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Internal Assessment Test 3 – Nov. 2019

Sub:	**CONTROL ENGINEERING**				Sub Code:	15ME73	Branch:	ME
Date:	19/11/19	Duration:	90 min's	Max Marks: 50	Sem / Sec:	**7^{th}/A & B**		OBE

	Section A (Answer all questions)	MARKS	CO	RBT
1	For a unity feedback system : G(s) = $\frac{K(s+1)}{s(s-1)(s+6)}$ Determine: i) Range of K for system to be stable. ii) Marginal value of K. iii) Location of roots for marginal value of K.	[10]	CO4	L2
2	G(s)H(s) = $\frac{K(s+1)}{s(s-1)(s^2+4s+16)}$ Draw root locus. Determine for damping ratio = 0.6: i) Closed loop dominant poles ii) Damped natural frequency iii) Gain K	[20]	CO5	L2
	Section B (Answer ANY ONE question)			
3	The open loop transfer function of a certain unity feedback system is given by G(s) = $\frac{K}{s(s+2)(s+20)}$ Construct Bode Plots and determine: i) Limiting value of K for system to be stable. ii) Value of K for gain margin to be 10dB. iii) Value of K for phase margin to be 50^0.	[20]	CO4	L2
4	For a system G(s)H(s)= $\frac{242(s+5)}{s(s+1)(s^2+ 5s+121)}$ Sketch the Bode Plot and determine gain margin, phase margin, gain crossover frequency, phase crossover frequency. Comment on stability of the system.	[20]	CO4	L2

CI CCI HOD

Q1. $G(s)H(s) = \dfrac{k(s+1)}{s(s-1)(s^2+4s+16)}$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k(s+1)}{s(s-1)(s^2+4s+16)} = 0$$

$$s(s-1)(s^2+4s+16) + k(s+1) = 0$$

$$s^4 + 3s^3 + 12s^2 - 16s + ks + k = 0$$

$$s^4 + 3s^3 + 12s^2 + s(k-16) + k = 0$$

Routh's array.

s^4	1	12	k
s^3	4	$(k-16)$	0
s^2	$\left(\frac{52-k}{3}\right)$	k	0
s^1	$\dfrac{\left(\frac{52-k}{3}\right)(k-16)-3k}{\left(\frac{52-k}{3}\right)}$	0	0
s^0	k		0

or stability,

i) From s^0 row; $k > 0$

ii) From s^1 row; $\left(\frac{52-k}{3}\right)(k-16) - 3k > 0$

$$(52-k)(k-16) - 9k > 0$$

$$(52k - 832 - k^2 + 16k - 9k) > 0$$

$$k^2 - 59k + 832 < 0$$

$\therefore$ Roots are $k > 23.315$ & $k < 35.684$

iii) From s^2 row $\quad \frac{52-k}{3} > 0$

$$k < 52$$

$0 < k < 23.315$ - unstable

$23.315 < k < 35.685$ - stable

$k > 35.685$ - unstable

overall marginally stable.

2) $G(s)H(s) = \dfrac{k(s+1)}{s(s-1)(s^2+4s+16)}$

open-poles at $s = 0, 1, -2 + 3.464i, -2 - 3.464i$

$\therefore P = 4$

$Z = -1$

$P > Z$

$P - Z = 4 - 1 = 3$ branches = No: of asymptodes.

Angle of asymptodes $\theta_A = \dfrac{(2q+1)180^\circ}{P-Z}$

$\theta_{A_1} = 60^\circ, \theta_{A_2} = 180^\circ, \theta_{A_3} = 300^\circ$

Centroid $\sigma_A = \dfrac{\Sigma R.P \text{ of poles} - \Sigma R.P \text{ of zeroes}}{P-Z}$

$\sigma_A = \dfrac{0+1-2-2-(-1)}{3} = -0.6$

1. For S_1 between 0 & 1; $P_r + Z_r = 1+0 = 1$ (odd) RL ✓
2. For S_2 between -1 & 0; $P_r + Z_r = 2+0 = 2$ (even) RL ✗
3. For S_3 between $-\infty$ to -1, $P_r + Z_r = 2+1 = 3$ (odd) RL ✓.

Characterstic equation of system

$1 + G(s)H(s) = 0$

$1 + \dfrac{k(s+1)}{s(s-1)(s^2+4s+16)} = 0$

$s^4 + 3s^3 + 12s^2 - 16s + k(s+1) = 0$

$K = -\dfrac{(s^4+3s^3+12s^2-16s)}{(s+1)}$

$\dfrac{dk}{ds} = \dfrac{(s+1)(-1)(4s^3+9s^2+24s-16) + (s^4+3s^3+12s^2-16s)}{(s+1)^2} = 0.$

$3s^4 + 10s^3 + 21s^2 + 24s - 16$

$S = 0.448,\ -2.26,\ -0.76 + 2.16i,\ -0.76 - 2.16i$

Valid points $S = 0.448,\ -2.26$

Characteristic equation..

$$s^4 + 3s^3 + 12s^2 - 16s + ks + k = 0$$

Routh's array.

s^4	1	12	k
s^3	3	$k-16$	0
s^2	$\frac{52-k}{3}$	k	0
s^1	$\frac{-k^2+59k-832}{52-k}$	0	0
s^0	k	0	0

$$\frac{-k^2 + 59k - 832}{52-k} = 0 \qquad k = 35.68 \text{ \& } 23.21.$$

at $k = 35.68$, $\quad S = \pm 2.56j$

$k = 23.31$, $\quad S = \pm 1.56i$

$\phi_{P_1} = 180° - \tan^{-1}\frac{3.464}{3} = 130.89°$

$\phi_{P_2} = 180° - \tan^{-1}\frac{3.464}{2} = 120$

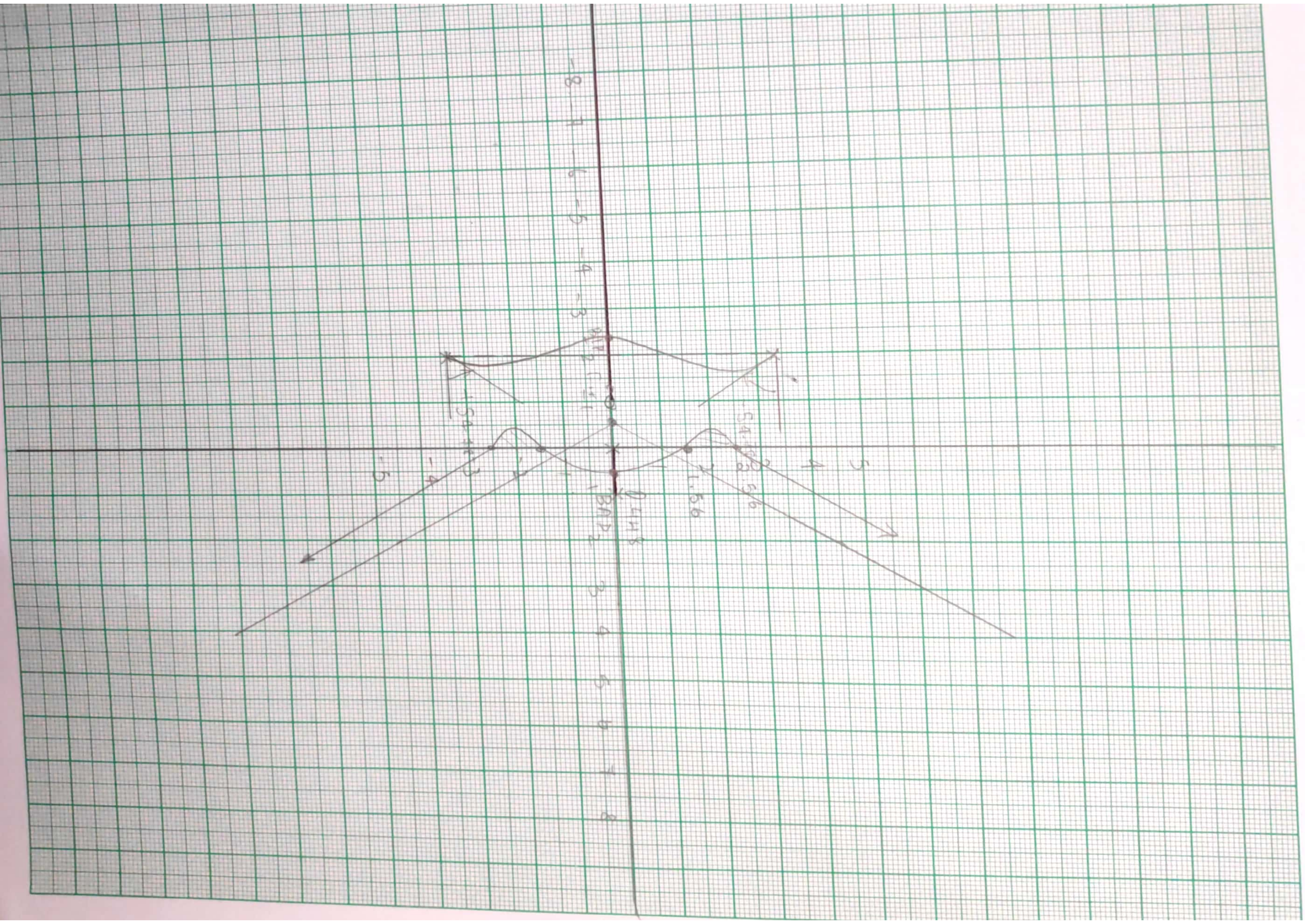
$\phi_{P_3} = 90$ $\qquad \phi = \Sigma\phi_P - \Sigma\phi_z$

$\phi_{z_1} = 106.11°$ $\qquad \phi = 234.78.$

$\Sigma\phi_P = 340.89°$ $\qquad$ Angle of departure $= 180 - 234.78$

$\Sigma z = 106.11°$ $\qquad = -54.78$ at $-2 + 3.464i$

$+54.78$ at $-2 - 3.464i$



3) $G(s) = \frac{k}{s(s+2)(s+20)}$

From standard form $G(s)H(s) = \frac{k}{s(s+2)(s+20)}$

Here $H(s) = 1$.

$$G(s)H(s) = \frac{k}{40s(1+\frac{1}{2}s)(1+\frac{1}{20}s)} = \frac{k}{40s(1+0.5s)(1+0.05s)}$$

$$G(i\omega)H(i\omega) = \frac{A}{i\omega(1+0.5i\omega)(1+0.05\omega)} \quad \text{where } \frac{k}{40} = A.$$

$$20\log|G(i\omega)H(i\omega)| = 20\log A - 20\log(\omega) - 20\log\sqrt{1+(0.5\omega)^2} - 20\log\sqrt{1+(0.05\omega)^2}$$

$$\omega_{c_1} = \frac{1}{0.5} = 2 \text{ rad/s} \qquad \omega_{c_2} = \frac{1}{0.05} = 20 \text{ rad/s}.$$

Range Rad/s	slope in (dB/ decand)
0.1 – 2	−20
2 – 20	−40
20 – ∞	−60

Phase table.

ω	$\frac{1}{i\omega}$ $-\tan^{-1}\left(\frac{\omega}{0}\right)$	$\frac{1}{(1+0.5i\omega)}$ $-\tan^{-1}\left(\frac{0.5\omega}{1}\right)$	$\frac{1}{(1+0.05i\omega)}$ $-\tan^{-1}\left(\frac{0.05\omega}{1}\right)$	$\Sigma\theta_R$
0.2	−90	−5.71	−0.57	−96.28
0.8	−90	−21.8	−2.29	−114.09
2	−90	−45	−5.71	−140.71
10	−90	−78.69	−26.56	−195.25
20	−90	−84.28	−45	−219.28
100	−90	−88.85	−78.69	−257.54
∞	−90	−90	−90	−270

i) $20 \log A < 28$

$\log A < 1.4$

$A < 10^{1.4}$

$A < 25.11$.

$K < 25.11 \times 40 = \underline{1004.75}^\circ$

ii) For gain margin to be 10dB

$28dB - 10 = 18dB$

$20 \log A < 18$

$A < 7.943$

$K < \underline{317.79}^\circ$

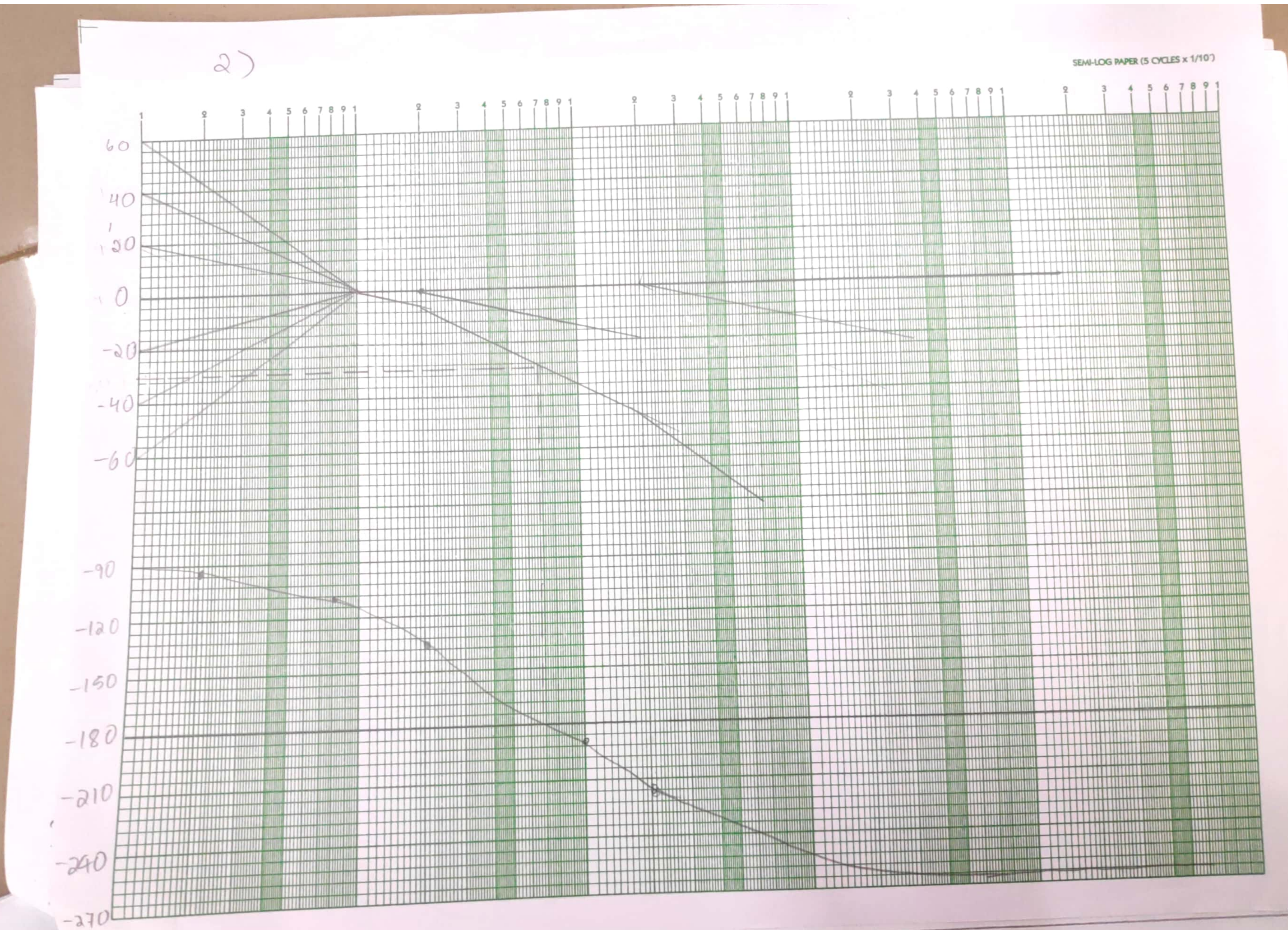
For phase margin to be 50°

$20 \log A < 6$

$A < 1.995$

$K < \underline{79.81}$.

2)



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4) $G(s)H(s) = \dfrac{48.4(1+0.2i\omega)}{i\omega(1+i\omega)(1+0.04i\omega-8.26\times10^{-3}\omega^2)}$

$20\log|G(i\omega)H(i\omega)| = 20\log 48.4 + 20\log\sqrt{1+(0.2\omega)^2} - 20\log\omega - 20\log\sqrt{(1-84.3\times10^{-3}\omega^2)^2+(0.04\omega)^2}$

Corner frequencies $\omega_c = \dfrac{1}{0.2} = 5\,rad/s$

$\omega_{c_2} = \dfrac{1}{0.04} = 25\,rad/s$ $\qquad \omega_{gc} = 16\,rad/s$

Phase margin $= -30^\circ$

$\omega_{c_3} = \omega_n$

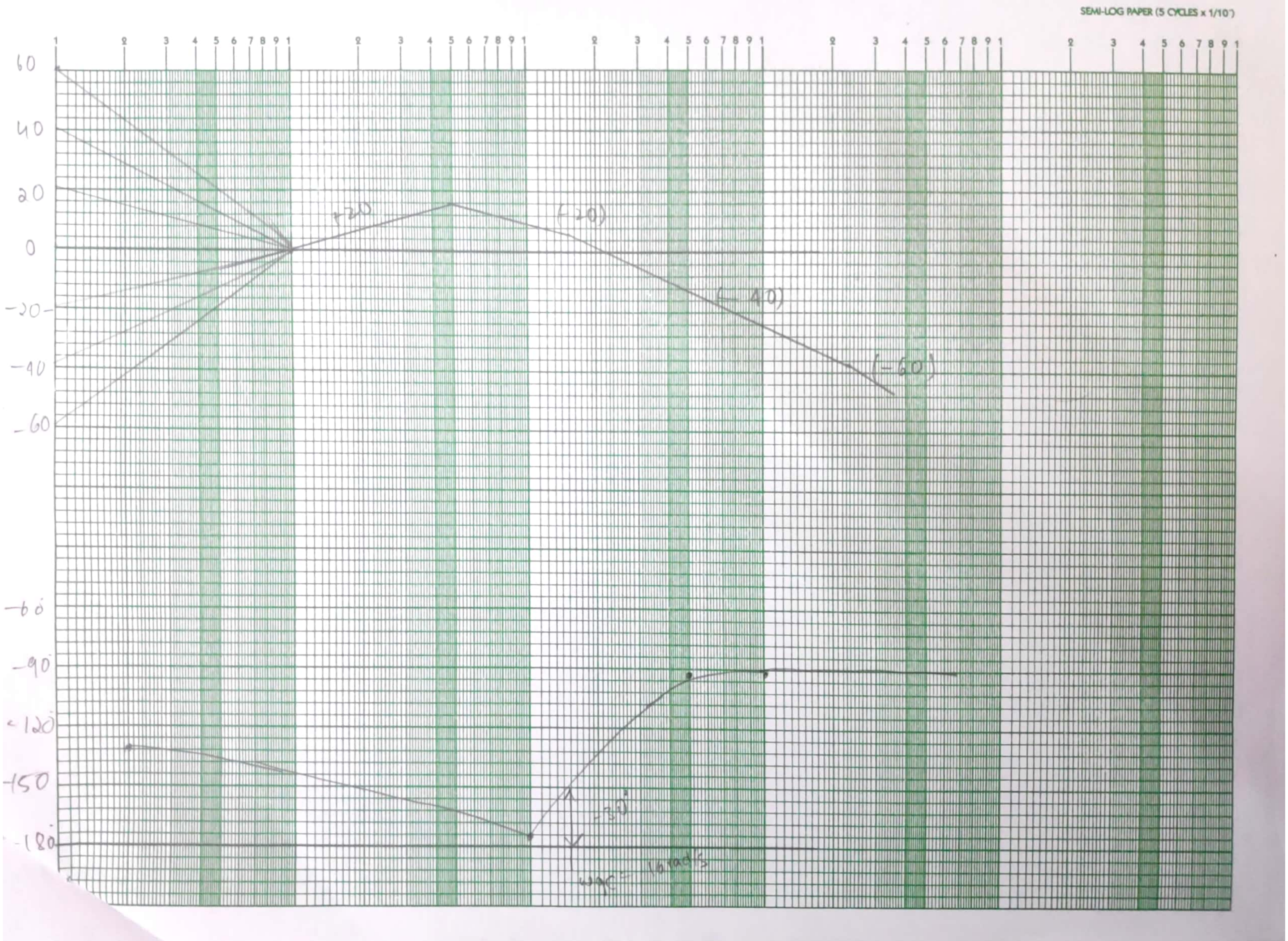
Now, $\dfrac{1}{\omega_n^2} = 8.26\times10^{-3}$

$\omega_n = 11\,rad/s = \omega_{c_3}$

Range (rad/s)	Slope (dB/dec)
0-5	20
5-11	-20
11-25	-40
25-∞	-60

$\phi = \tan^{-1}\left[\dfrac{\frac{2\xi\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$

ω	$\frac{1}{i\omega}$ $\tan^{-1}\left(\frac{\omega}{0}\right)$	$+\tan^{-1}\left(\frac{0.2\omega}{1}\right)$	$-\tan^{-1}\left(\frac{\omega}{1}\right)$	$-\tan^{-1}\left(\frac{0.04\omega}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right)$	ϕ_R
0.2	-90	2.29	-11.303°	-0.45°	-99.9
10	↓	63.43	-84.28°	-66.54°	-177
50		84.28	-88.85°	5.8°	-88
100		87.137	-89.427	2.8°	-89.97
∞	-90°	90°	-90°	0	-90


SEMI-LOG PAPER (5 CYCLES x 1/10")
60
40
20
0
-20
-40
-60
-60
-90
-120
-150
-180
+20
(-20)
(-40)
(-60)
-30°
ωgc = 16 rad/s