CBCS SCHEME

17MATDIP41

USN | | | | | | | | | | |

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020

Additional Mathematics – II

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Find the rank of the matrix:
$$A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$$ by elementary row transformations. **(08 Marks)**

b. Solve by Gauss elimination method
$$2x + y + 4z = 12$$
$$4x + 11y - z = 33$$
$$8x - 3y + 2z = 20$$ **(06 Marks)**

c. Find all the eigen values for the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **(06 Marks)**

OR

2 a. Reduce the matrix
$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$ into its echelon form and hence find its rank. **(06 Marks)**

b. Applying Gauss elimination method, solve the system of equations
$$2x + 5y + 7z = 52$$
$$2x + y - z = 0$$
$$x + y + z = 9$$ **(06 Marks)**

c. Find all the eigen values for the matrix $A = \begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ **(08 Marks)**

Module-2

3 a. Solve $\dfrac{d^4y}{dx^4} - \dfrac{2d^3y}{dx^3} + \dfrac{d^2y}{dx^2} = 0$ **(06 Marks)**

b. Solve $\dfrac{d^2y}{dx^2} - \dfrac{6dy}{dx} + 9y = 5e^{-2x}$ **(06 Marks)**

c. Solve $\dfrac{d^2y}{dx^2} + y = \sec x$ by the method of variation of parameters. **(08 Marks)**

OR

4 a. Solve $\dfrac{d^3y}{dx^3} + y = 0$ **(06 Marks)**

b. Solve $y'' + 3y' + 2y = 12x^2$ **(06 Marks)**

c. Solve by the method of undetermined coefficients :
$y'' - 4y' + 4y = e^x$ (08 Marks)

Module-3

5 a. Find the Laplace transforms of $\sin 5t \cos 2t$ (06 Marks)
b. Find the Laplace transforms of $(3t + 4)^3$ (06 Marks)
c. Express $f(t) = \begin{cases} \sin 2t & 0 < t < \pi \\ 0 & t > \pi \end{cases}$.
in terms of unit step function and hence find $L[f(t)]$. (08 Marks)

OR

6 a. Find the Laplace transforms of $\dfrac{\sin^2 t}{t}$ (06 Marks)
b. Find the Laplace transform of $2^t + t \sin t$ (06 Marks)
c. If $f(t) = t^2$, $0 < t < 2$ and $f(t + 2) = f(t)$, for $t > 2$, find $L[f(t)]$. (08 Marks)

Module-4

7 a. Find the Laplace Inverse of
$$\frac{1}{(s+1)(s-1)(s+2)}$$ (08 Marks)
b. Find the inverse Laplace transform of $\dfrac{3s+7}{s^2 - 2s - 3}$. (06 Marks)
c. Solve $y'' + 2y' - 3y = \sin t$, $y(0) = 0$, $y'(0) = 0$. (06 Marks)

OR

8 a. Find the inverse Laplace transform of
$$\log\left(\frac{s+a}{s+b}\right)$$ (06 Marks)
b. Find the inverse Laplace transform of $\dfrac{4s-1}{s^2+25}$ (06 Marks)
c. Find the inverse Laplace of $y'' - 5y' + 6y = e^t$ with $y(0) = y'(0) = 0$. (08 Marks)

Module-5

9 a. State and prove Addition theorem on probability. (05 Marks)
b. A student A can solve 75% of the problems given in the book and a student B can solve 70%. What is the probability that A or B can solve a problem chosen at random. (06 Marks)
c. Three machines A, B, C produce 50%, 30% and 20% of the items in a factory. The percentage of defective outputs of these machines are 3, 4 and 5 respectively. If an item is selected at random, what is the probability that it is defective? If a selected item is defective, what is the probability that it is from machine A? (09 Marks)

OR

10 a. Find the probability that the birth days of 5 persons chosen at random will fall in 12 different calendar months. (05 Marks)
b. A box A contains 2 white balls and 4 black balls. Another box B contains 5 white balls and 7 black balls. A ball is transferred from box A to box B. Then a ball is drawn from box B. Find the probability that it is white. (06 Marks)
c. State and prove Baye's theorem. (09 Marks)

* * * * *

Fourth Semester B.E. Degree Exam - Dec'19/Jan 20
Additional Maths - II (17MATDIP41)
(Solutions)

Module - I

Q1. (a) $A = \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 4 & 8 & 13 & 12 \end{bmatrix}$

$R_3 \to R_3 - 2R_1$ $\quad A \sim \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \end{bmatrix}$

$R_3 \to R_3 - R_1$

$A \sim \begin{bmatrix} 2 & 3 & 5 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad \therefore \rho(A) = 2$

(b) $C = \left[\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 4 & 11 & -1 & 33 \\ 8 & -3 & 2 & 20 \end{array}\right]$ $\quad R_2 \to R_2 - 2R_1$, $\; R_3 \to R_3 - 4R_1$

$C \sim \left[\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & 9 & -9 & 9 \\ 0 & -7 & -14 & -28 \end{array}\right]$ $\quad R_2 \to R_2/9, \; R_3 \to -R_3/7$

$C \sim \left[\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 2 & 4 \end{array}\right]$ $\quad R_3 \to R_3 - R_2$

$C \sim \left[\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 3 & 3 \end{array}\right]$ $\Rightarrow$
$2x + y + 4z = 12$
$y - z = 1$
$3z = 3$

$\Rightarrow z = 1, \; y = 2, \; x = 3$ Ans

c) The characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0 \Rightarrow (8-\lambda)\{(7-\lambda)(3-\lambda) - 16\} + 6\{-6(3-\lambda) + 8\}$$
$$+ 2\{(24) - 2(7-\lambda)\} = 0$$

$$(8-\lambda)\{21 - 10\lambda + \lambda^2 - 16\} + 6\{-18 + 6\lambda + 8\} + 2\{24 - 14 + 2\lambda\} = 0$$

$$(8-\lambda)\{\lambda^2 - 10\lambda + 5\} + 6(6\lambda - 10) + 2(2\lambda + 10) = 0$$

$$\Rightarrow 8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda - 60 + 4\lambda + 20 = 0$$

$$\Rightarrow -\lambda^3 + 18\lambda^2 - 45\lambda = 0 \qquad \lambda^3 - 18\lambda^2 + 45\lambda = 0 \quad \lambda\{\lambda^2 - 18\lambda + 45) = 0$$

$\lambda = 0, 3, 15$

$\therefore$ The eigen values of given matrix are $0, 3, 15$

Q2.(a) $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ $\quad R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$ $\quad R_3 \to R_2 + R_1$

$A \sim \begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad \therefore \rho(A) = 2$

SN.

2. b) $2x + 5y + 7z = 52$
$2x + y - z = 0$
$x + y + z = 9$

Augmented matrix is $(A:B) = \left(\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 2 & 1 & -1 & 0 \\ 1 & 1 & 1 & 9 \end{array}\right)$

$R_2 \to R_2 - R_1$, $R_3 \to 2R_3 - R_1$

$$\sim \left(\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 0 & -4 & -8 & -52 \\ 0 & -3 & -5 & -34 \end{array}\right)$$

$R_2 \to -\frac{1}{4} R_2$

$$\sim \left(\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 0 & 1 & 2 & 13 \\ 0 & -3 & -5 & -34 \end{array}\right)$$

$R_3 \to 3R_2 + R_3$

$$\sim \left(\begin{array}{ccc|c} 2 & 5 & 7 & 52 \\ 0 & 1 & 2 & 13 \\ 0 & 0 & 1 & 5 \end{array}\right)$$

$\Rightarrow z = 5$
$y + 2z = 13$
$2x + 5y + 7z = 52.$

By back substitution,

$y + 2(5) = 13$
$y + 10 = 13$
$\Rightarrow y = 3$

$2x + 5(3) + 7(5) = 52$
$2x + 15 + 35 = 52$
$2x = 52 - 50$
$\Rightarrow x = 1.$

$\therefore x = 1$
$y = 3$
$z = 5$

c) $A = \begin{pmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{pmatrix}$

Characteristic eq^n is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (7-\lambda)\left[(6-\lambda)(5-\lambda) - 4\right] + 2\left[-2(5-\lambda) + 0\right] = 0$$

$$\Rightarrow (7-\lambda)(30 - 6\lambda - 5\lambda + \lambda^2 - 4) - 4(5-\lambda) = 0.$$

$$\Rightarrow (7-\lambda)(\lambda^2 - 11\lambda + 26) - 20 + 4\lambda = 0.$$

$$\Rightarrow 7\lambda^2 - 77\lambda + 182 - \lambda^3 + 11\lambda^2 - 26\lambda + 4\lambda - 20 = 0.$$

$$\Rightarrow -\lambda^3 + 18\lambda^2 - 99\lambda + 162 = 0.$$

$\lambda = 3, 9, 6$, eigenvalues of A.

Module - 2.

3. a) $\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = 0.$

$\Rightarrow (D^4 - 2D^3 + D^2)y = 0$ where $D = \frac{d}{dx}$.

Auxiliary eqn $m^4 - 2m^3 + m^2 = 0$

$\Rightarrow m^2(m^2 - 2m + 1) = 0.$

$\Rightarrow m^2(m-1)^2 = 0$

$\Rightarrow m = 0, 0, 1, 1.$

$\therefore$ The solution is,

$y = (c_1 + c_2x)e^{0x} + (c_3 + c_4x)e^x.$

$= (c_1 + c_2x) + (c_3 + c_4x)e^x$; where

c_1, c_2, c_3, c_4 are constants.

b) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 5e^{-2x}.$

$\Rightarrow (D^2 - 6D + 9)y = 5e^{-2x}$ where $D = \frac{d}{dx}$.

Auxiliary eqn. is $m^2 - 6m + 9 = 0$

$\Rightarrow (m-3)^2 = 0.$

$\Rightarrow m = 3, 3.$

$\therefore$ Complimentary fn. is $y_c = (c_1 + c_2x)e^{3x}$, where

c_1 and c_2 are constants.

Particular integral, $y_p = \frac{1}{D^2 - 6D + 9} \cdot 5e^{-2x}.$

$= \frac{5e^{-2x}}{(-2)^2 - 6(-2) + 9}$

$= \frac{1}{5}e^{-2x}.$

$\therefore$ General soln $y = y_c + y_p = (c_1 + c_2x)e^{3x} + \frac{1}{5}e^{-2x}.$

c) $y'' + y = \sec x$

$(D^2+1)y = \sec x.$

$m^2+1=0 \Rightarrow m = \pm i$

$\therefore\ y_c = c_1 \cos x + c_2 \sin x$

Let $y = A(x)\cos x + B(x)\sin x$ — (1)

be the complete solution of the given equation, where $A(x)$, $B(x)$ are to be found.

we have

$y_1 = \cos x$ $\qquad$ $y_2 = \sin x$

$y_1' = -\sin x$ $\qquad$ $y_2' = \cos x$

$W = y_1 y_2' - y_2 y_1' = 1$ $\qquad$ Also $\phi(x) = \sec x$

$A' = \dfrac{-y_2\phi(x)}{W}$ $\qquad$ $B' = \dfrac{y_1\phi(x)}{W}$

$A' = \dfrac{-\sin x \sec x}{1}$, $\qquad$ $B' = \dfrac{\cos x \sec x}{1}$

$A' = -\tan x$ $\qquad$ $B' = 1$

$A = -\log(\sec x) + K_1$, $\qquad$ $B = x + K_2$

substituting in (1) we have

$y = K_1 \cos x + K_2 \sin x - \cos x \log(\sec x) + x \sin x.$

4 (a) Solve $\dfrac{d^3y}{dx^3} + y = 0$

$\therefore$ A.E $\quad m^3 + 1 = 0$

$\Rightarrow \quad (m+1)(m^2 - m + 1) = 0$

$m = -1, \quad m = \dfrac{1 \pm i\sqrt{3}}{2}$

Thus, $y = c_1 e^{-x} + e^{x/2}\left\{ c_2 \cos\frac{\sqrt{3}}{2}x + c_3 \sin\frac{\sqrt{3}}{2}x \right\}$.

4 (b) Solve $y'' + 3y' + 2y = 12x^2$

as we have $\quad (D^2 + 3D + 2)y = 12x^2$

A.E is, $\quad m^2 + 3m + 2 = 0$ or $(m+1)(m+2) = 0$

$\Rightarrow m = -1, -2$

$\therefore y_c = c_1 e^{-x} + c_2 e^{-2x}$

$y_p = \dfrac{12x^3}{D^2 + 3D + 2}$

we need to divide for obtaining the PI.

$$
\begin{array}{r|l}
 & 6x^2 - 18x + 21 \\
\hline
2 + 3D + D^2 & 12x^3 \\
 & 12x^2 + 36x + 12 \\
\hline
 & -36x - 12 \\
 & -36x - 54 \\
\hline
 & 42 \\
 & 42 \\
\hline
 & 0
\end{array}
$$

Hence $y_p = 6x^2 - 18x + 21$

Complete solution $y = y_c + y_p$

Thus $\boxed{y = c_1 e^{-x} + c_2 e^{-2x} + 6x^2 - 18x + 21}$

4(c) $y'' - 4y' + 4y = e^x$ --- (1)

$\Rightarrow$ we have $(D^2 - 4D + 4)y = e^x$

$\therefore$ AE $m^2 - 4m + 4 = 0$

$(m-2)^2 = 0 \Rightarrow m = 2, 2$

$\Rightarrow$ $y_c = (c_1 + c_2 x) e^{2x}$

P.I is of the form $y = ae^x$

$y' = ae^x, \; y'' = ae^x$ --- (2)

substituting these in the (1), we get

$ae^x - 4ae^x + 4ae^x = e^x$

$ae^x = e^x \Rightarrow a = 1$

$\therefore$ (2) becoms P.I $= y_p = 1e^x$

$\therefore$ General solution $y = y_c + y_p$

$$\boxed{y = (c_1 + c_2 x) e^{2x} + e^x}$$

//

Scanned by CamScanner

CMR

5(a) $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\sin 5t \cos 2t = \frac{1}{2}[\sin 7t + \sin 3t]$

$L[\sin 5t \cos 2t] = \frac{1}{2}\left[\frac{7}{s^2+49} + \frac{3}{s^2+9}\right]$

(b) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$(3t+4)^3 = (3t)^3 + 4^3 + 3 \cdot 3t \cdot 4(3t+4)$

$(3t+4)^3 = 27t^3 + 64 + 108t^2 + 144t$ $\qquad L[t^n] = \frac{n!}{s^{n+1}}$

$L[3t+4]^3 = 27 \cdot \frac{3!}{s^4} + \frac{64}{s} + 108\frac{2!}{s^3} + 144\frac{1}{s^2}$

(c) $f(t) = \begin{cases} \sin 2t, & 0 < t < \pi \\ 0, & t > \pi \end{cases}$

WKT If $f(t) = \begin{cases} f_1(t), & t \leq a \\ f_2(t), & t > a \end{cases}$ then $f(t) = f_1(t) + [f_2(t) - f_1(t)]u(t-a)$

$\therefore f(t) = \sin 2t + [(0 - \sin 2t)u(t-\pi)] \Rightarrow L[f(t)] = L[\sin 2t] - L[\sin 2t\, u(t-\pi)]$

Let us find $L[\sin 2t\, u(t-\pi)]$

Taking $F(t-\pi) = \sin 2t$, $F(t) = \sin(2+\pi) = \sin(2\pi + 2t)$

$\therefore F(t) = \sin 2t \quad \therefore \bar{F}(s) = 2/s^2 + 4$

$\therefore L[F(t-\pi)u(t-\pi)] = e^{-\pi s}\bar{F}(s) = \frac{2e^{-\pi s}}{s^2+4}$

$\therefore L[f(t)] = \frac{2}{s^2+4} - \frac{2e^{-\pi s}}{s^2+4}$

6(a) $f(t) = \sin^2 t = \frac{1}{2}(1 - \cos 2t)$, $\bar{f}(s) = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$

$L\left[\frac{f(t)}{t}\right] = \frac{1}{2}\int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+4}\right) ds$

$L\left[\frac{\sin^2 t}{t}\right] = \frac{1}{2}\left[\log s - \frac{1}{2}\log(s^2+4)\right]_s^\infty = \frac{1}{2}\log\left[\frac{s}{\sqrt{s^2+4}}\right]_s^\infty = \lim_{s\to\infty}\frac{1}{2}\log\left[\frac{s}{s\sqrt{1+(4/s^2)}}\right] - \frac{1}{2}\log\frac{s}{\sqrt{s^2+4}}$

$= \frac{1}{2}\left\{\log 1 - \log\left(\frac{s}{\sqrt{s^2+4}}\right)\right\} = \frac{1}{2}\log\left(\frac{\sqrt{s^2+4}}{s}\right)$

⑥ (b) Given $f(t) = 2^t + t\sin t$

Consider $f_1(t) = 2^t$

$$\therefore L[f_1(t)] = L[2^t] = L[e^{\log_e 2^t}]$$

$$= L[e^{t\log 2}]$$

$$\therefore L[2^t] = \frac{1}{s-\log 2}$$

Let $f_2(t) = t\sin t$

Since, $L[\sin t] = \frac{1}{s^2+1}$

$$L[f_2(t)] = L[t\sin t] = (-1)\frac{d}{ds}\left(\frac{1}{s^2+1}\right)$$

$$= (-1)\frac{(-1)}{(s^2+1)^2}\cdot 2s$$

$$\therefore L[t\sin t] = \frac{2s}{(s^2+1)^2}$$

$$\boxed{\therefore L[2^t + t\sin t] = \frac{1}{s-\log 2} + \frac{2s}{(s^2+1)^2}}$$

⑥ (c) Given that $f(t)$ is a periodic function with period 2. Therefore,

$$L[f(t)] = \frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$$

$$= \frac{1}{1-e^{-2s}}\left(\int_0^2 e^{-st} t^2\,dt\right)$$

$$= \frac{1}{1-e^{-2s}}\left[t^2 \frac{e^{-st}}{-s}\Big|_0^2 - \int_0^2 2t\,\frac{e^{-st}}{-s}\,dt \right]$$

$$= \frac{1}{1-e^{-2s}}\left[\frac{-4e^{-2s}}{s} + 0 + \frac{2}{s}\left(t\frac{e^{-st}}{-s}\Big|_0^2 - \int_0^2 1\,\frac{e^{-st}}{-s}\,dt \right) \right]$$

$$= \frac{1}{1-e^{-2s}}\left(-\frac{4e^{-2s}}{s} + \frac{2}{s}\left[\frac{-2e^{-2s}}{s} + 0 + \frac{1}{s}\int_0^2 e^{-st}\,dt \right] \right)$$

$$= \frac{1}{1-e^{-2s}}\left[\frac{-4e^{-2s}}{s} - \frac{4e^{-2s}}{s^2} + \frac{2}{s^2}\left(\frac{e^{-st}}{-s} \right)_0^2 \right]$$

$$= \frac{1}{1-e^{-2s}}\left(\frac{-4e^{-2s}}{s} - \frac{4e^{-2s}}{s^2} + \frac{2e^{-2s}}{s^3} \right)$$

$$= \frac{-2e^{-2s}}{s^3(1-e^{-2s})}\left(2s^2+2s+1\right)$$

$$\boxed{\therefore\ L[f(t)] = \frac{-(2s^2+2s+1)\,2e^{-2s}}{s^3(1-e^{-2s})}}$$

—— x ——

7 a Let $f(s) = \dfrac{1}{(s+1)(s-1)(s+2)}$

$$= \frac{A}{s+1} + \frac{B}{s-1} + \frac{C}{s+2}$$

$$= -\frac{1}{2(s+1)} + \frac{1}{4(s-1)} + \frac{1}{3(s+2)}$$

$$\therefore L^{-1}(f(s)) = L^{-1}\left(-\frac{1}{2(s+1)} + \frac{1}{4(s-1)} + \frac{1}{3(s+2)}\right)$$

$$= -\frac{1}{2} L^{-1}\left[\frac{1}{s+1}\right] + \frac{1}{4} L^{-1}\left(\frac{1}{s-1}\right) + \frac{1}{3} L^{-1}\left(\frac{1}{s+2}\right)$$

$$= -\frac{1}{2} e^{-t} + \frac{1}{4} e^{t} + \frac{1}{3} e^{-2t}$$

$$\therefore L^{-1}\left(\frac{1}{(s+1)(s-1)(s+2)}\right) = -\frac{e^{-t}}{2} + \frac{1}{4} e^{t} + \frac{1}{3} e^{-2t}$$

7 b Let $F(s) = \dfrac{3s+7}{s^2-2s-3}$

$$= \frac{3s+7}{s^2-2s+1-4}$$

$$= \frac{3s+7}{(s-1)^2-4}$$

$$= \frac{3(s-1)}{(s-1)^2-4} + \frac{10}{(s-1)^2-4}$$

$$\therefore L^{-1}[F(s)] = L^{-1}\left(\frac{3(s-1)}{(s-1)^2-2^2}\right) + L^{-1}\left[\frac{10}{(s-1)^2-4}\right]$$

$$= 3e^t L^{-1}\left[\frac{s}{s^2-2^2}\right] + \frac{10}{2}e^t L^{-1}\left[\frac{2}{s^2-2^2}\right]$$

$$\therefore L^{-1}[F(s)] = 3e^t . \sinh 2t + 5e^t \cosh 2t .$$

—— X ——

7c) Solve $y'' + 2y' - 3y = \sin t$ $\quad y(0) = 0 \; ; \; y'(0) = 0$

$$L\{y''(t)\} + 2L\{y'(t)\} - 3L\{y(t)\} = L\{\sin t\}$$

$$\{s^2 L\{y(t)\} - sy(0) - y'(0)\}$$
$$+ 2\{sL\{y(t)\} - y(0)\} - 3L\{y(t)\} = \frac{1}{s^2+1}$$

Using given initial conditions

$$(s^2 + 2s - 3) \, L\{y(t)\} = \frac{1}{s^2+1}$$

$$L\{y(t)\} = \frac{1}{(s^2+1)(s^2+2s-3)} = \frac{1}{(s^2+1)(s-1)(s+3)}$$

$$y(t) = L^{-1}\left[\frac{1}{(s^2+1)(s-1)(s+3)}\right]$$

$$\frac{1}{(s^2+1)(s-1)(s+3)} = \frac{As+B}{s^2+1} + \frac{C}{s-1} + \frac{D}{s+3}$$

(ie) $\quad 1 = (As+B)(s-1)(s+3) + C(s^2+1)(s+3) + D(s^2+1)(s-1)$

Put $s=1$ $\Rightarrow$ $1 = 8C$ $\Rightarrow$ $C = 1/8$

Put $s=-3$ $\Rightarrow$ $1 = D(-40)$ $\Rightarrow$ $D = -1/40.$

Put $s=0$ $\Rightarrow$ $1 = B(-3) + C(3) + D(-1)$

$\Rightarrow$ $1 = -3B + 3/8 + 1/40$ $\Rightarrow$ $B = -1/5$

equating coeff of s^3 on both sides we get

$0 = A + C + D$ $\Rightarrow$ $A = -1/10$

$$\therefore \quad y(t) = \frac{-1}{10} L^{-1}\left(\frac{s}{s^2+1}\right) - \frac{1}{5} L^{-1}\left\{\frac{1}{s^2+1}\right\}$$

$$+\frac{1}{8} L^{-1}\left\{\frac{1}{(s-1)}\right\} - \frac{1}{40} L^{-1}\left(\frac{1}{s+3}\right)$$

$$\therefore y(t) = \frac{-\cos t}{10} - \frac{1}{5}\sin t + \frac{1}{8}e^{t} - \frac{1}{40}e^{-3t}.$$

Scanned by CamScanner

8) a) find inverse L.T of $\log\left(\frac{s+a}{s+b}\right)$

$\bar{f}(s) = \log\left(\frac{s+a}{s+b}\right) = \log(s+a) - \log(s+b)$

$-\bar{f}'(s) = -\left\{\frac{1}{s+a} - \frac{1}{s+b}\right\}$

$-\bar{f}'(s) = \frac{1}{s+b} - \frac{1}{s+a}$

$L^{-1}\left\{-\bar{f}'(s)\right\} = L^{-1}\left\{\frac{1}{s+b}\right\} - L^{-1}\left\{\frac{1}{s+a}\right\}$

$\Rightarrow t\,f(t) = e^{-bt} - e^{-at}$

$\Rightarrow f(t) = \frac{e^{-bt} - e^{-at}}{t}$

8) b) $L^{-1}\left(\frac{4s-1}{s^2+25}\right) = ?$

$= 4\,L^{-1}\left(\frac{s}{s^2+5^2}\right) - L^{-1}\left(\frac{1}{s^2+5^2}\right)$

$= 4\cos 5t - \frac{1}{5}\sin 5t$

8) c) $y'' - 5y' + 6y = e^t$ with $y(0)=0$; $y'(0)=0$

$L\{y''(t)\} - 5L\{y'(t)\} + 6L\{y(t)\} = L\{e^t\}$.

$\{s^2 L\{y(t)\} - sL(y(0)) - y'(0)\} - 5\{sL\{y(t)\} - y(0)\}$
$+ 6L\{y(t)\} = \frac{1}{s-1}$

$(s^2 - 5s + 6) L\{y(t)\} = \frac{1}{s-1}$

$L\{y(t)\} = \frac{1}{(s-1)(s-3)(s-2)}$

$$\frac{1}{(s-1)(s-2)(s-3)} = \frac{A}{(s-1)} + \frac{B}{(s-2)} + \frac{C}{(s-3)}$$

$1 = A(s-2)(s-3) + B(s-1)(s-3) + C(s-1)(s-2)$

Put $s=2$ $\Rightarrow$ $1 = B(1)(-1)$ $\Rightarrow$ $B = -1$

Put $s=3$ $\Rightarrow$ $1 = 2 \times 1\, C$ $\Rightarrow$ $C = 1/2$

Put $s=1$ $\Rightarrow$ $1 = A(-1)(-2)$ $\Rightarrow$ $A = 1/2$

$\therefore y(t) = \frac{1}{2} L^{-1}\left(\frac{1}{s-1}\right) - 1\, L^{-1}\left(\frac{1}{s-2}\right) + L^{-1}\left(\frac{1}{s-3}\right)$

$y(t) = \frac{1}{2} e^t - e^{2t} + e^{3t}$.

9. a) Addition theorem of probability

The probability of the happening of one or the other mutually exclusive events = sum of the probabilities of the two events.

i.e., if A, B are mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B).$$

Pf :- Let the total no of exhaustive, mutually exclusive and equally possible cases in the trials be 'n'. Out of these let m_1 cases be favourable to A and m_2 cases to B.

$\therefore$ No of favourable cases to either A or B $= m_1 + m_2$

$$\therefore P(A \text{ or } B) = \frac{m_1}{n} + \frac{m_2}{n}.$$

$$= P(A) + P(B).$$

Hence the proof.

9 b) The probability that student A solve problems in Book is $P(A) = 0.75$

The probability that student B solve problems in book is $P(B) = 0.70$

$\therefore$ We have $P(\bar{A}) = 0.25$ and $P(\bar{B}) = 0.30$

The probability that A or B can solve a problem choosen at random is given by $P(A \cup B)$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\because$ A and B are independent $P(A \cap B) = P(A) \cdot P(B)$

$\therefore$ $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

$\therefore$ $P(A \cup B) = 0.75 + 0.70 - (0.75)(0.70)$

$= 1.45 - 0.525$

$\therefore$ $\boxed{P(A \cup B) = 0.925}$

9c) Machine A produces 50% of the items of the factory and out of these 3% are defective.

Let D denote the event of selecting a defective item.

$\therefore$ $P(A) = 0.5$ and $P(D/A) = 0.03$

$P(B) = 0.3$ and $P(D/B) = 0.04$

$P(C) = 0.2$ and $P(D/C) = 0.05$

We have $P(D) = P(A)P(D/A) + P(B)P(D/B) + P(C)P(D/C)$

$\therefore$ $P(D) = (0.5)(0.03) + (0.3)(0.04) + (0.2)(0.05)$

$\boxed{P(D) = 0.037}$

Thus the probability of selecting a defective item is 0.037.

Next, we shall find the that the probability that the defective item is from A.

That is to find P(A/D) and we have by Bayer's theorem

$$P(A/D) = \frac{P(A)\,P(D/A)}{P(D)} = \frac{(0.5)(0.03)}{0.037}$$

$$\boxed{P(A/D) = 0.4054}$$

10 a) The sample space of birth days of 5 persons choosen at random will fall in 12 different calendar months $= 12 \times 12 \times 12 \times 12 \times 12 = 12^5$

favourable event $= 12 \times 11 \times 10 \times 9 \times 8$

$\therefore$ The probability $= \frac{12 \times 11 \times 10 \times 9 \times 8}{12^5}$

$= 0.38194$

10b) Total number of balls in box A = 2W + 4B = 6

Total number of balls in box B = 5W + 7B = 12

Since a ball is transferred from A to B, two cases arise.

<u>case(i)</u> suppose the transferred ball is white.

Probability of the transferer of a white ball is $\frac{2}{6} = \frac{1}{3}$

Then box B will have 6W and 7B = 13 balls.

Hence probability of getting a white ball from B after the transfer is $\frac{6}{13}$.

$\therefore$ Probability of transferring a white ball and getting white from B is $\frac{1}{3} \times \frac{6}{13} = \frac{2}{13}$.

<u>case (ii)</u> suppose the transferred ball is black,

Probability of transfer is $4/6 = 2/3$.

Then box B will have 5W and 8B = 13 balls.

Hence the probability of getting a white ball after the transfer is $5/13$.

$\therefore$ Probability of transferring a black ball and getting a white from B is $\frac{2}{3} \times \frac{5}{13} = \frac{10}{39}$

Either of these two cases are favourable to the event

Thus the required probability by addition theorem is $\frac{2}{13} + \frac{10}{39} = \frac{6+10}{39} = \frac{16}{39}$

10 c) <u>Baye's theorem</u> on <u>conditional probability</u>.

Let $A_1, A_2, \ldots A_n$ be a set of exhaustive and mutually exclusive events of the sample space S with $P(A_i) \neq 0$ for each i. If A is any other event associated with A_i, ($A \subset \bigcup_{i=1}^{n} A_i$) with $P(A) \neq 0$

then $$P(A_i/A) = \frac{P(A_i)\,P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A/A_i)}$$

<u>Proof</u>: We have $S = A_1 \cup A_2 \cup \cdots \cup A_n$ and $A \subset S$

$$\therefore A = S \cap A = (A_1 \cup A_2 \cup \cdots \cup A_n) \cap A$$
$$= (A_1 \cap A) \cup (A_2 \cap A) \cup \cdots \cup (A_n \cap A)$$

$\because A_i \cap A$ for $i = 1$ to n are mutually exclusive, we have by applying the addition rule of probability.

$$P(A) = P(A_1 \cap A) + P(A_2 \cap A) + P(A_3 \cap A) + \cdots + P(A_n \cap A)$$

Now applying multiplication rule onto each term in the RHS we have.

$$P(A) = P(A_1)\,P(A/A_1) + P(A_2)\,P(A/A_2) + \cdots + P(A_n)\,P(A/A_n)$$

ie, $$P(A) = \sum_{i=1}^{n} P(A_i)\,P(A/A_i).$$

The conditional probability of A_i for any i given A, is defined by

$$P(A_i/A) = \frac{P(A_i \cap A)}{P(A)} = \frac{P(A_i)\,P(A/A_i)}{P(A)}$$

$$\therefore \boxed{P(A_i/A) = \frac{P(A_i)\,P(A/A_i)}{\sum_{i=1}^{n} P(A_i)\,P(A/A_i)}}$$

This proves Baye's theorem for conditional probability