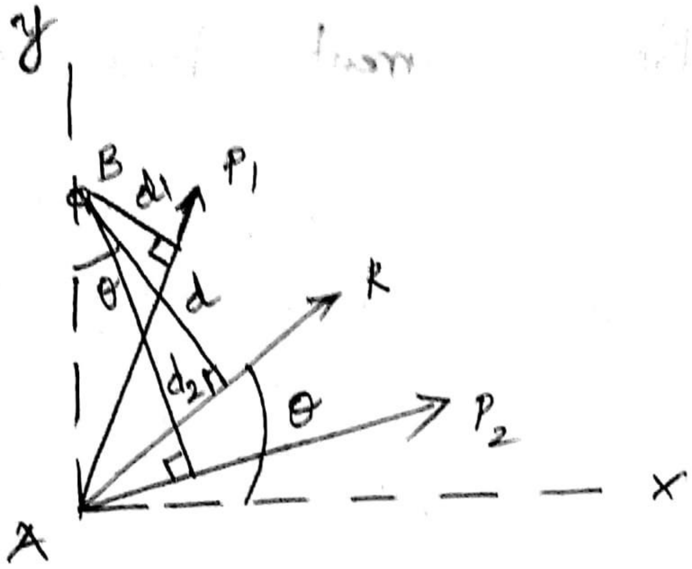
Proof



$$R \times d = R\sin\theta \times 0 + R\cos\theta \times AB$$

$$R \times d = R_x \cdot AB$$

$$P_1 \cdot d_1 = P_{1x} \; AB$$

$$P_2 \, d_2 = P_{2x} \; AB$$

$$P_1 \cdot d_1 + P_2 \, d_2 = AB\,(\underbrace{P_{1x} + P_{2x}})$$

$$= AB\,(R_x)$$

$$= Rd$$

$$\Longrightarrow \underline{\underline{P_1 d_1 + P_2 d_2 = Rd}}$$

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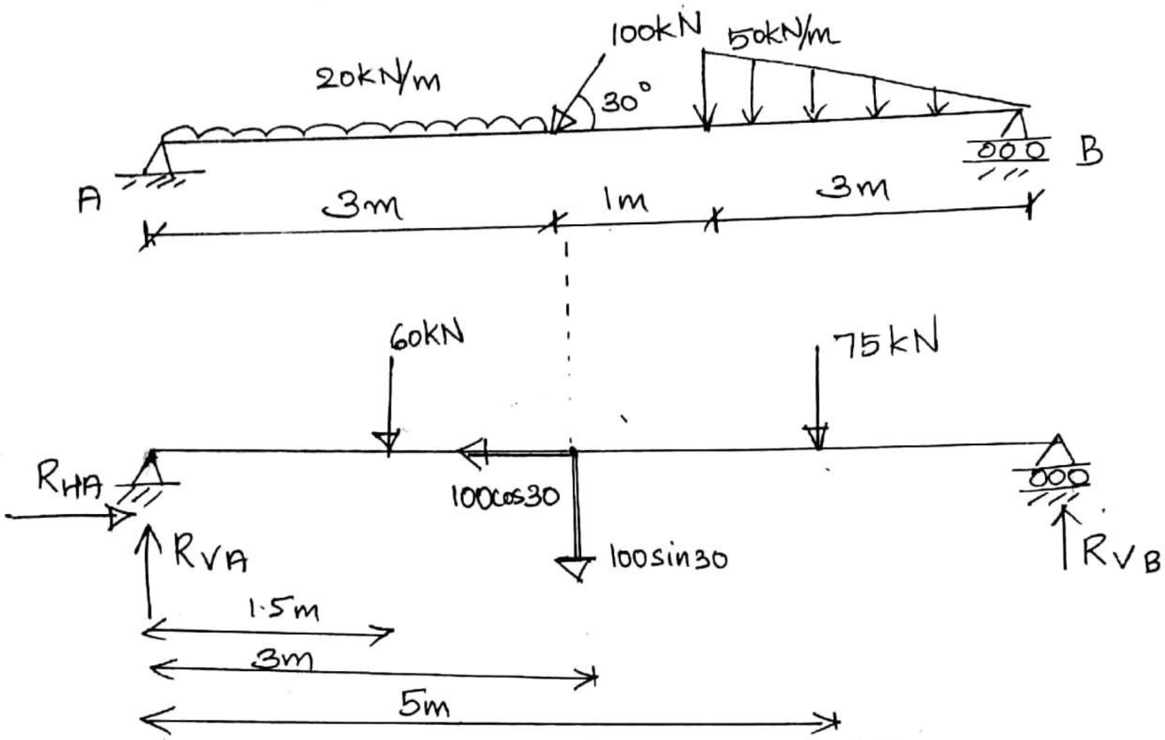

$\Sigma F_x = 0 \Rightarrow R_{HA} = 100\cos 30$

$= \underline{\underline{86.6\,kN}}$

$\Sigma F_y = 0 \Rightarrow R_{VA} - 60 - 100\sin 30 - 75 + R_{VB} = 0$

$R_{VA} + R_{VB} = 185\,kN$

$\Sigma M_A = 0 \Rightarrow (60 \times 1.5) + (100\sin 30 \times 3) + (75 \times 5)$

$(-R_{VB} \times 7) = 0$

$\Rightarrow R_{VB} = 87.85\,kN$

$R_{VA} = 185 - R_{VB}$

$= 185 - 87.85$

$= \underline{\underline{97.14\,kN}}$

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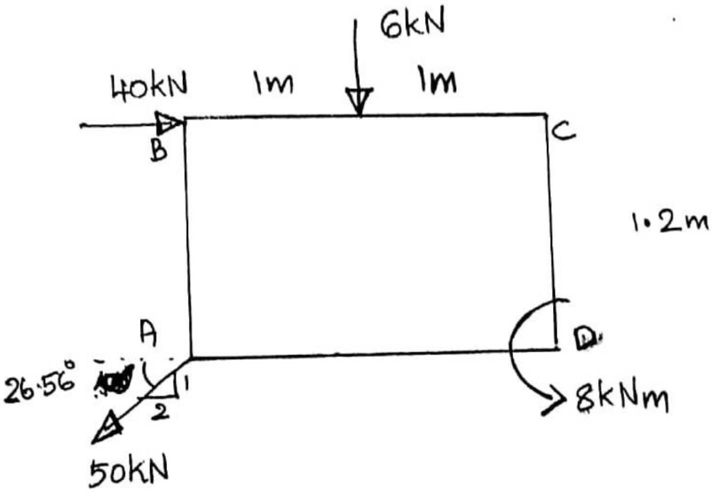


$$\Sigma F_x = 40 - 50\cos 26.56 = -4.72\,kN$$

$$\Sigma F_y = -6 - 50\sin 26.56 = -28.36\,kN$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$= \sqrt{-4.72^2 + (-28.36)^2}$$

$$= \underline{\underline{28.75\,kN}}$$

$$\theta = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x} = \tan^{-1}\frac{-28.36}{-4.72}$$

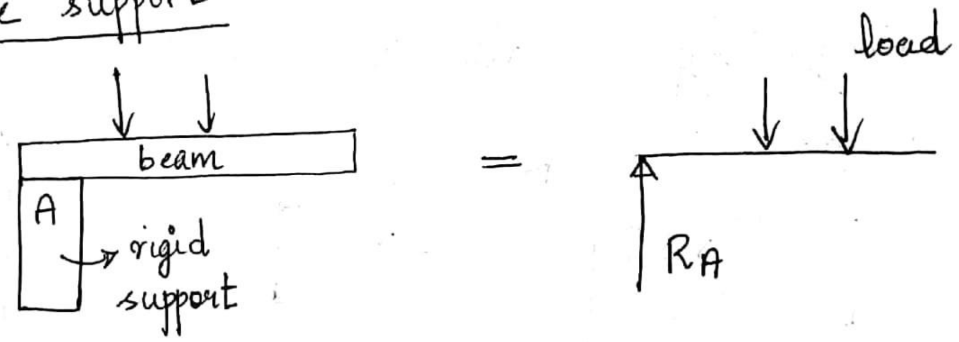
$$= \underline{\underline{80.55^\circ}}$$

$$\Sigma M_D = 40 \times 1.2 - 6 \times 1 + (50 \times \cos 26.56 \times 0)$$
$$- 50 \times \sin 26.56 \times 2 - 8 = -10.71\,kNm$$

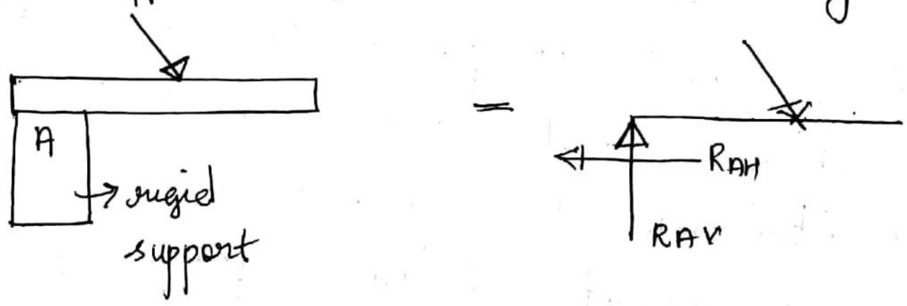
$$d = \frac{|\Sigma M_D|}{|R|} = \frac{10.71}{28.75} = \underline{\underline{0.37\,m}}$$

Types of Beam Supports

1. Simple support



Here the beam is simply placed on a rigid support. Here the reaction will be perpendicular to the rigid support opposite to the loading direction.



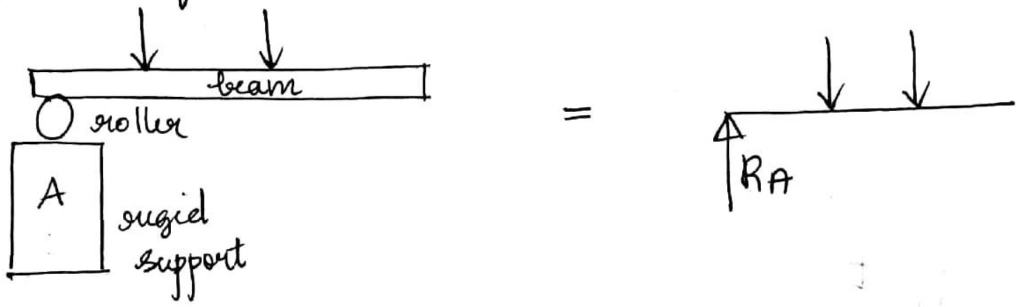
This R_{AH} is actually due to friction at point of contact b/w beam and the rigid support. Since this point of contact is very small if inclined loads come on the beam we will opt for other supports. (hinged support).

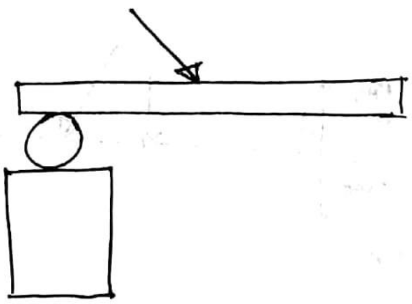
eg: beams simply placed on columns in buildings

Note: Rotation at point A is allowed, ie there is no moment reaction at A. as a result of which the beam deflects as shown.

2. Roller Support

The beam is placed on a rigid support with a roller in between to remove the friction at point of contact between the beam and the support





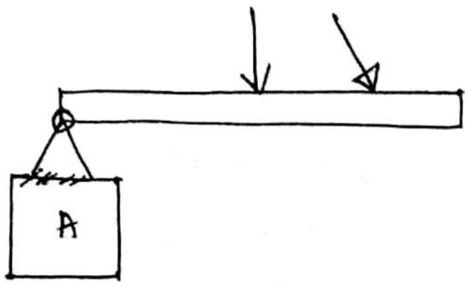
- unstable
as no horizontal reaction can be developed at A
⇒ the beam slips of the support.

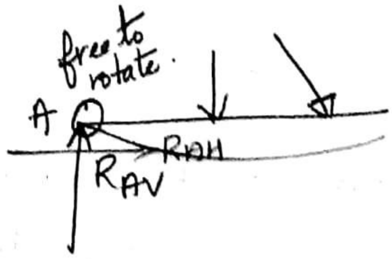
eg: bridge decks.

Note: Rotation at point A is allowed like the previous case.

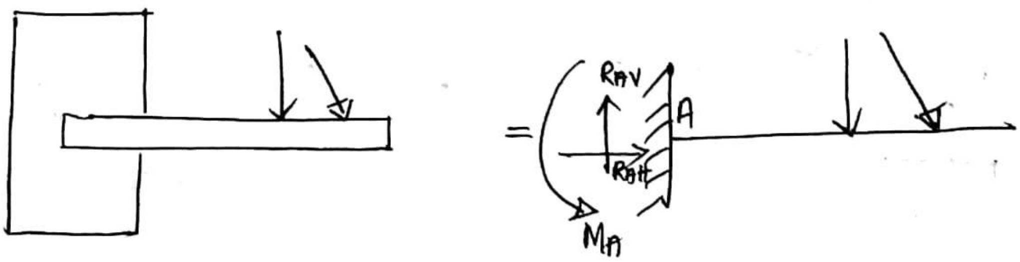
Hinged or Pinned support.

The beam is hinged / pinned on to a rigid support. This is achieved by mechanical devices. The reactions developed vertical reaction (R_{AV}), horizontal reaction R_{AH} in case of inclined loads. This support does not produce a moment reaction and the beam is free to rotate.



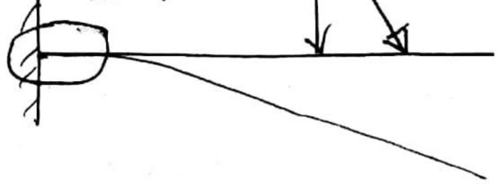


Fixed Support



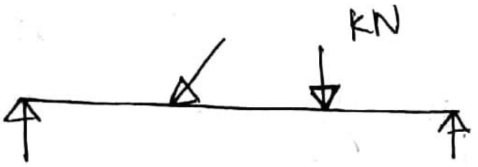
This support restricts the beam from translation in vertical and horizontal direction as well as from rotation.

eg: When reinforcements in beams are taken into columns which makes the support fixed.

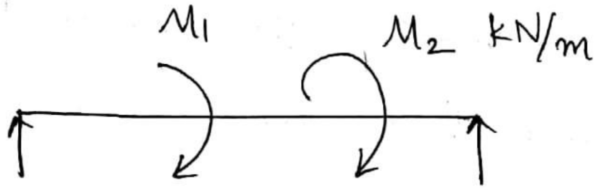


Types of Loads on Beams

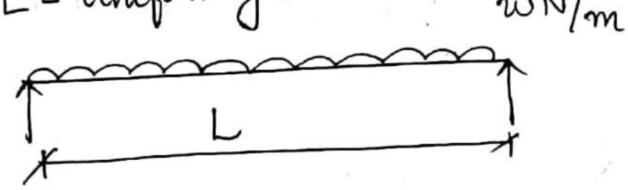
1) Point loads or Concentrated loads.



) Couple moments



) UDL - Uniformly Distributed Load



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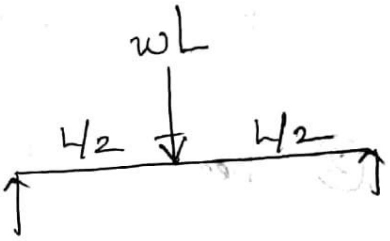
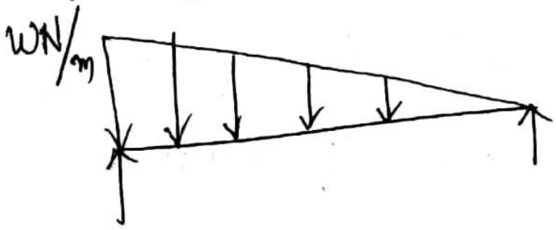


eg: weight of beam always acts like a UDL.

Uniformly Varying load (UVL)



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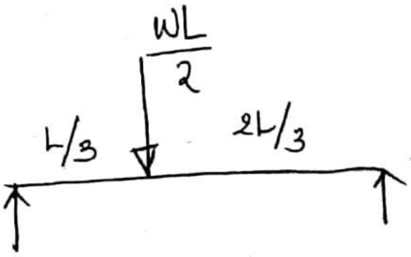

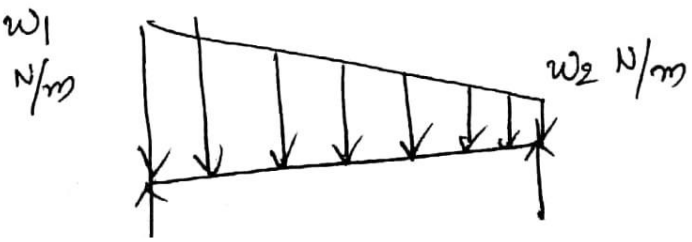

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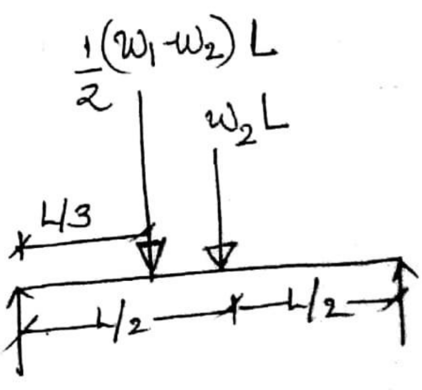
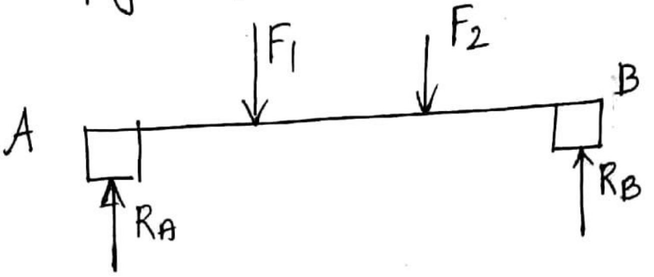
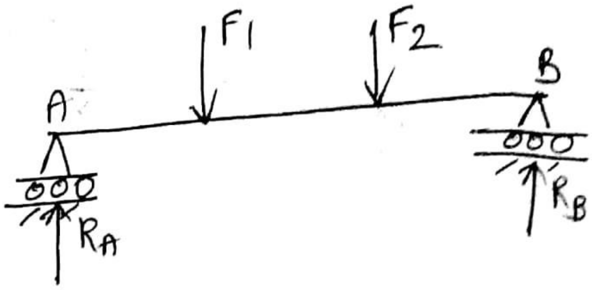


Types of Beams

① Simply supported beams and Beams on roller support



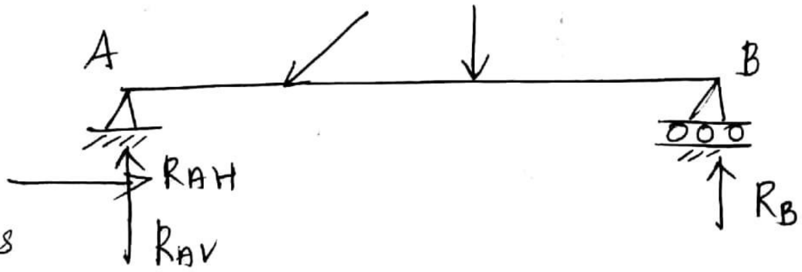


2 eqns
2 unknowns

The support reactions R_A and R_B must be found by $\boxed{\Sigma F_y = 0 \text{ and } \Sigma M = 0}$. If inclined loads come on the beam then the beam will become unstable.

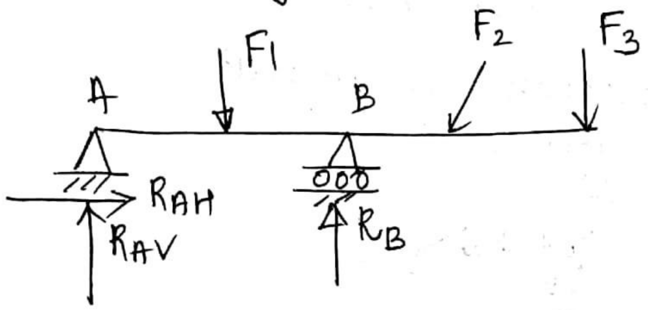
② Beams with one end hinged and the other roller.

3 eqns
3 unknowns



The reactions R_{AH}, R_{AV}, R_B can be found by $\boxed{\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma M = 0}$. The problem will simplify to the first case if inclined loads are not present.

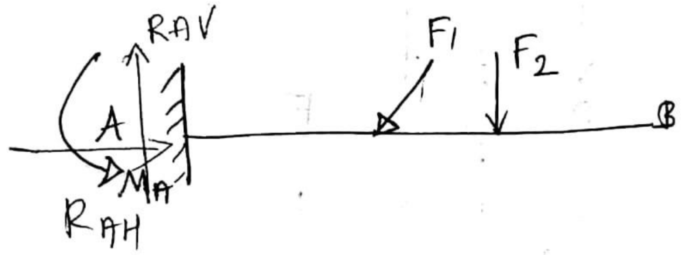
③ Overhanging beams



Cannot be stable if both supports are rollers.

R_{AH}, R_{AV}, R_B can be solved by $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$
3 equations and 3 unknowns.

④ Cantilever Beams.



R_{AV}, R_{AH}, M_A can be solved by $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$
3 equations and 3 unknowns.

In all the above case the number of unknown is equal to the number of equations. Hence all the above type of beams are <u>Statically determinate</u>

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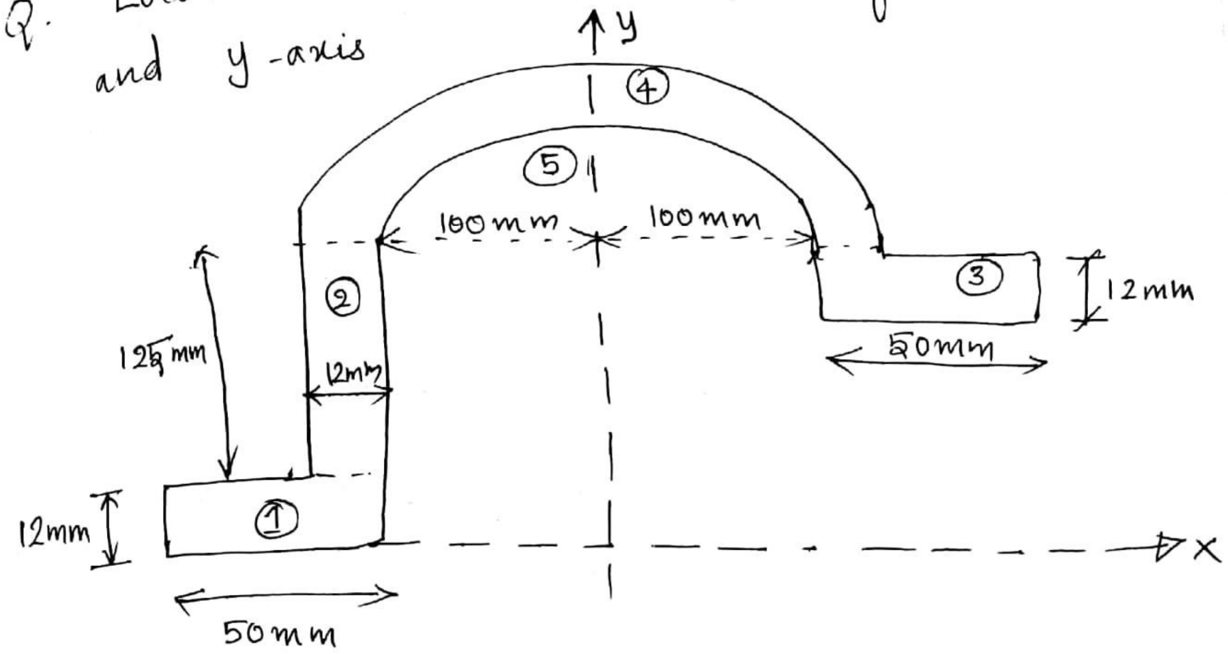
Q. Locate the centroid with reference to x and y-axis



Component	Area mm^2	x (mm)	y (mm)
1	$50 \times 12 = 600$	$-\left(100 + \frac{50}{2}\right) = -125$	$\frac{12}{2} = 6$
2.	$125 \times 12 = 1500$	$-(100 + 12/2) = -106$	$12 + 125/2 = 74.5$
3.	$50 \times 12 = 600$	$100 + 50/2 = 125$	$125 + 12/2 = 131$
~~4~~ 5	$-\frac{\pi \times (100)^2}{2} = 15707.9$	0	$125 + 12 + \frac{4 \times 100}{3\pi} = 179.44$
~~5~~ 4	$+\frac{\pi \times (\cancel{88}\,112)^2}{2} = 19704.07$ ~~+12164.25~~	0	$125 + 12 + \frac{4 \times \cancel{88}\,112}{3\pi} = 184.53$

$A = \cancel{60}\ 6696.17 \, mm^2$

$$\bar{X} = \frac{(600 \times -125) + (1500 \times -106) + (600 \times 125) - (15707.9 \times 0) + (19704.07 \times 0)}{6696.17}$$

$$= -23.75 \text{ mm}$$

$$\bar{Y} = \frac{(600 \times 6) + (1500 \times 74.5) + (600 \times 131) + -(15707.9 \times 179.44) + (19704.07 \times 184.83)}{6696.17}$$

$$= 151.03 \text{ mm}$$

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A}$$

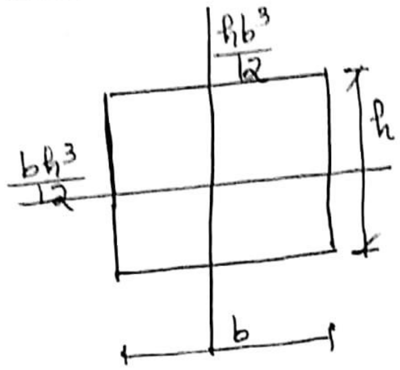
$$= \frac{(120 \times 10) \times \frac{120}{2} + 100 \times 12 \times \frac{12}{2} + 180 \times 10 \times \frac{180}{2}}{(120 \times 10) + (100 \times 12) + (180 \times 10)}$$

$$= \frac{241200}{4200} = \underline{\underline{57.43\,mm}}$$

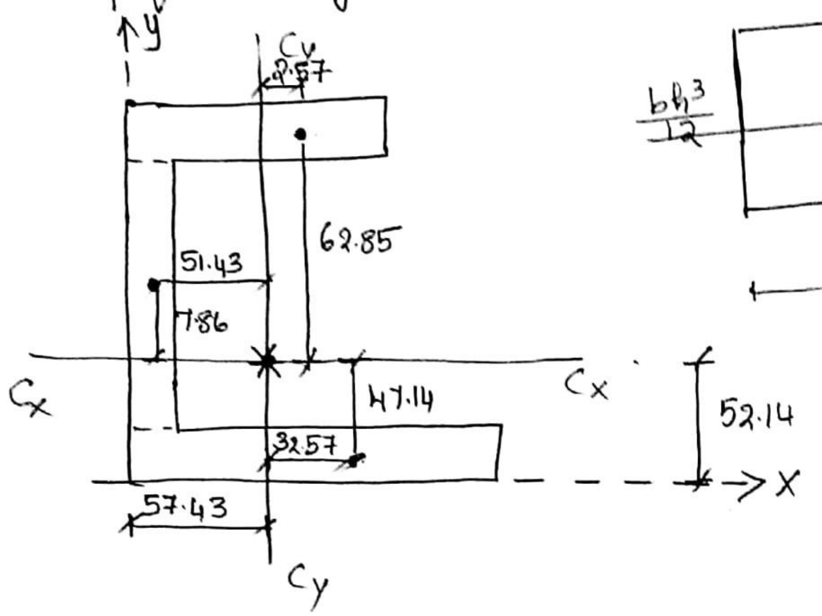
$$\bar{Y} = \frac{120\times10\times\left[100+10+10/2\right] + 100\times12\times\left[\frac{100}{2}+10\right] + 180\times10\times\frac{10}{2}}{4200}$$

$$= \underline{\underline{52.14\ mm}}$$

MI of a rectangle



Simplified diagram



For each figure ①, ②, ③ transfer MI from centroidal axis to the main centroidal axis using parallel axis theorem.

$$I_{cx} = \overbrace{I_{xi}}^{\text{centroidal of individual figure}} + A\cdot y^2$$

$$I_{cy} = \overbrace{I_{yi}}^{\text{centroidal of individual figure}} + A\, x^2$$

$$I_{cx} = \left[\frac{120\times10^3}{12} + \left(120\times10\times62.85^2\right)\right] + \left[\frac{100^3\times12}{12} + \left(100\times12\times7.86^2\right)\right] + \left[\frac{180\times10^3}{12} + \left(180\times10\times47.14^2\right)\right]$$

$$\underline{\underline{I_{cx} = 9.84 \times 10^6 \, mm^4}}$$

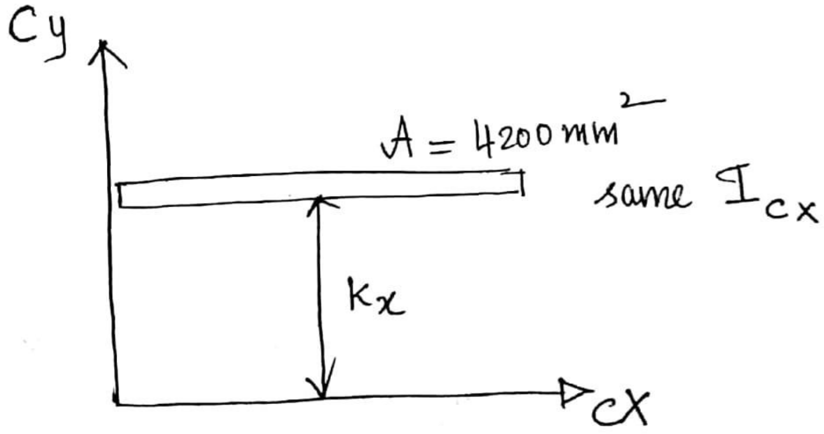
Ilrly

$$I_{cy} = \left[\frac{10 \times 120^3}{12} + (10 \times 120 \times 2.57^2)\right] + \left[\frac{12^3 \times 100}{12} + (12 \times 100 \times 51.43^2)\right] + \left[\frac{180^3 \times 10}{12} + (180 \times 10 \times 32.57^2)\right]$$

$$= \underline{\underline{11.405 \times 10^6 \, mm^4}}$$

The least Moment of inertia is I_{cx}

$= 9.84 \times 10^6 \, mm^4$



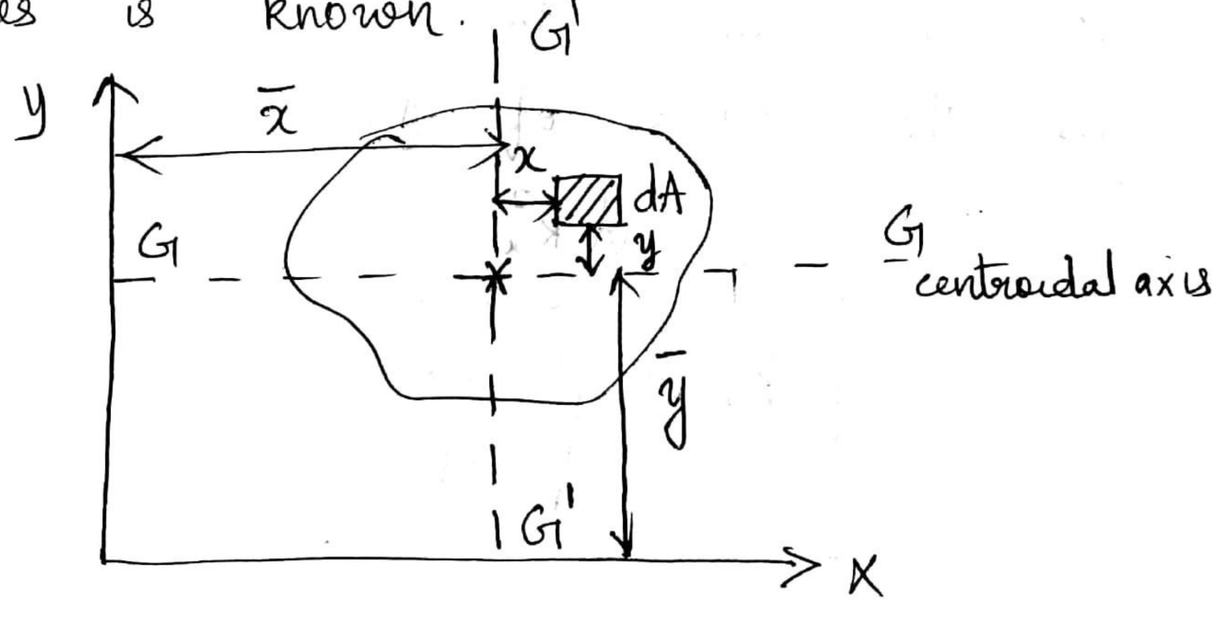
$$A \cdot k_x^2 = I_{cx}$$

$$k_x = \sqrt{\frac{I_{cx}}{A}} = \sqrt{\frac{9.84 \times 10^6}{4200}} = \underline{\underline{48.40 \, mm}}$$

minimum radius of gyration

PARALLEL AXIS THEOREM

This theorem is used to find the moment of inertia about an axis parallel to the centroidal axis. If the area moment of inertia about centroidal axes is known.



$$I_{GG} = \int y^2 \, dA$$

$$I_{xx} = \int (y+\bar{y})^2 \, dA$$

$$= \int y^2 dA + \bar{y}^2 \int dA + \underbrace{2\bar{y} \int y \, dA}_{0}$$

$$\boxed{I_{xx} = I_{GG} + \bar{y}^2 A}$$

$\int y \, dA \rightarrow$ moment of area about centroidal axis

$= 0$

Similarly

$$\boxed{I_{yy} = I_{G'G'} + \bar{x}^2 A}$$

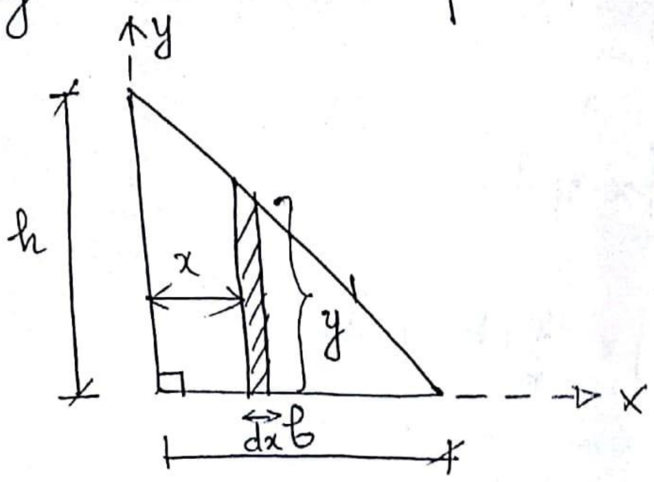
Statement of Theorem:

Moment of Inertia of an area about an axis in the plane of area is equal to the moment of inertia about an axis passing through the centroid and parallel to the given axis plus the product of the area and square of the distance between the two parallel axes.

The second term is always positive because of the square term and hence I_{xx} is always greater than I_{GG}.

$\Rightarrow$ Moment of inertia about centroidal axis is the least

Now, using a vertical strip



From similar triangles

$$\frac{y'}{b-x} = \frac{h}{b}$$

$$y' = \frac{h}{b}(b-x)$$

Area of the shaded element

$$dA = y'dx$$

$$A = \int dA = \int_0^b y'dx$$

$$= \int_0^b \frac{h}{b}(b-x)\,dx$$

$$= \frac{h}{b}\left(bx - x^2/2\right)_0^b$$

$$= \frac{h}{b}\left(b^2 - b^2/2\right)$$

$$= \frac{h}{b} \times \frac{b^2}{2} = \underline{\underline{\frac{bh}{2}}}$$

$$= \frac{\int x\, y\, dx}{A}$$

$$= \frac{\int_0^b x\, \frac{h}{b}(b-x)\, .dx}{\frac{1}{2}bh}$$

$$= \frac{\frac{h}{b}\int_0^b bx - x^2\, dx}{\frac{1}{2}bh}$$

$$= \frac{\frac{h}{b}\left(b\frac{x^2}{2} - \frac{x^3}{3}\right)_0^b}{\frac{1}{2}bh}$$

$$= \frac{h}{b} \times \frac{b^3}{2} - \frac{b^3}{3} \times \frac{2}{bh}$$

$$= \frac{\cancel{h}}{\cancel{b}} \times \frac{b^{\cancel{3}}}{6} \times \frac{2}{\cancel{b}\cancel{h}}$$

$$\bar{X} = \frac{b}{3}$$

$$\bar{y} = \frac{\int y\, dA}{A}$$

y → distance of centroid of subdivision from X-axis

$$= \frac{1}{A}\int_0^b \frac{h}{b}(b-x) \times \frac{h}{b}(b-x)\, dx \times \frac{1}{2}$$

$\frac{1}{2}$ → centroidal distance $= y = y'/2$

$$= \frac{1}{A}\, \frac{h^2}{b^2}\int_0^b \frac{(b^2 + x^2 - 2bx)}{2}\, dx$$

$$= \frac{1}{2A}\, \frac{h^2}{b^2}\left(b^2x + \frac{x^3}{3} - bx^2\right)_0^b = \frac{h^2}{2b^2A}\left(\cancel{b^3} + \frac{b^3}{3} - \cancel{b^3}\right)$$

$$\bar{Y} = \frac{h^2}{b^2} \times \frac{b^3}{3} \times \frac{2}{bh} \times \frac{1}{2} = \underline{\underline{h/3}}$$