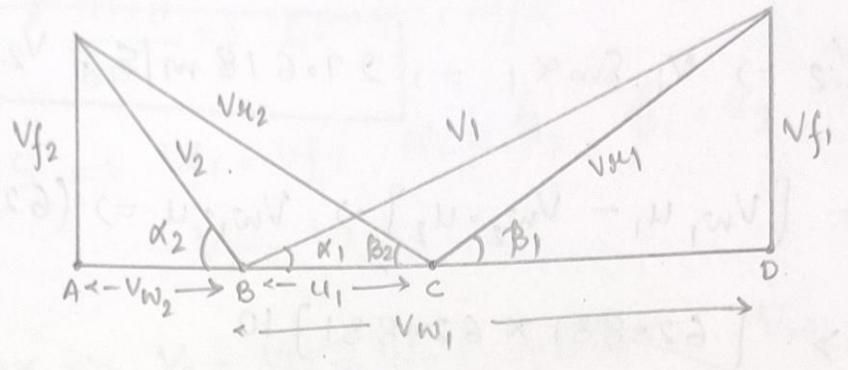
# TURBOMACHINES

## IAT 3-NOVEMBER 2019 SOLUTIONS




We know Euler Equation

$u(V_{w_1} + V_{w_2})$.

$AD \Rightarrow V_{w_1} + V_{w_2} \Rightarrow AC + CD$. —— (1)

$\Rightarrow AC \Rightarrow \cos\beta_2 = \frac{AC}{V_{r_2}} \Rightarrow AC \Rightarrow V_{r_2}\cos\beta_2$. —— (2)

$\Rightarrow CD \Rightarrow \cos\beta_1 \Rightarrow \frac{CD}{V_{r_1}} \Rightarrow CD \Rightarrow V_{r_1}\cos\beta_1$. —— (3).

putting eq (2) and (3) in (1).

we have.

$V_{w_1} + V_{w_2} \Rightarrow V_{r_1}\cos\beta_1 + V_{r_2}\cos\beta_2$.

$\Rightarrow V_{r_1}\cos\beta_1\left[1 + \frac{V_{r_2}}{V_{r_1}} \cdot \frac{\cos\beta_2}{\cos\beta_1}\right]$.

we know $\frac{V_{r_2}}{V_{r_1}} = C_b \quad \frac{\cos\beta_2}{\cos\beta_1} = K =$ constant.

$\Rightarrow V_{r_1}\cos\beta_1\left[1 + C_b \cdot K\right]$ —— (4)

Now $\cos\alpha_1 \Rightarrow \dfrac{u + C_D}{V_1} \Rightarrow C_D \Rightarrow V_1\cos\alpha_1 - u$.

$V_{r1}\cos\beta_1 = V_1\cos\alpha_1 - u$. —— (5)

putting eq (5) in (4)

we have.

$\Rightarrow (V_1\cos\alpha_1 - u).(1 + C_b.K)$.

putting the above eqn in euler's eqn. we have.

$\Rightarrow u\,(V_1\cos\alpha_1 - u)(1 + C_b \cdot K)$.

$\Rightarrow V_1 u\,(\cos\alpha_1 - u/V_1)\,(1 + C_b \cdot K)$. —— (6)

putting eq (6) in W.D eq. we have.

$$W.D \Rightarrow \frac{\cancel{V_1}\,u(\cos\alpha_1 - \phi)(1 + C_b \cdot K)}{1/2\, V_1^{\cancel{2}}} \qquad (u/V_1 = \phi)$$

$W.D \Rightarrow 2 \cdot \dfrac{u}{V} \cdot (\cos\alpha_1 - \phi)(1 + C_b \cdot K)$.

$W.D \Rightarrow$ ~~2φ cos~~ $2\phi[\cos\alpha_1 - \phi][1 + C_b \cdot K]$.

for maximum efficiency $\Rightarrow \dfrac{d\eta_b}{d\phi} = 0$.

$$\frac{d\left[2\phi(\cos\alpha_1 - \phi)(1 + C_b \cdot K)\right]}{d\phi} = 0.$$

$\phi(-1) + (\cos\alpha_1 - \phi) = 0$

$\phi = \cos\alpha_1 / 2$.

putting back in equation we have.

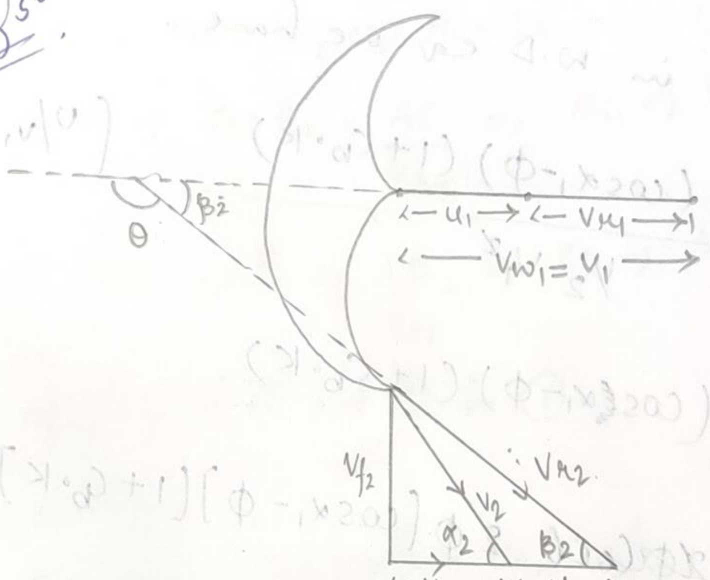
$$\cancel{2}\left(\cos\alpha_1/\cancel{2}\right)\left(\cos\alpha_1 - \cos\alpha_1/2\right)\left(1+C_b\cdot K\right).$$

$$\Rightarrow \frac{\cos\alpha_1^2}{2}\left(1+C_b\cdot K\right)$$

Now if $C_b = 1$ and $K = 1$. then.

$$\boxed{\eta_{max} = \cos\alpha_1^2}$$

Q3.




from the outlet velocity Δ we have.

$$\cos\beta_2 \Rightarrow \frac{V_{w_2}+u}{V_{r_2}} \Rightarrow V_{r_2}\cos\beta_2 - u = V_{w_2}.$$

Euler eqnation $\Rightarrow E \Rightarrow u(V_{w_1} \pm V_{w_2})$.

Assuming $V_{r_1} = V_{r_2}$.

$V_{w_2} = V_{r_1} \cos \beta_2$

$\Rightarrow V_{r_1} = V_1 - u_1$

$\Rightarrow V_{w_2} = (V_1 - u) \cos \beta_2$

$\Rightarrow u [V_1 + (V_1 - u) \cos \beta_2]$.

Now we know bucket coefficient $= C_b = V_{r_2}/V_{r_1}$

Now to find out efficiency of max condition we have.

$$\eta \Rightarrow \frac{u [V_1 + (V_1 - u) \cos \beta_2]}{\frac{1}{2} V_1^2}$$

$$\frac{d\eta_w}{du} \Rightarrow 0$$

Solving the above equation we have.

$\Rightarrow 2u = V$ or $\frac{1}{2} V = u \Rightarrow \boxed{u = 0.5V}$

Hence Bucket Speed is half of jet velocity.

**Q9.** $D = 1.2\,m$ $N = 3000$ rpm $\alpha = 18^\circ$ $\phi = 0.42$.

$\frac{V_{r_2}}{V_{r_1}} = 0.9$ $\beta_2 = \beta_1 - 3^\circ$ $\dot{m} = 5$ Kg/s.

$u = \frac{\pi D N}{60} \Rightarrow \frac{\pi \times 1.2 \times 3000}{60} \Rightarrow 188.49\,m/s$.

$u/V_1 = \phi$ $V_1 = u/\phi \Rightarrow 188.49/0.42 = 448.79\,m/s$

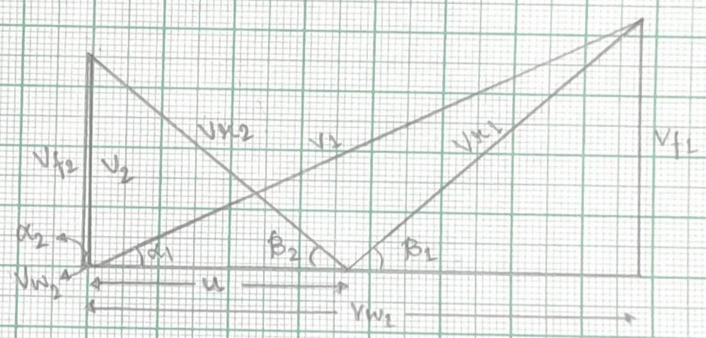
Q.5

$\alpha_1 = 18^\circ$

$\beta_1 = 46^\circ$

$\beta_2 = \beta_1 - 3^\circ$

$\beta_2 = 46^\circ - 3^\circ = 43^\circ$




Scale

1 cm = 50 m/s.

$u = 3.769 \times 50$
$= 188.49$ m/s

$V_1 = 8.9758 \times 50$
$= 448.79$ m/s

$V_{r1} = 6 \times 50$
$\Rightarrow 300$ m/s

$V_{f1} \Rightarrow 4.3 \times 50$
$\Rightarrow 215$ m/s

$V_{r2} = 0.9 \times V_{r1}$
$\Rightarrow 0.9 \times 300$
$V_{r2} = 270$ m/s

$\Delta V_w \Rightarrow [V_{w1} \pm V_{w2}]$

$\Delta V_w = 79 \times 50$
$\Rightarrow 395$ m/s

$V_{f2} = 3.5 \times 50$
$\Rightarrow 175$ m/s

$V_{r_2} = 0.9 \times V_{r_1} \Rightarrow V_{r_2}\ 0.9 \times 300 \Rightarrow 270\ m/s$

(1) Velocity of whirl $\Rightarrow \Delta V_w \Rightarrow [V_{w_1} \pm V_{w_2}]$

$\Rightarrow \cancel{8\ 400\ m/s}\ 395\ m/s.$

~~(b) Axial Thrust~~ $\cancel{\Rightarrow \dot{m}\ [V_{f_1} - V_{f_2}]}$

$\cancel{\Rightarrow 5\ [215 - 170] = [5][25]}$

$\cancel{\Rightarrow 125\ N.}$

~~(c) Blade angles~~ $\cancel{\Rightarrow \beta_1}$

(b) Axial Thrust $\Rightarrow \dot{m}\ (V_{f_1} - V_{f_2})$.

$\Rightarrow 5\ [215 - 175] \Rightarrow \underline{\underline{200\ N.}}$

(c) Blade angles ; $\beta_1 = 46°$.

$\beta_2 = \beta_1 - 3°$

$\beta_2 = 46 - 3° = 43°$.

(d) Power Developed

$\Rightarrow \dot{m}\, u\, (\Delta V_w)$

$\Rightarrow \dot{m} \times u \times \Delta V_w \Rightarrow 5 \times 188.49 \times 395.$

$\Rightarrow 372267.75\ W.$

$\Rightarrow 372.267\ kW.$

Q6. $H = 70\,m$, $N = 600\,RPM$, $S.P = 370\,kW$, $O.E = 0.80$.

$H.E = 0.95$, $\psi = 0.25$, $B_1/D_1 = 0.1$, $D_2 = 2D_1$

$t = 0.1\,\pi D$ $\quad V_{f1} = V_{f2} = V_2$ $\quad \alpha_2 = 90^\circ$ $\quad V_{w_2} = 0$.

P(A) $\eta_0 = \dfrac{P}{\rho \cdot g \cdot Q \cdot H}$ $\Rightarrow 0.8 = \dfrac{370 \times 10^3}{1000 \times 9.81 \times Q \times 70}$.

$\Rightarrow Q \Rightarrow 0.6735\ m^3/s$.

$\eta_h = \dfrac{\cancel{\rho Q}(V_{w_1} u_1 \pm \cancel{V_{w_2} u_2}^{\,0})}{\rho \cdot g \cdot \cancel{Q} \cdot H}$ $\qquad 0.95 \times 9.81 \times 70 = V_{w_1} \cdot u_1$

$\Rightarrow V_{w_1} \cdot u_1 = 652.365$.

$V_{f1} = \psi \sqrt{2gh}$

$V_{f1} = 0.25 \sqrt{2 \times 9.81 \times 70}$.

from inlet $\Delta$. we have.

$V_{f1} = 9.26 = V_{f2}$.

$\tan \alpha_1 = \dfrac{V_{f1}}{V_{w_1}}$

$Q = A_{f1} \times V_{f1}$.

$Q = \cancel{\frac{\pi}{4}(D_1)^2}\ \pi C \cdot D_1 \cdot B_1 \times V_{f1}$

$u_1 \Rightarrow \dfrac{\pi D_1 N}{60}$

$Q \Rightarrow \pi(1) D_1 (0.1 D_1)\, 9.26$

$u_1 \Rightarrow 15.11\ m/s$.

$0.6735 \Rightarrow \pi(0.1 D_1^2)(9.26)$

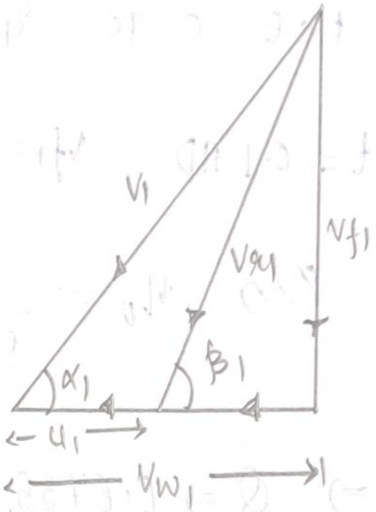
$D_1 = 0.481\ m$

$u_2 = \dfrac{\pi D_2 N}{60}$

$D_2 = 0.9623\ m$.

$u_2 \Rightarrow 30.23\ m/s$.

$V_{w_1} = \frac{652 \cdot 365}{u_1} \Rightarrow 43.1743.$




$V_{w_1} > u_1$

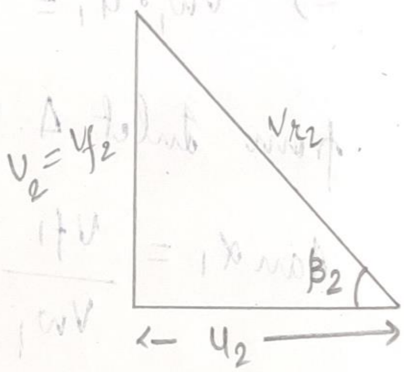
$\tan \alpha_1 = \frac{V_{f1}}{V_{w_1}}$

$\alpha_1 = \tan^{-1} \frac{9.26}{43.174}$

$\alpha_1 = 12.105^\circ$

$\tan \beta_1 \Rightarrow \frac{V_{f1}}{V_{w_1} - u_1} \Rightarrow \frac{9.26}{43.174 - 15.11} = 18.26^\circ.$

$\tan \beta_2 \Rightarrow \frac{V_{f2}}{u_2}.$




$\beta_2 \Rightarrow \tan^{-1} \frac{9.26}{30.23}$

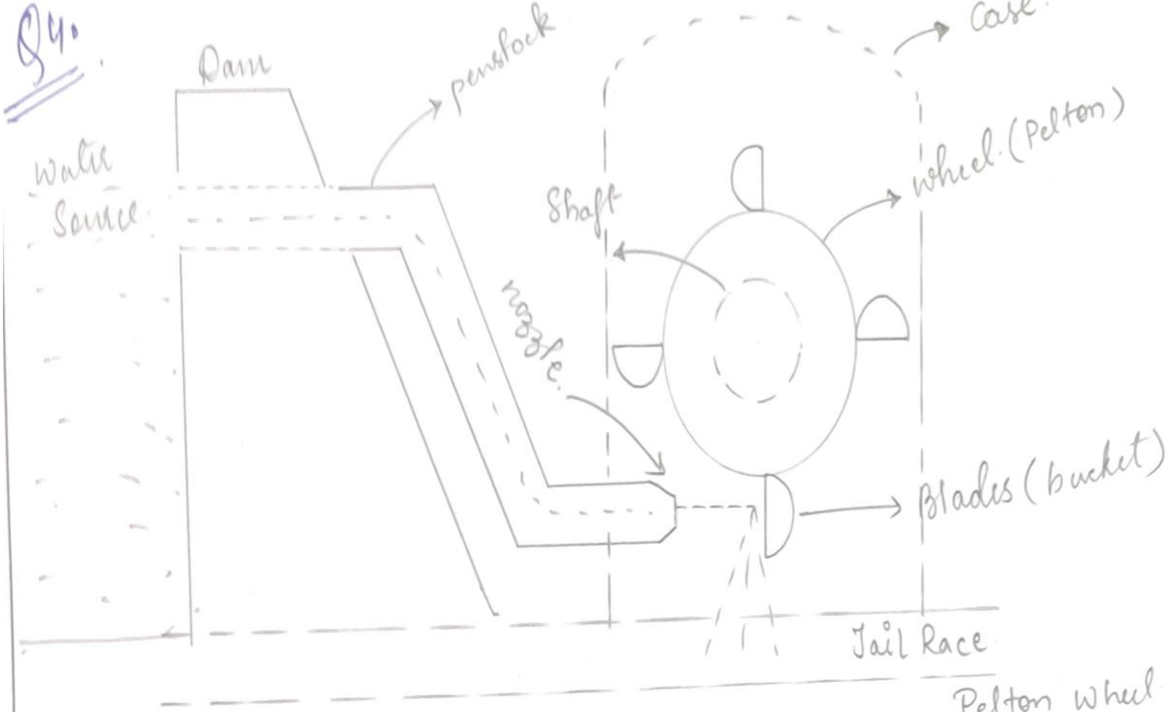
$\beta_2 = 17.03^\circ$

Width of the wheel $\Rightarrow t \Rightarrow 0.1 \pi D$

$t \Rightarrow 0.1(\pi \times 0.481).$

$\boxed{t \Rightarrow 0.15 \text{ m}}$

Q4.



Pelton wheel

In the pelton wheel mechanism the water is provide from a water Source near the dam from a certain height. The water is transported with the help of penstok. This penstock carries the water at high velocity and low pressure.

This high velocity low pressure water hits the blades of the pelton wheel (Buckets) which cause the wheel to rotates. Thus the the rotation of the wheel causes the movement of generator shaft which in turn produces electricity.