CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test – 3

| Sub: Mechanics of Materials | | | | Code: 18ME32 |
|---|---|---|---|---|
| Date: 18/11/2019 | Duration: 90 mins | Max Marks: 50 | Sem: 3 | Branch (sections): ME (A) |

Answer any **FOUR** questions. Good luck!

| | | Marks | OBE | |
|---|---|---|---|---|
| | | | CO | RBT |
| 1 | A beam of I section consists of 180 mm x 15 mm flanges and a web of 280 mm depth x 15 mm thickness. It is subjected to a bending moment of 120 kN-m and shear force of 60 kN. Sketch the bending and shear stress distributions along the depth of the section. | [12.5] | CO5 | L3 |
| 2 | Derive an expression for strain energy stored in an elastic bar subjected to axial load, torque and bending moment. | [12.5] | CO3 | L3 |
| 3 | The internal and external diameters of a thick cylinder are 300mm and 500mm respectively. It is subjected to an external pressure of 4MPa. Find the internal pressure that can be applied if the permissible stress in cylinder is limited to 13MPa. Sketch radial and hoop stresses distribution across the section. | [12.5] | CO3 | L3 |
| 4 | A bolt is subjected to an axial pull of 12kN together with a transverse shear of 6kN. Determine the diameter of the bolt by using maximum shear stress theory. Take elastic limit in tension = 300 N/mm$^2$; factor of safety =3; Poisson's ratio=0.3. | [12.5] | CO3 | L3 |
| 5 | List various theories of failure and explain any two. | [12.5] | CO3 | L3 |
| 6 | Prove that $\frac{M}{I} = \frac{\sigma}{Y} = \frac{E}{R}$ with usual notations. Also list all the assumptions in simple bending theory. | [12.5] | CO3 | L3 |

**CI** **CCI** **HOD**

**Q NO 1:**

(i) Bending stress distribution

For symmetrical 'I' section,

Distance of NA from the bottom layer $\bar{y} = \frac{310}{2} = 155\,mm$

$$MI \text{ about } NA = \frac{180 \times 310^3 - 165 \times 280^3}{12} = 1.45 \times 10^8\,mm^4$$

From bending equation $\frac{M}{I} = \frac{\sigma}{y}$ | $y_{max} = y_{tmax} = \bar{y} = y_{ang} = 155\,mm$

For symmetrical 'I' section.

Max. bending stress in the topmost fibre = Bending stress in the bottom most fibre.

$$\therefore \sigma_{max} = \frac{M}{I} y_{max} = \frac{120 \times 10^6}{1.45 \times 10^8} \times 155 = 128.3\,N/mm^2$$

(ii) Shear stress distribution:

(a) Shear stress distribution in the flange

Shear stress at the top edge of the flange = 0

Shear stress at the bottom edge of the flange $= \frac{F}{IB} \times A\bar{y}$

where, A = Area of flange = $180 \times 15 = 2700\,mm^2$

$\bar{y}$ = Distance of C.G from NA (flange) $= 155 - \frac{15}{2}$
$= 147.5\,mm$

B = Width of flange = 180 mm

I = Total MI of the section $= 1.45 \times 10^8\,mm^4$

$$\therefore \text{Shear stress} = \frac{60 \times 10^3}{1.45 \times 10^8 \times 180} \times 2700 \times 147.5 = 0.916\,N/mm^2$$

(b) Shear stress distribution in the ~~flow~~ web.

Shear stress at the junction $= 0.916 \times \frac{B}{b}$

$= 0.916 \times \frac{180}{15}$  $b$ = width of web $= 15\ mm$

$= 11\ N/mm^2$

Maximum shear stress at the NA $= \frac{F}{Ib} \times A\bar{y}$

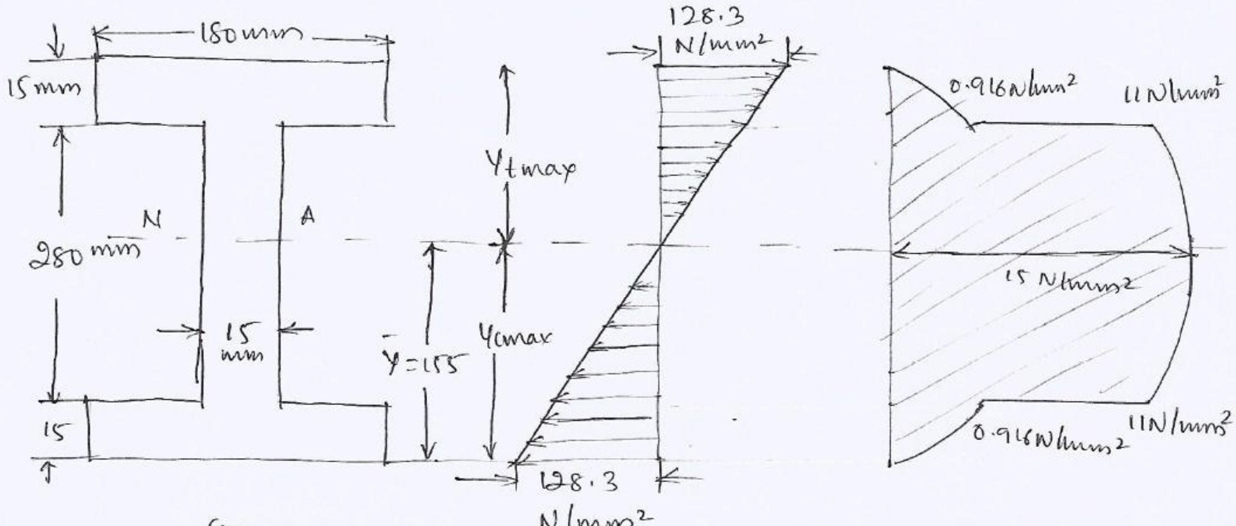
where,

$b$ = width of beam = 15 mm

$A\bar{y}$ = Moment of flange area about NA + Moment of web area above the NA about NA

$= (180 \times 15)\left(155 - \frac{15}{2}\right) + (15 \times 140)\left(\frac{140}{2}\right) = 545250\ mm^3$

$\therefore\ \tau_{max}$ or $q_{max} = \frac{60 \times 10^3}{1.45 \times 10^8 \times 15} \times 545250 = 15\ N/mm^2$




(a) (b) Bending stress distribution (c) Shear stress distribution

**Q NO. 2:**

* Internal strain energy stored within an elastic bar subjected to an axial tensile force 'P'

Consider a bar shown in fig(a) is subjected to an axial force 'P'. The deformation in the bar due to the applied force 'P' is equal to 'δ'.

Now, $\delta = \frac{PL}{AE}$, where A = Cross-sectional area of the bar
l = Length of bar
E = Young's modulus and
P = Applied force.

According to this expression, the relation between force and deformation is a linear one i.e., Force-displacement diagram will be linear as shown in fig(b).

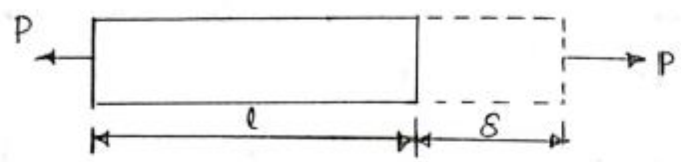



Fig. (a)

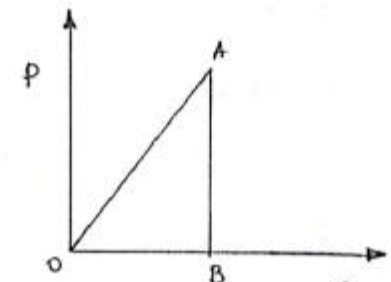



Fig. (b)

When the force has reached a specific value such as that indicated by point A, it will have done +ve work done indicated by the shaded area OAB.

∴ Work done by the force = Area of $\Delta^{le}$ OAB

$$= \frac{1}{2} P\delta = \frac{1}{2} P\left(\frac{PL}{AE}\right)$$

$$= \frac{1}{2}\frac{P^2L}{AE} = \frac{P^2L}{2AE}$$

This work done by the external force is stored within the bar as internal strain energy, denoted by 'U'.

$$\therefore U = \frac{P^2L}{2AE}$$

* Internal strain energy stored within an elastic bar subjected to a pure bending moment 'M'.

- Consider a beam initially straight is subjected to pure bending moment M. Due to the action of this bending moment M, the beam deforms or bend into a circular arc of radius of curvature 'R' as shown in fig (a).

The applied bending moment $M = \frac{EI}{R}$ $\quad (\because \frac{M}{I} = \frac{E}{R})$

where, M = Applied bending moment, E = Young's modulus, I = Moment of inertia, R = Radius of curvature of beam

Length of Arc is equal to the product of the angle subtended by the arc and the radius of curvature.

i.e., Length of arc = $R\theta$. But, length of arc = length of beam

$$\text{i.e., } L = R\theta \quad \therefore \quad R = \frac{L}{\theta}$$

$$\text{i.e., } M = \frac{EI}{L/\theta} = \frac{EI\theta}{L}, \quad \therefore \quad \theta = \frac{ML}{EI}$$

According to this expression, the relation between bending moment and angle subtended is linear i.e., Moment - Angle subtended diagram will be linear as shown in fig (b).

When the moment has reached a specific value such as that indicated by point A, it will have the work done indicated by the shaded area OAB.

$\therefore$ Workdone by the moment = Area of the $\Delta^{le}$ OAB

$$= \tfrac{1}{2} M\theta$$

$$= \tfrac{1}{2} M\left(\frac{ML}{EI}\right) = \frac{M^2 L}{2EI}$$

This workdone by the external moment is stored within the beam as Internal strain energy, denoted by U

$$\therefore \quad U = \frac{M^2 L}{2EI}$$

Determine the internal strain energy stored within an elastic bar subjected to a torque 'T'.

Consider a bar as shown in fig(a) is subjected to a torque 'T'. The angle of twist in the bar due to the applied 'T' is equal to 'θ'.

Now $\theta = \frac{Tl}{GJ}$ $\left(\because \frac{T}{J} = \frac{G\theta}{l}\right)$

where, θ = Angle of twist, T = Torque applied, l = length of bar, G = Modulus of rigidity, J = Polar moment of inertia.

According to this expression, the relation between torque and angle of twist is a linear one i.e., Torque-Angle of twist diagram will be linear as shown in fig.(b)

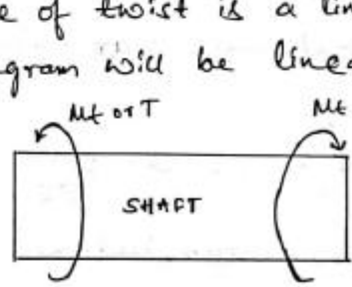



Fig (a).

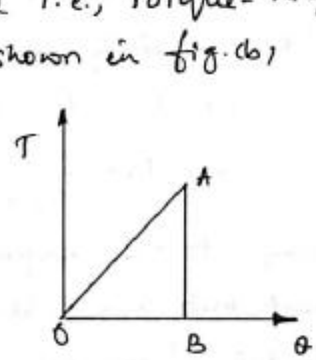



Fig (b)

When the torque has reached a specific value such as that indicated by point A, it will have +ve work done indicated by the shaded area OAB.

∴ Work done by the torque = Area of the $\Delta^{le}$ OAB

$$= \frac{1}{2}T\theta = \frac{1}{2}T\left(\frac{Tl}{GJ}\right) = \frac{T^2 l}{2GJ}$$

This work done by the external torque is stored within the bar as internal strain energy, denoted by 'U'.

$$\therefore U = \frac{T^2 l}{2GJ}$$

If the torque 'T' varies along the length of the bar, then in elemental length 'dx' of the bar, strain energy is $dU = \frac{T^2 dx}{2GJ}$

For entire bar, $U = \int_0^l \frac{T^2 dx}{2GJ}$

**Q NO 4:**

Solution: Data: $\sigma_e = 300\ N/mm^2$, F.O.S = 3, $\mu = 0.3$

Let the diameter of bolt be 'd'. Then the direct stress

$$\sigma = \frac{Load}{Area} = \frac{12 \times 10^3}{\frac{\pi d^2}{4}} = \frac{48 \times 10^3}{\pi d^2}$$

Shear stress at centre of bolt, $\tau = \frac{4}{3} \times \tau_{av}$

$$= \frac{4}{3} \times \frac{6 \times 10^3}{\frac{\pi}{4} d^2}$$

$$= \frac{32 \times 10^3}{\pi d^2}$$

$\therefore$ Principal stresses are, $\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$

$$= \frac{24 \times 10^3}{\pi d^2} + \sqrt{\left(\frac{24 \times 10^3}{\pi d^2}\right)^2 + \left(\frac{32 \times 10^3}{\pi d^2}\right)^2}$$

$$= \frac{24 \times 10^3}{\pi d^2}\left(1 + \sqrt{1 + \left(\frac{32}{24}\right)^2}\right.$$

$$= \frac{24 \times 10^3}{\pi d^2}(1 + 1.66667)$$

$$= \frac{20371.83}{d^2} \quad - (i)$$

$$\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

$$= \frac{24 \times 10^3}{\pi d^2}(1 - 1.66667)$$

$$= -\frac{5092.984}{d^2} \quad - (ii)$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$$

$$= \frac{24\times10^3}{\pi d^2} \times 1.66667$$

$$= \frac{12732.421}{d^2} \quad \text{— (iii)}$$

(i) From maximum principal stress theory:

Permissible stress in tension $= \frac{300}{3} = 100\ N/mm^2$

$$\therefore \quad \sigma_1 = 100\ N/mm^2$$

$$\text{i.e} \quad \frac{20371.833}{d^2} = 100$$

$$\therefore \quad d = \underline{14.273\,mm} \quad \text{(Ans)}$$

(ii) Maximum shear stress theory:

The design condition is $\tau_{max} = \frac{\text{Shear stress at elastic limit}}{\text{Factor of safety}}$

$$\frac{12732.421}{d^2} = \frac{300/2}{3}$$

$$\therefore \quad d = 15.958\ mm \quad \text{(Ans)}$$

Hence maximum shear stress theory governs the design. Use at least 15.958 mm diameter bolt. The practical size is 16 mm diameter. (Ans)

**Q NO 5:**

The following five theories of failure can be used for predicting failure load in any structural element.

1. Maximum Principal stress theory
2. Maximum Shear stress theory
3. Maximum Principal strain theory
4. Maximum strain energy theory
5. Maximum distortion energy theory.

1. Maximum Principal stress theory (Rankine's theory):

This theory states that a material in complex state of stress fails, when the maximum principal stress in it reaches the value of stress at elastic limit in simple tension.

Thus in two-dimensional stress condition failure criteria is:

$$\sigma_1 = \sqrt{\frac{\sigma_x + \sigma_y}{2}} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \sigma_e \quad -(1)$$

where, $\sigma_e$ = Stress at elastic limit in uniaxial tension test.
$\simeq f_y$, Yield stress.

* This theory is found to be reasonably good for brittle material

The design eqn is $\sigma_{max} \leq \frac{\sigma_{yt}}{FOS}$ ; $\frac{\sigma_{yt}}{FOS} = \sigma_e$

2. Maximum Shear stress theory (Coulomb's or Guest & Tresca theory

According to this theory, a material in complex state of stress fails when the maximum shearing stress in it reaches the value of shearing stress at elastic limit in uniaxial tension test.

In general two dimensional stress system maximum shearing stress is given by

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

The design equation is $\tau_{max} \leq \frac{\tau_{yt}}{FOS}$

$\tau_{yt}$ = Shear stress at yield/elastic limit

In uniaxial tension test, maximum shearing stress at elastic limit

$$\tau_{max} = \sqrt{\left(\frac{\sigma_e - 0}{2}\right)^2 + 0} = \frac{\sigma_e}{2}$$

$\therefore$ According to this theory, failure criteria is

$$\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} = \frac{\sigma_e}{2}$$

* This theory gives better results for ductile materials with elastic limit same in tension and in compression.

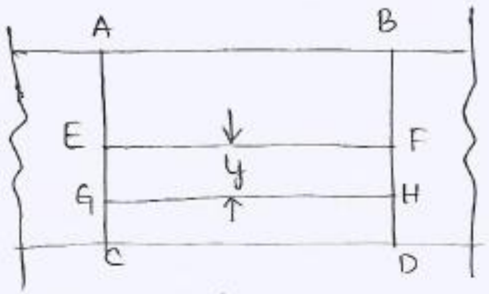
**Q NO 6:**

Assumptions in Simple Theory of Bending.

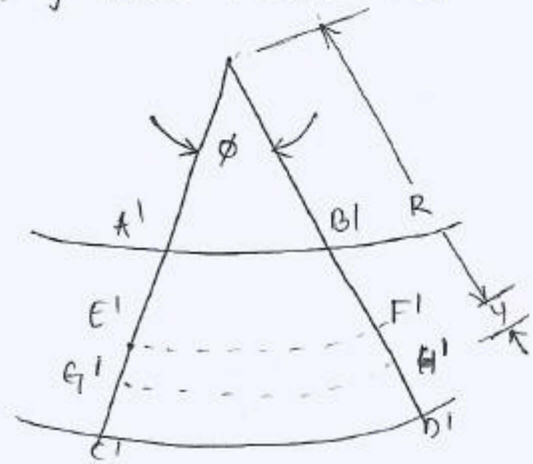
The following assumptions are made in theory of Simple bending.

1. The beam is initially straight and every layer of it is free to expand or contract
2. The material is homogeneous and isotropic
3. Young's modulus is same in tension and compression.
4. Stresses are within elastic limit.
5. Plane section remains plane even after bending.
6. The radius of curvature is large compared to depth of beam.

Consider a portion of beam between Sections AC and BD as shown in fig (a). Let EF be the neutral axis and GH an element at a distance 'y' from neutral axis.




(a) Before bending.




(b) After bending.

Fig (b) shows the same portion after bending. Let 'R' be the radius of curvature and $\phi$ be the angle subtended by C'A' and D'B' at centre of radius of curvature.

Since 'EF' is neutral axis, there is no change in its length. (at neutral axis stresses are zero)

$$EF = E'F' = R\phi$$

Now, strain in GH = $\dfrac{\text{Final length} - \text{Original length}}{\text{Original length}}$

$$= \frac{G'H' - GH}{GH}$$

But $GH = EF = R\phi$ and $G'H' = (R+y)\phi$

Hence strain in layer GH = $\dfrac{(R+y)\phi - R\phi}{R\phi} = \dfrac{y}{R}$

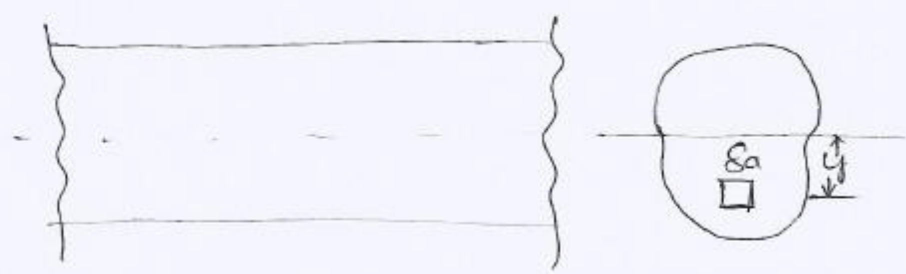
If '$\sigma_b$' is the bending stress and 'E' is the Young's - modulus then strain is $\frac{\sigma_b}{E}$ and hence

$$\frac{\sigma_b}{E} = \frac{y}{R} \quad \text{(or)} \quad \frac{\sigma_b}{y} = \frac{E}{R}$$

$$\text{(or)} \quad \underline{\sigma_b = \frac{E}{R} \times y}$$

Thus bending stress varies linearly across the depth.

Consider an elemental area '$\delta a$' at a distance '$y$' from neutral axis in the beam, the cross section of which is as shown in fig. below.




Now stress '$\sigma_b$' on this element is given by

$$\sigma_b = \frac{E}{R} y$$

$\therefore$ Force on this elemental area $= \sigma_b \cdot \delta a$

$$= \frac{E}{R} y \cdot \delta a$$

Moment of this resisting force about neutral axis

$$= \frac{E}{R} y\, \delta a\, y = \underline{\frac{E}{R} y^2 \delta a}$$

$\therefore$ Total moment of resistance ($M'$) of the cross-sectional area, $M' = \Sigma \frac{E}{R} y^2 \delta a$ ; $M' = \underline{\frac{E}{R} \Sigma y^2 \delta a}$

From the definition of moment of inertia, which is Second moment of area about centroid, we can write

$$I = \Sigma y^2 \delta a$$

where,
I is centroidal moment of inertia.

$$\therefore \; M' = \frac{E}{R} I$$

For equilibrium moment of resistance (M') should be equal to applied moment i.e, $\underline{M' = M}$.

Hence, we get $M = \frac{E}{R} I$

(or) $\underline{\frac{M}{I} = \frac{E}{R}}$ — (1)

From eqn, $\frac{\sigma_b}{y} = \frac{E}{R}$ and equation (1)

We can write the bending equation as

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$

where,

M - Bending moment

I - Moment of inertia about centroidal axis

$\sigma_b$ - Bending stress

y - Distance of the fibre from neutral axis

E - Young's modulus

and R - Radius of curvature.