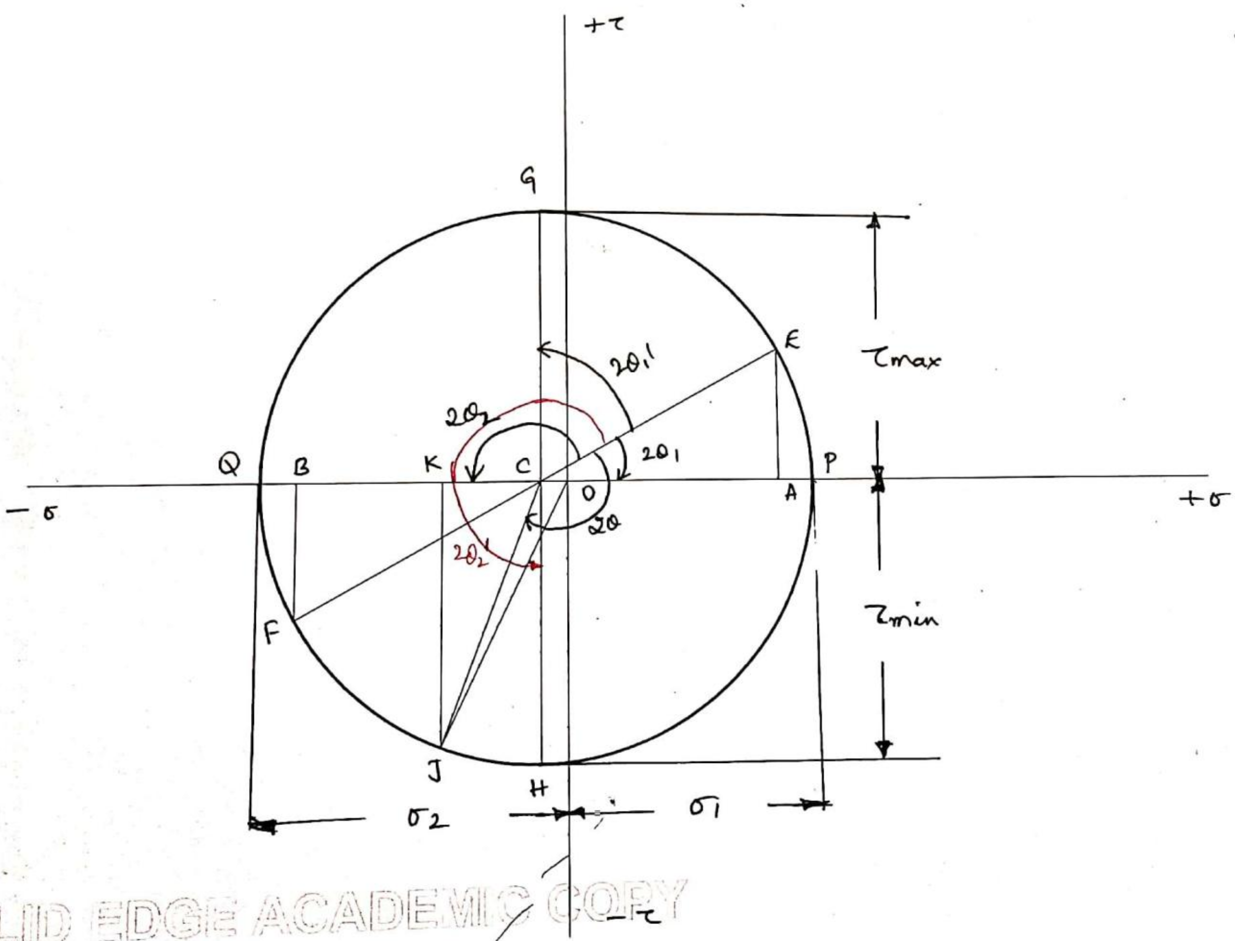
Normal stress on inclined plane: $\sigma_n = OK \times \text{Scale}$

$= 2.5 \times 20 = -50 \text{ N/mm}^2$

Tangential stress: $\sigma_t = JK \times \text{Scale} = -4.8 \times 20 = \underline{-96 \text{ N/mm}^2}$

Resultant stress: $\sigma_R = OJ \times \text{Scale} = 5.4 \times 20 = \underline{108 \text{ N/mm}^2}$

Scale : 20 N/mm² = 1cm



Q. 4. Data: $\sigma_x = -100\ N/mm^2$; $\sigma_y = -50\ N/mm^2$, $\tau_{xy} = +25\ N/mm^2$

$\theta = 30^\circ$ (CCW).

From Mohr's circle:

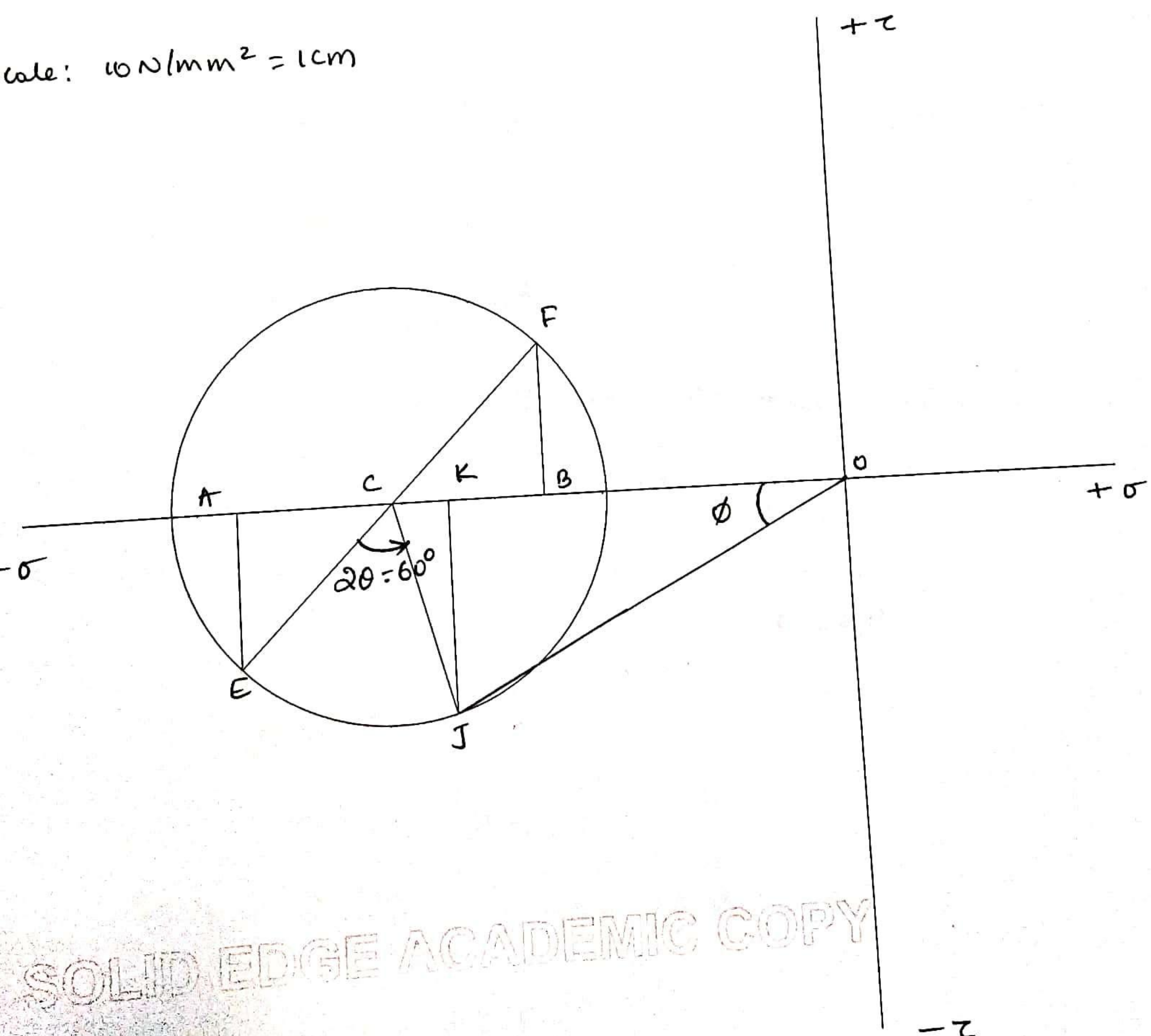
(1) Normal stress, $\sigma_n = OK \times Scale = 6.6 \times 10 = 66\ N/mm^2$

(2) Tangential stress, $\sigma_t = JK \times Scale = -3.4 \times 10 = -34\ N/mm^2$

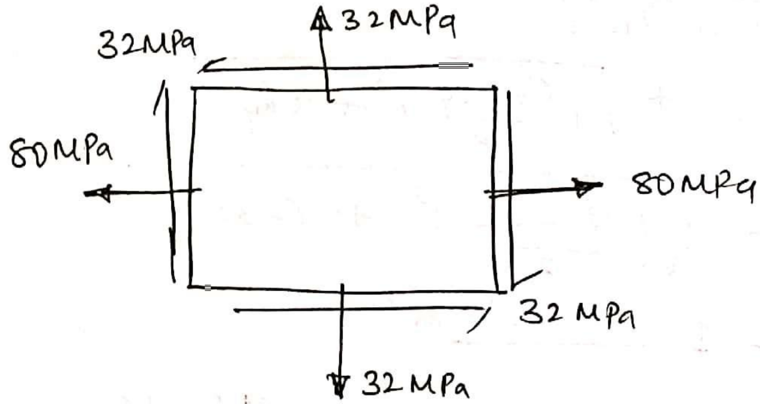
(3) Resultant stress, $\sigma_R = OJ \times Scale = 7.42 \times 10 = 74.2\ N/mm^2$

(4) Angle of obliquity $\phi = J\hat{O}K = -27.4^\circ$ (CW)

Scale: $10\ N/mm^2 = 1cm$



Q.5



Data: $\sigma_x = 32\ MPa$; $\sigma_y = 80\ MPa$; $\tau_{xy} = -32\ MPa$

(1) Major principal stress (or) max. normal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80 + 32}{2} + \sqrt{\left(\frac{80 - 32}{2}\right)^2 + (-32)^2}$$

$$= 56 + 40$$

$$\therefore \sigma_1 = \underline{96\ N/mm^2}$$

Minor principal stress (or) min. normal stress

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80 + 32}{2} - \sqrt{\left(\frac{80 - 32}{2}\right)^2 + (-32)^2}$$

$$\therefore \sigma_2 = 16\ N/mm^2$$

Location of principal planes:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}; \quad \theta = \frac{1}{2} \tan^{-1} \frac{2(-32)}{80 - 32} = \underline{-26.565^\circ}$$

$$\therefore \theta_1 = \underline{-26.565^\circ}$$

$$\theta_2 = \theta_1 + 90^\circ = -26.565^\circ + 90^\circ = \underline{63.435^\circ}$$

(2) Max (or) min shear stress

$$\tau_{max} \text{ or } \tau_{min} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \pm\sqrt{\left(\frac{80-32}{2}\right)^2 + (-32)^2}$$

$$= \pm \underline{40 \ N/mm^2}$$

(OR) τ_{max} or τ_{min} $= \frac{\sigma_1 - \sigma_2}{2} = \frac{96-16}{2} = \underline{40 N/mm^2}$

Location of shear stress planes:

$\theta_1' = \theta_1 + 45^\circ = -26.565 + 45^\circ = \underline{18.435^\circ}$

$\theta_2' = \theta_1 + 135^\circ = -26.565^\circ + 135^\circ = \underline{108.435^\circ}$

(3) Normal stress acting on shear stress plane.

$\sigma_{avg} = \frac{\sigma_1 + \sigma_2}{2} = \frac{96+16}{2} = \underline{56 \ N/mm^2}$

(OR)

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{80+32}{2} = \underline{56 \ N/mm^2}$