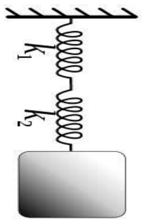
2020-21 ODD SEM VTU QN PAPER

SCHEME OF EVALUATION

1.A Expression for Spring Constant for Series Combination



Consider a load suspended through two springs with spring constants k_1 and k_2 in series combination. Both the springs experience same stretching force. Let Δx_1 and Δx_2 be their elongation.

Total elongation is given by

$$\Delta X = \Delta X_1 + \Delta X_2 = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{F}{k_{eqv}} = \frac{F}{k_1} + \frac{F}{k_2}$$

$$\frac{1}{k_{eqv}} = \frac{1}{k_1} + \frac{1}{k_2}$$

Period $T = \dfrac{2\pi}{\sqrt{\dfrac{k}{m}}}$

1B

Mach number: M = Velocity of fluid/velocity of sound.

Ultrasonic wave : Sound waves of frequency greater than 20,000Hz.

Subsonic waves: These are sound waves with Mach number less than 1. Velocity of the object is less than velocity of sound.

Ex: Low intensity shock waves produced during the motion of ordinary aircrafts.

Super sonic waves: These are shock waves with Mach number greater than 1. Velocity of the object is greater than velocity of sound.

Ex: shock waves produced during the motion of jet planes, bullets etc.

Hypersonic waves: These are shock waves with Mach number greater than 5.

Mach angle: Shock waves propagate as a cone. The semi vertical angle of the cone of shock waves is known as Mach angle (μ). $\mu = \sin^{-1}(1/M)$

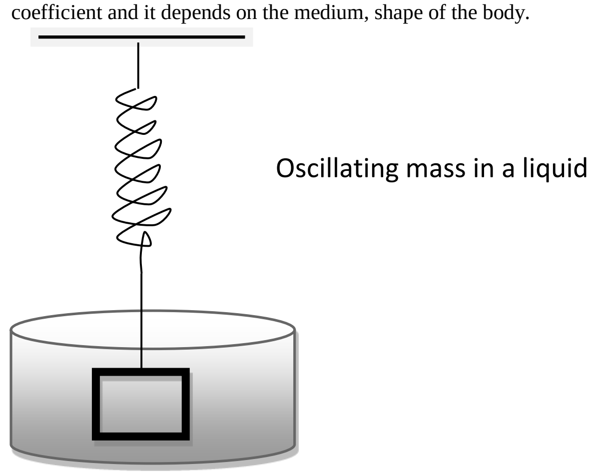
1C

$$f = \frac{\omega}{2\pi} = \frac{\sqrt{\dfrac{k}{m}}}{2\pi} = 5Hz$$

2A

Mechanical Case:

In a damped harmonic oscillator, the amplitude decreases gradually due to losses such as friction, impedance etc. The oscillations of a mass kept in water, charge oscillations in a LCR circuit are examples of damped oscillations. Let us assume that in addition to the elastic force F =-kx, there is a force that is opposed to the velocity, F = b v where b is a constant known as resistive coefficient and it depends on the medium, shape of the body.



For the oscillating mass in a medium with resistive coefficient b, the equation of motion is given by

$$m\frac{d^2x}{dt^2} + kx + b\frac{dx}{dt} = 0$$

This is a homogeneous, linear differential equation of second order.

The auxiliary equation is $D^2 + \dfrac{b}{m}D + \dfrac{k}{m} = 0$

The roots are $D_1 = -\dfrac{b}{2m} + \dfrac{1}{2m}\sqrt{b^2 - 4mk}$ and

$$D_2 = -\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}$$

The solution can be derived as

$$x(t) = Ce^{-\left(\frac{b}{2m} - \frac{1}{2m}\sqrt{b^2 - 4mk}\right)t} + De^{-\left(\frac{b}{2m} + \frac{1}{2m}\sqrt{b^2 - 4mk}\right)t} \quad \text{.......(1)}$$

Note: This can be expressed as x(t) = $Ae^{-\frac{b}{2m}t}\cos(\omega t - \phi)$ where

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$A = \sqrt{C^2 + D^2} \quad \phi = \tan^{-1}(D/C)$$

Here, the term $Ae^{-\frac{b}{2m}t}$ represents the decreasing amplitude and (ωt-ϕ) represents phase

Applying following boundary conditions in (1) : 1. At t = 0 x = x_o 2.At t = 0 $\frac{dx}{dt} = 0$

Simpify

$$C = \frac{x_0}{2}\left(1 - \frac{b}{\sqrt{b^2 - 4mk}}\right)$$

$$D = \frac{x_0}{2}\left(1 + \frac{b}{\sqrt{b^2 - 4mk}}\right)$$

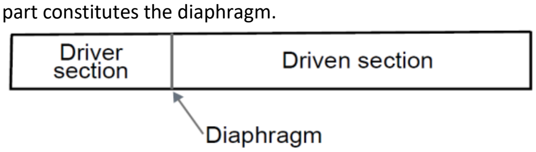
Case : $b^2 = 4mk$ CRITICAL DAMPING

2 B

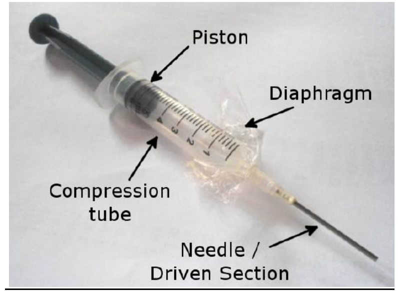
Reddy shock tube:

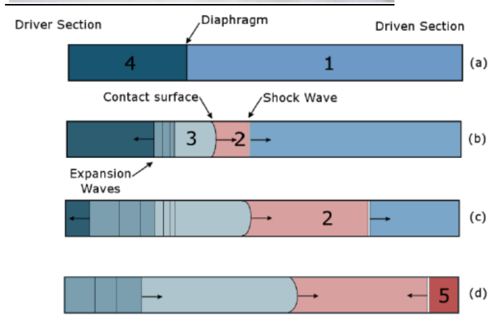
A shock tube is a device used to study the changes in pressure & temperature which occur due to the propagation of a shock wave. A shock wave may be generated by an explosion caused by the buildup of high pressure which causes diaphragm to burst.

It is a hand driven open ended shock tube. It was conceived with a medical syringe. A plastic sheet placed between the plastic syringe part and the needle part constitutes the diaphragm.



- A high pressure (driver) and a low pressure (driven) side separated by a diaphragm.
- When diaphragm ruptures, a shock wave is formed that propagates along the driven section.
- Shock strength is decided by driver to driven pressure ratio, and type of gases used.



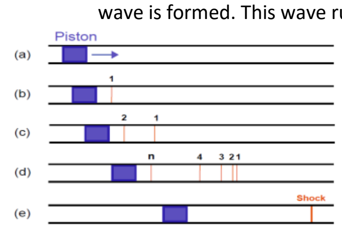


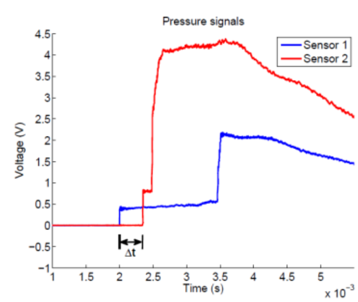
Working:

- The piston is initially at rest and accelerated to final velocity V in a short time t.
- The piston compresses the air in the compression tube. At high pressure, the diaphragm ruptures and the shock wave is set up. For a shock wave to form, $V_{piston} > V_{sound}$.

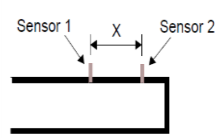
Formation of shock wave:

As the piston gains speed, compression waves are set up. Such compression waves increase in number. As the piston travels a distance, all the compression waves coalesce and a single shock wave is formed. This wave ruptures the diaphragm.





$$U_S = \frac{X}{\Delta t}$$



2C $\omega_f = \omega_0$

$$A_{RESONANCE} = \frac{\frac{F_o}{m}}{\sqrt{0 + \frac{b^2}{m^2}\omega_f^2}} = 0.03m$$

3. A

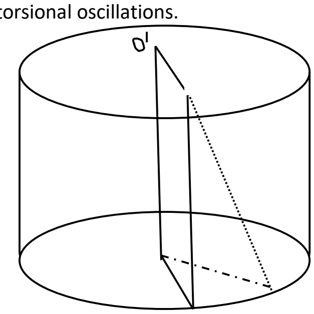
Rigidity Modulus of Elasticity =Tangential stress / shear Strain $= \frac{F}{A\theta}$

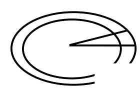
Young's Modulus of Elasticity = Longitudinal stress / Linear Strain $= \frac{FL}{Al}$

Bulk Modulus of elasticity = Normal stress / Volume Strain $= \frac{FV}{Av}$

COUPLE PER UNIT TWIST OF A SOLID CYCLINDER

Oscillations of an object about an axis along which it is suspended constitute torsional oscillations.





Consider a cylindrical rod of rigidity modulus n, length l, radius r fixed at one end and twisted at the other end through an angle θ by a couple. Imagine the cylinder to be made of large number of coaxial cylinders of increasing radius. Consider a cylinder of radius x and thickness dx. For a given couple, the displacement at its rim is maximum. On twisting, the point B shifts to B¹.

$$BB^1 = l\phi = x\theta$$

$$\phi = \frac{x\theta}{l}$$

$$n = \frac{S}{\phi} \Rightarrow Stress\ S = \frac{nx\theta}{l}$$

This stress is acting on the area 2.πx.dx

Total force is

$$F = \frac{nx\theta}{l} 2\pi x.dx$$

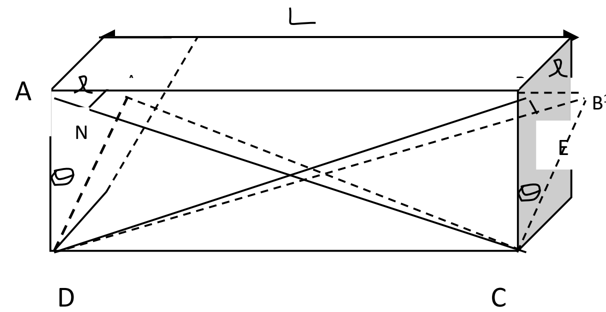
Moment of force along OO¹ $= couple = F.x = \frac{nx^3\theta}{l} 2\pi.dx$

Total twisting couple C

$$= \int_0^R \frac{nx^3\theta}{l} 2\pi.dx = \frac{2\pi n\theta}{l}\frac{R^4}{4}$$

Couple per unit twist $\frac{C}{\theta} = \frac{\pi n}{l}\frac{R^4}{2}$

3B



From the triangle BEB¹, $B\hat{B}^1E = 45^0$

Here EB is the perpendicular from B to DB¹ . $\therefore DB = DE$

$$\cos 45 = \frac{EB^1}{BB^1} \Rightarrow EB^1 = \frac{l}{\sqrt{2}}$$

$$DE = \sqrt{2}L$$

Extension strain along DE = $\frac{EB^1}{DE} = \frac{l}{2L} = \frac{\theta}{2}$ where θ is the shearing strain

From the triangle AA¹N, $AA^1N = 45^0$

$$\cos 45 = \frac{AN}{AA^1} \Rightarrow AN = \frac{l}{\sqrt{2}}$$

$$CN = \sqrt{2}L$$

Compression strain along AC = $\frac{AN}{CN} = \frac{l}{2L} = \frac{\theta}{2}$

Elongation strain + Compression strain = θ/2 +θ/2 =θ

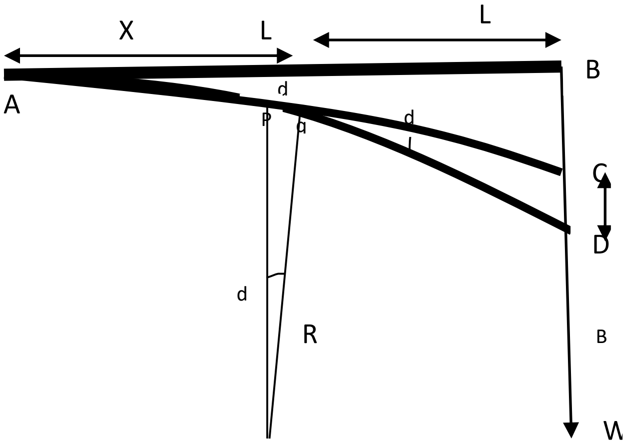
3C

$$Y = \frac{\frac{F}{A}}{\frac{\Delta X}{L}} = 27.7x10^{10} N/m^2$$

4A

CANTILEVER

It is a beam fixed horizontally at one end and loaded at the other.



Let AB be the neutral axis of the cantilever of length L fixed at A and loaded at B. Consider a section P of the beam at a distance x from A.

Bending moment = W .PC =W (L-X)= $= Y\dfrac{I}{R} = Ya\dfrac{k^2}{R}$

Here R is the radius of curvature of neutral axis at P. The moment of the load increases towards the point A, the radius of curvature is different at different points and decreases towards A. For a point Q at a distance dx from P, it is same as at P.

PQ = dx = R.dΘ

Bending moment W (L-X) $= Y\dfrac{ak^2 d\theta}{dx}$

Draw tangents to the neutral axis at P and Q meeting the vertical line at C and D. The angle subtended by them is dΘ. The depression of Q below P is given by

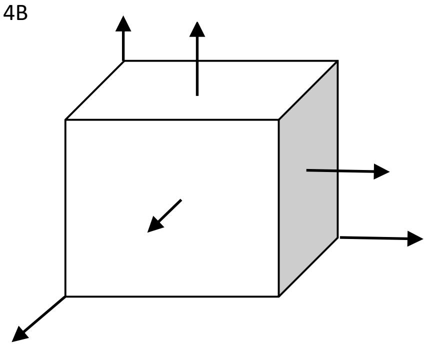
$$dy = (L - X)d\theta = W\frac{(L-X)^2}{Yak^2}.dx$$

Total depression BB^1 of the loaded end

$$\int dy = \int_0^L W\frac{(L-X)^2}{Yak^2}.dx = W\frac{L^3}{3YI}$$

$$Y = \frac{WL^3}{3yI}$$

4B



Let stresses T_X, T_Y and T_Z act perpendicular to faces of a unit cube as shown in the figure .Let α be the increase per unit length per unit stress (linear strain) along the force, β be the lateral contraction (lateral strain) per unit length per unit stress perpendicular to force.

Elongation produced along X axis = $T_X .\alpha.1$

Contraction produced along X axis $= (T_y.\beta.1 + T_Z.\beta.1)$

Change in Length of AB $= T_x\alpha - T_y\beta - T_Z\beta$

Change in Length of BG $= T_y\alpha - T_x\beta - T_z\beta$

Change in Length of BC $= T_z\alpha - T_y\beta - T_x\beta$

Length of AB $= 1 + T_x\alpha - T_y\beta - T_Z\beta$

Length of BG $= 1 + T_y\alpha - T_x\beta - T_z\beta$

Length of BC

$$= 1 + T_z\alpha - T_y\beta - T_x\beta$$

Volume of cube = $(1 + T_x\alpha - T_y\beta - T_Z\beta)$ X $(1 + T_y\alpha - T_x\beta - T_z\beta)$ x $(1 + T_z\alpha - T_y\beta - T_x\beta)$

$$= 1 + T_X\alpha + T_Y\alpha - T_Z\beta - T_X\beta - T_y\beta + T_Z\alpha - T_x\beta - T_Y\beta - T_Z\beta$$

$$= 1 + (\alpha - 2\beta)(T_X + T_Y + T_Z)$$

neglecting terms containing α.β,α^2 ,β^2

$= 1 + (\alpha - 2\beta)(3T)$ if $T_X = T_Y = T_Z$

Increase in volume = $= 1 + (\alpha - 2\beta)(3T)$ - 1

If Inward pressure is applied, **reduction in volume**

$$= 1 + (\alpha - 2\beta)(3P)$$

Bulk Modulus = $\dfrac{P}{3P(\alpha - 2\beta)} = \dfrac{1}{3(\alpha - 2\beta)}$

$$K = \frac{1}{3(\alpha - 2\beta)}$$

$$K = \frac{1}{3\alpha(1-2\sigma)}$$

$$K = \frac{Y}{3(1-2\sigma)}$$

4C

$$\tau = \frac{\pi\eta R^4\theta}{2L} = 0.7 rad$$

5.A

Del operator: $$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

Divergence: It represents the magnitude of a physical quantity emerging or converging at a point. For example tip of a fountain head is a source of divergence. Electric fields are said to be divergent in nature. Mathematically it is obtained by differentiating components of a vector function F (F_x ,F_y, F_z) with respect to position coordinates x,y,z respectively.

$$\nabla \bullet \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

Ex: Volume charge density enclosed in a closed surface is expressed as

$$\nabla \bullet \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

Right hand side in the above expression is a scalar. Divergence operation on vector yields a scalar function. Divergence of vector is zero if there is no outflow or inflow. Magnetic fields form closed loops and their divergence is zero. $\nabla \bullet B = 0$

Diverging electric field lines at a positive chargeis an example for Positive divergence.

converging electric field lines at a negative charge is an example of negative divergence.

5B

Attenuation

Attenuation is the loss of power suffered by the optical signal as it propagates through the fiber.If P_{in} is the incident energy and P_O is the energy at a distance L, then

$$P_O = P_{in} e^{-\alpha L}$$

$$-\alpha z \ln_e e = \ln \frac{P_O}{P_{in}}$$

$$\alpha = \frac{1}{z}\ln\left(\frac{P_{in}}{P_O}\right) \text{bel/km}$$

$$\alpha = \frac{1}{z}\ln\left(\frac{P_{in}}{P_O}\right) \text{ per km}$$

For easier representation of loss percentage, the following expression is used

Attenuation constant $\alpha = \frac{10}{z}\log\left(\frac{P_{in}}{P_{out}}\right) \text{dB/Km}$

Different loss mechanisms:

1. Absorption losses:

In this case, the loss of signal power occurs due to absorption of photons by the impurities and defects present in glass .Impurities such as Ge- 0 , B-O, absorb in the range of 1-2 µm, chromium and copper ions absorb at 0.6µm.,Fe ions absorb at 1.1µm. Hydoxy ions absorb at 1.38µm. Better techniques of making glass with reduced water content can minimize these losses.Tominimise the absorption loss, impurity content has to be less than 1 part in 10^9.

2. Scattering losses:

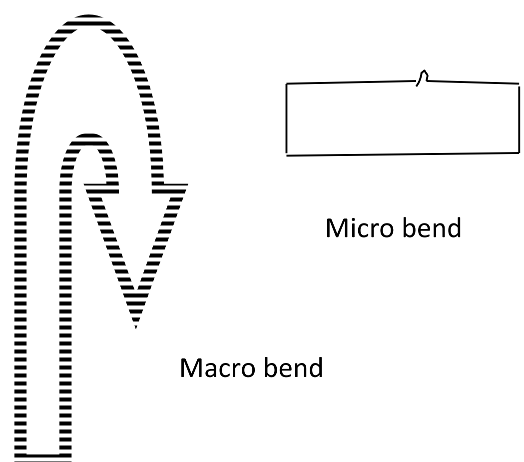
This occurs due to the Rayleigh scattering of the signal caused by variations in refractive index of the glass due to changes in composition, defects, presence of air bubbles, strains etc. The scattered light moves in random direction and escapes from the fiber reducing the intensity. These losses decrease at higher wavelengths.Hence, this loss is minimized by operating at high wavelengths.

Scattered Intensity $\alpha \frac{1}{\lambda^4}$

3. Radiation losses:

Radiative losses occur due to bending of fiber.

Macroscopic Bends:This refers to the bends having radii that are large compared to the fibre diameter.These losses are reduced by using lower wavelength and lower numerical aperture. This loss is high at 1550nm.



Microscopic bends:

These are repetitive small scale fluctuations in the linearity of the fibre axis.

5C

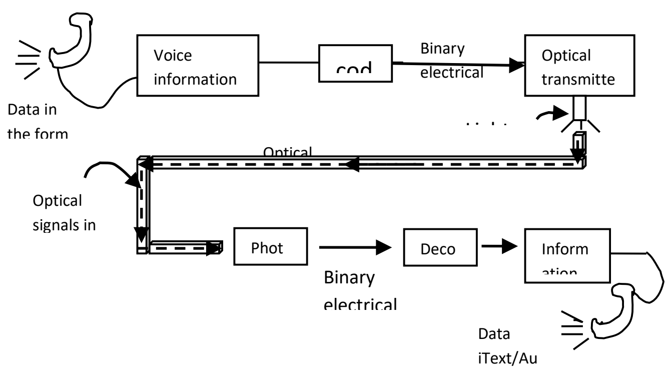
$$N = \frac{V_{number}{}^{2}}{2} = \frac{(\frac{2\pi R}{\lambda}.NA)^2}{2} = 4481$$

6A

<u>Point to point communication system using optical fibers</u>

This system is represented through a block diagram as follows.

The information in the form of voice/ picture/text is converted to electrical signals through the transducers such as microphone/video camera. The analog signal is converted in to binary data with the help of coder. The binary data in the form of electrical pulses are converted in to pulses of optical power using Semiconductor Laser. This optical power is fed to the optical fiber. Only those modes within the angle of acceptance cone will be sustained for propagation by means of total internal reflection. At the receiving end of the fiber, the optical signal is fed in to a photo detector where the signal is converted to pulses of current by a photo diode. Decoder converts the sequence of binary data stream in to an analog signal . Loudspeaker/CRT screen provide information such as voice/ picture.



6 B

According to this law, the magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through it. The induced emf will be in a direction which opposes the change which causes it.

Induced emf e = - $\frac{d\varphi}{dt}$ where Φ is the flux linking with the circuit.

Differential form of Faradays Law:

From Stokes theorem $\oint E.dl = \int\int_s \left(\nabla x \vec{E}\right).ds$

The above equation becomes

$$\int\int_s \left(\nabla x \vec{E}\right).ds = -\int\int \frac{d\vec{B}}{dt}.\vec{ds}$$

$$\therefore \nabla x\vec{E} = -\frac{d\vec{B}}{dt}$$

6C

$$\nabla \bullet A = 6x + yz$$

7a.

<u>Time independent Schrödinger equation</u>

A matter wave can be represented in complex form as

$$\Psi = A\sin kx(\cos wt + i\sin wt)$$

$$\Psi = A\sin kxe^{iwt}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = kA\cos kxe^{iwt}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A\sin kxe^{iwt} = -k^2\Psi \text{ ……………………. (1)}$$

From debroglie's relation

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\Pi p}{h}$$

$$k^2 = 4\Pi^2 \frac{p^2}{h^2} \text{ ………………………. (2)}$$

Total energy of a particle E = Kinetic energy + Potential Energy

$$E = \frac{1}{2} m v^2 + V$$

$$E = \frac{p^2}{2m} + V$$

$$p^2 = (E - V)2m$$

Substituting in (2)

$$k^2 = \frac{4\Pi^2 (E-V)2m}{h^2}$$

$\therefore$ From (1)

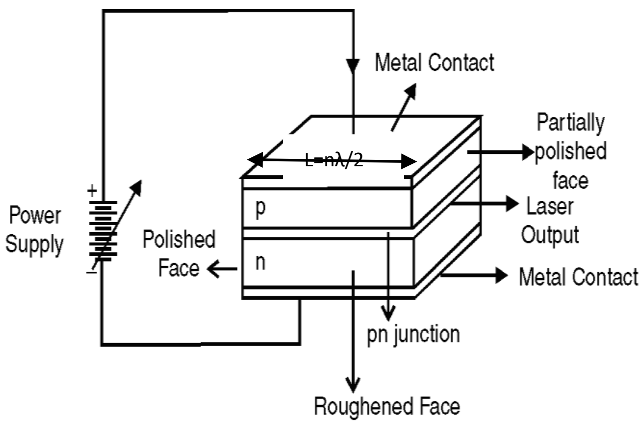
$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m(E-V)\Psi}{h^2}} = 0$$

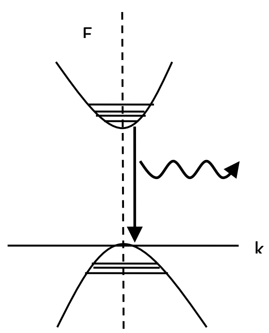
7B

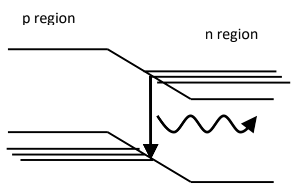
Semiconductor laser :

It is the only device which can be used for amplification in the infrared and optical ranges.

Amplification is possible if the population of the valence and conduction bands could be inverted as shown in the diagram.







The first laser action was observed in a GaAs junction(8400Å) which is a direct gap semiconductor.

When a heavily doped junction is forward biased, electrons from n side are injected into p side causing population inversion. They combine with holes on the p side releasing photons. The junction region is the active region .The optical cavity is formed by the faces of the crystal itself which are taken on the cleavage plane and are then polished. The wavelength of the radiation depends on temperature. The wavelength of laser increases as the temperature increases as the energy gap decreases. The frequency can be increased to the optical region by alloying with phosphor according to the relation $Ga(As)_{1-x}P_x$

.

If E_g is the energy gap, then $E_g = eV_{forward} = \frac{hc}{\lambda}$

7C

$$\Delta E = \frac{h}{4\pi.\Delta t} = 3.76x10^{-25}\,J$$

8A.

TO SHOW THAT ELECTRON DOES NOT EXIST INSIDE THE NUCLEUS:

We know that the diameter of the nucleus is of the order of 10^{-14}m.If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

- $\Delta x \le 10^{-14}\,m$

Now the uncertainty in momentum is

$$\Delta p \ge \frac{h}{4\pi.\Delta x}$$

$$\Delta p \ge \frac{6.62\times10^{-34}}{4\pi\times10^{-14}}$$

$$\Delta p \ge 0.5\times10^{-19}\,Ns$$

Then the momentum of the electron can atleast be equal to the uncertainty in momentum.

$$p \ge 0.5\times10^{-19}\,Ns$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by (for small velocities -non-relativistic-case)

$$E = \frac{p^2}{2m} = \frac{(0.5\times10^{-19})^2}{2\times9.1\times10^{-31}} = 1.37\times10^{-11}\,J = 8.5MeV$$

. The beta decay experiments have shown that the kinetic energy of the beta particles (electrons) is only a fraction of this energy. This indicates that electrons do not exist within the nucleus. They are produced at the instant of decay of nucleus ($n \rightarrow p + \overset{-}{e} + \nu$ / $p \rightarrow n + \overset{+}{e} + \nu$).

8.B

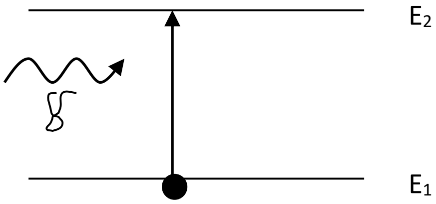
8B

Expression for energy density:

Induced absorption:

It is a process in which an atom at a lower level absorbs a photon to get excited to the higher level.

Let E_1 and E_2 be the energy levels in an atom and N1 and N_2 be the number density in these levels respectively. Let U_γ be the energy density of the radiation incident..



$$\gamma = \frac{E_2 - E_1}{h}$$

Rate of absorption is proportional to the number of atoms in lower state and also on the energy density U_{γ}.

Rate of absorption = $B_{12}\, N_1\, U_{\gamma}$

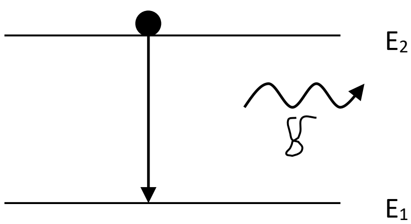
Here B_{12} is a constant known as Einsteins coefficient of spontaneous absorption.

Spontaneous emission:

It is a process in which ,atoms at the higher level voluntarily get excited emitting a photon. The rate of spontaneous emission representing the number of such deexcitations is proportional to number of atoms in the excited state.

Rate of spontaneous absorption = $A_{21}\, N_2$

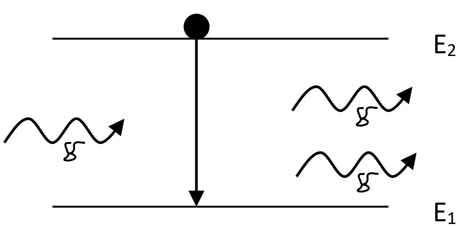
Here B_{12} is a constant known as Einsteins coefficient of spontaneous emission.



$$\gamma = \frac{E_2 - E_1}{h}$$

Stimulated emission:

In this process, an atom at the excited state gets deexcited in the presence of a photon of same energy as that of difference between the two states.



$$\gamma = \frac{E_2 - E_1}{h}$$

The number of stimulated emissions is proportional to the number of atoms in higher state and also on the energy density U_{γ}.

Rate of stimulated emission = $B_{21}\, N_2\, U_{\gamma}$

Here B_{21} is the constant known as Einsteins coefficient of stimulated emission.

At thermal equilibrium,

Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12} N_1 U_{\gamma} = A_{21} N_2 + B_{21} N_2 U_{\gamma}$$

$$U_{\gamma} = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

Rearranging this, we get

$$U_{\gamma} = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1}\right]$$

From Boltzmans law , $\frac{N_1}{N_2} = e^{\frac{h\gamma}{kT}}$

Hence

$$U_{\gamma} = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}} e^{\frac{h\gamma}{kT}} - 1}\right]$$

From Planck's radiation law,

$$U_{\gamma} = \frac{8\pi h\gamma^3}{c^3}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]} - 1}\right]$$

Comparing these expressions, we get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\gamma^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1$$

.

8C

$$\frac{N_2}{N_1} = e^{-\frac{hc}{\lambda kT}}$$

$$\lambda = 655nm$$

9A

<u>Conductivity of Intrinsic semiconductors:</u>

Current density $J = n\, e\, V_d = I/A$

CURRENT – n e A Vd

For a semiconductor, J = J (e)+ J (holes) $= n_e\, e\, V_d(e) + n_h\, e\, V_d(h)$ ....(1)

But drift velocity $V_d = \mu E = \mu . J/\sigma$ mu = Vd/E J = sigma . E

Using (1), $\sigma = n_e\, e\, \mu_e + n_h\, e\, \mu_h$

In an intrinsic semiconductor, number of holes is equal to number of electrons.

, $\sigma_{int} = n_e e[\mu_e + \mu_{hole}]$

n_e is the electron concentartion

n_p is the hole concentration

μ_e is the mobility of electrons

μ_h is the mobility of holes

9.B.

INTERNAL FIELDS IN A DIELECTRIC:
It is the resultant of the applied field and the field produced due to all the dipoles.

The internal electric field is $E_i = E_a + \dfrac{1.2\mu}{\pi\varepsilon d^3}$

$$E_i = E_a + \frac{1.2\mu}{\pi\varepsilon d^3} \qquad \because \mu = \alpha E_a$$

C CLAUSIUS – MOSOTTI RELATION:

This expression relates dielectric constant of an insulator (ε) to the polarization of individual atoms (α) comprising it.

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha}{3\varepsilon_0}$$

where N is the number of atoms per unit volume

α is the polrisability of the atom

ε_r is the relative permittivity of the medium

ε_o is the permittivity of free space.

Proof: If there are N atoms per unit volume, the electric dipole moment per unit volume – known as polarization is given by

$$P = N\alpha E_i$$

By the definition of polarization P, it can be shown that

$$P = \varepsilon_0 E_a (\varepsilon_r - 1) = N\alpha E_i$$

$$\varepsilon_0 \varepsilon_r E_a - \varepsilon_0 E_a = N\alpha E_i$$

$$\varepsilon_r = 1 + \frac{N\alpha E_i}{\varepsilon_0 E_a} \qquad \text{......................(1)}$$

The internal field at an atom in a cubic structure (γ =1/3) is of the form

$$E_i = E_a + \frac{p}{3\varepsilon_0} = E_a + \frac{N\alpha E_i}{3\varepsilon_0}$$

$$\frac{E_i}{E_a} = \frac{1}{\left[1 - \left(\frac{N\alpha}{3\varepsilon_0}\right)\right]}$$

Substituting for $\dfrac{E_i}{E_a}$ in equation (1)

$$\varepsilon_r = 1 + \frac{N\alpha}{\varepsilon_0}\left[\frac{1}{\left(1 - \frac{N\alpha}{3\varepsilon_0}\right)}\right] = \frac{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right] + \frac{N\alpha\varepsilon_0}{\varepsilon_0}}{\varepsilon_0\left[1 - \frac{N\alpha}{3\varepsilon_0}\right]} = \frac{1 + \frac{2}{3}\left(\frac{N\alpha}{\varepsilon_0}\right)}{1 - \frac{1}{3}\left[\frac{N\alpha}{\varepsilon_0}\right]}$$

$$\left[\frac{\varepsilon_r - 1}{\varepsilon_r + 2}\right] = \frac{\dfrac{1 + (2/3)\frac{N\varepsilon}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} - 1}{\dfrac{1 - (2/3)\frac{N\alpha}{\varepsilon_0}}{1 - (1/3)\frac{N\alpha}{\varepsilon_0}} + 2} = \frac{N\alpha}{3\varepsilon_0}$$

9.C.

$$f(E) = \frac{1}{e^{\frac{E - E_F}{kT}} + 1}$$

$$Case1: f(E) = 0.0.239$$

$$Case2: f(E) = 0.35$$

10.A

Fermi energy (E_F):

It is the highest energy possessed by an electron at zero Kelvin.

$$E_F = \frac{h^2}{8m}\left(\frac{3n}{\Pi}\right)^{\frac{2}{3}}$$

Fermi probability factor: It represents the probability of occupation of an energy level.

$$f(E) = \frac{1}{e^{(\frac{E - E_F}{kT})} + 1}$$

Density of energy of states:

It represents the number of energy levels per unit energy range per unit volume.

$$g(E) = 8\Pi\sqrt{2}m^{\frac{3}{2}}\frac{E^{\frac{1}{2}}}{h^3}$$

To show that energy levels below Fermi energy are completely occupied:

For E < E_F, at T = 0,

$$f(E) = \frac{1}{e^{(\frac{E - E_F}{kT})} + 1} = 1$$

To show that energy levels above Fermi energy are empty:

For E > E_F , at T=0

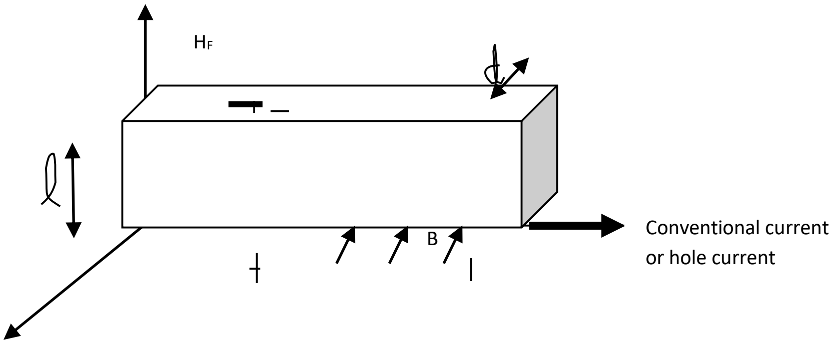
$$f(E) = \frac{1}{e^{(\frac{E - E_F}{kT})} + 1} = 0$$

At ordinary temperatures, for E = E_F,

f(E) = $\frac{1}{2}$

10B

Hall effect: When a conductor carrying current is placed in magnetic field, an electric field is produced inside the conductor in a direction normal to both current and the magnetic field.



Here B is along –X , V is along –Y axis

Lorentz force= = e (VXB)

$$-e\left(-\hat{j}\;\; X - \hat{i}\right) = +\hat{k}$$

So the electron is deflected along + Z axis

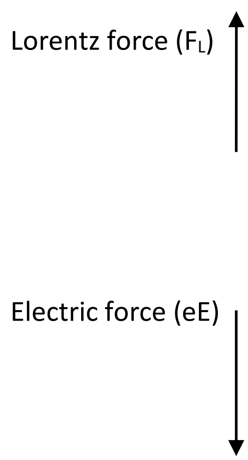
Consider a rectangular slab of an n type semiconductor carrying a current I along + X axis. Magnetic field B is applied along –Z direction. Now according to Fleming's left hand rule, the Lorentz force on the electrons is along +Y axis. As a result the density of electrons increases on the upper side of the material and the lower side becomes relatively positive. The develops a potential V_H- Hall voltage between the two surfaces. Ultimately, a stationary state is obtained in which the current along the X axis vanishes and a field E_y is set up.

Expression for electron concentration:

At equilibrium, Lorentz force is equal to force due to applied electric field

Bev =- e

E_H



Hall Field E_H = Bv

$$Current\ density\ J = I / A = neAV / A = -n_e ev$$

$$v = \frac{J}{n_e e}$$

$$E_H = B\frac{-J}{n_e e}$$

Hence

$$\frac{E_H}{JB} = -\frac{1}{n_e e} = R_H$$

R_H is known as Hall coefficient. It is negative for n type and p type for positive semiconductors.

Electron concentration $n_e = \frac{BI}{V_H d}$

Hall voltage V_H = $E_H . l = Bvl$

E = dv/dx

10C

$P = \varepsilon_o(\varepsilon_r - 1)E = 6{,}1x10^{-8}$ C/m²