USN | | | | | | | | | | |

# Internal Assessment Test I – January 2021

| Sub: | Calculus and Linear Algebra | | | | | Sub Code: | 18MAT11 | |
|---|---|---|---|---|---|---|---|---|
| Date: | 28/01/2021 | Duration: | 90 mins | Max Marks: | 50 | Sem / Sec: | I / A to G (PHY CYCLE) and I to O (CHEM CYCLE) | OBE |

| | | MARKS | CO | RBT |
|---|---|---|---|---|
| 1. | Diagonalize: $\boldsymbol{A} = \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{3} & \mathbf{-1} \end{pmatrix}$. | [08] | CO6 | L3 |
| 2. | Find the rank of the matrix: $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{3} & \mathbf{-1} & \mathbf{2} \\ \mathbf{0} & \mathbf{11} & \mathbf{-5} & \mathbf{3} \\ \mathbf{2} & \mathbf{-5} & \mathbf{3} & \mathbf{1} \\ \mathbf{4} & \mathbf{1} & \mathbf{1} & \mathbf{5} \end{bmatrix}$. | [07] | CO6 | L3 |
| 3. | Investigate the values of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ such that the following system may have (i) unique solution, (ii) infinitely many solutions and (iii) no solution. $$\boldsymbol{x+y+z=6}$$ $$\boldsymbol{x+2y+3z=10}$$ $$\boldsymbol{x+2y+\lambda z=\mu}$$ | [07] | CO6 | L3 |
| 4. | Solve using Gauss Jordan Method: $$\boldsymbol{x+y+z=8}$$ $$\boldsymbol{-x-y+2z=-4}$$ $$\boldsymbol{3x+5y-7z=-14}$$ | [07] | CO6 | L3 |
| 5. | Find the dominant eigenvalue and the corresponding eigenvector of the matrix $\boldsymbol{A} = \begin{pmatrix} \mathbf{6} & \mathbf{-2} & \mathbf{2} \\ \mathbf{-2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} & \mathbf{3} \end{pmatrix}$ by power method taking the initial vector as $[1,1,1]^T$ **(perform only 4 iterations)**. | [07] | CO6 | L3 |
| 6. | Solve using Gauss Seidel method **(perform only 3 iterations)**: $$\boldsymbol{20x+y-2z=17}$$ $$\boldsymbol{3x+20y-z=-18}$$ $$\boldsymbol{2x-3y+20z=25}.$$ | [07] | CO6 | L3 |
| 7. | Solve using Gauss Elimination Method: $$\boldsymbol{x+2y+z=3}$$ $$\boldsymbol{2x+3y+2z=5}$$ $$\boldsymbol{3x-5y+5z=2}.$$ | [07] | CO6 | L3 |

# Internal Assessment Test - I
## (Solution)

Q1 Given

$$A = \begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}$$

The characteristic equation of A is

$|A - \lambda I| = 0$

That is, $\begin{vmatrix} (1-\lambda) & 1 \\ 3 & (-1-\lambda) \end{vmatrix} = 0$

$\therefore \quad (1-\lambda)(-1-\lambda) - 3 = 0$

$\Rightarrow \quad \lambda^2 - 4 = 0$

$\Rightarrow \quad \lambda = \pm 2$.

we form the system $[A - \lambda I][X] = [0]$

That is $\quad (1-\lambda)x + y = 0$

$\quad 3x - (1+\lambda)y = 0$

Case: I Let $\lambda = +2$

we obtain a ~~singular eq~~ single equation

$x = y \quad$ or $\quad \frac{x}{1} = \frac{y}{1}$

$\therefore\ X_1 = (1,1)'$ is the eigen vector for

$\lambda = \pm 2$

Case II Let $\lambda = -2$

we obtain a single equation,

$3x + y = 0 \Rightarrow 3x = -y$

or $\dfrac{x}{1} = \dfrac{y}{-3}$

$\therefore\ X_2 = (1,-3)'$ is the eigen vector

for $\lambda = -2$

Model Matrix, $P = [X_1\ X_2] = \begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$

$|P| = -4$ so $P^{-1} = \dfrac{1}{4}\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$

Now

$$P^{-1}AP = \frac{1}{4}\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -3 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 8 & 0 \\ 0 & -8 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

Thus $\boxed{P^{-1}AP = \text{Diag}(2,-2)}$

Q2. Given

$$A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$$

Apply Row-operations

$R_3 \to R_3 - 2R_1$

$R_4 \to R_4 - 4R_1$

$$A \sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & -11 & 5 & -3 \\ 0 & -11 & 5 & -3 \end{bmatrix}$$

Again by applying $R_3 \to R_3 + R_2$

$R_4 \to R_4 + R_2$

$$A \sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Furthere apply $R_2 \to \dfrac{R_2}{11}$

$$A \sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 1 & -5/11 & 3/11 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

i.e no. of non-zero rows = 2

$\therefore$ $\boxed{\rho(A) = 2}$

---

Q.3. Given

$$x+y+z=6$$
$$x+2y+3z=10$$
$$x+2y+\lambda z=\mu$$

Augmented Matrix

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{array}\right]$$

Apply $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & \mu-6 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$[A:B] \sim \begin{bmatrix} 1 & 1 & 1 & : & 6 \\ 0 & 1 & 2 & : & 4 \\ 0 & 0 & \lambda-3 & : & \mu-10 \end{bmatrix}$$

a) Unique Solution: we must have

$\rho[A] = \rho[A:B] = 3$, $\rho[A]$ will be 3

if $(\lambda-3) \neq 0$ or $\lambda \neq 3$ irrespective of

the value of $\mu$.

Hence, the system will have unique solⁿ if

$\boxed{\lambda \neq 3}$.

b) Infinite solutions: Here we have $n=3$

and we need $\rho(A) = \rho[A:B] = r < 3$

we must have $r=2$ since first row and

second row are non-zero.

$\therefore$ $\rho(A) = \rho[A:B] = 2$ only when the

last row of $[A:B]$ is completly zero.

This is possible if $\lambda-3=0$, $\mu=10$

Hence, the system will have infinite solution if $\boxed{\lambda = 3 \quad \text{and} \quad \mu = 10}$

c) No solution: we must have

$\rho(A) \neq \rho[A:B]$. By case (a)

$\rho(A) = 3$ if $\lambda = 3$ and

hence if $\lambda = 3$ we obtain $\rho(A) = 2$,

Hence, the system has no solution if

$\boxed{\lambda = 3 \quad \text{and} \quad \mu \neq 10.}$

---

Q 4. Given $x+y+z = 8$

$-x-y+2z = -4$

$3x+5y-7z = -14$

(Gauss-Jordan Method)

Augmented Matrix

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & : & 8 \\ -1 & -1 & 2 & : & -4 \\ 3 & 5 & -7 & : & -14 \end{bmatrix}$$

Apply $R_2 \to R_2 + R_1$

$R_3 \to R_3 - 3R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 0 & 3 & 4 \\ 0 & 2 & -10 & -38 \end{array}\right]$$

$$R_2 \longleftrightarrow R_3 \,, \quad R_3 \to \frac{R_3}{2}$$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 1 & -5 & -19 \\ 0 & 0 & 3 & 4 \end{array}\right]$$

$$R_1 \longrightarrow R_1 - R_2$$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 0 & 6 & 27 \\ 0 & 1 & -5 & -19 \\ 0 & 0 & 3 & 4 \end{array}\right]$$

$$R_1 \to R_1 - 2R_3 \,, \quad R_2 \to 3R_2 + 5R_3$$

$$\sim \left[\begin{array}{ccc:c} 1 & 0 & 0 & 19 \\ 0 & 3 & 0 & -37 \\ 0 & 0 & 3 & 4 \end{array}\right]$$

Now we have $x = 19 \,, \; 3y = -37 \,, \; 3z = 4$

Thus $\boxed{x = 19 \,, \; y = -\frac{37}{3} \,, \; z = \frac{4}{3}}$

Q 5. By data $x^{(0)} = [1, 1, 1]'$ is the initial eigen vector.

$$Ax^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6 \begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix}$$

$$= \lambda^{(1)} x^{(1)}$$

$$Ax^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 7.34 \\ -2.67 \\ 4.01 \end{bmatrix} = 7.34 \begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix}$$

$$= \lambda^{(2)} x^{(2)}$$

$$Ax^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 7.82 \\ -3.63 \\ 4.01 \end{bmatrix}$$

$$= 7.82 \begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \lambda^{(3)} x^{(3)}$$

$$Ax^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \begin{bmatrix} 7.94 \\ -3.89 \\ 3.99 \end{bmatrix}$$

$$= 7.94 \begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \lambda^{(4)} x^{(5)}$$

$$Ax^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 7.98 \\ -3.97 \\ 3.99 \end{bmatrix}$$

$$7.98 \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix} = \lambda^{(5)} x^{(5)}$$

Thus the dominant eigen value is $7.98 \approx 8$

and the corresponding eigen vector is

$[1, -0.5, 0.5]'$.

Q.6. Given

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

The equations are diagonally dominant and hence we first write them in the following form

$$x = \frac{1}{20}[17 - y - 2z]$$

$$y = \frac{1}{20}[-18 - 3x + z]$$

$$z = \frac{1}{20}[25 - 2x + 3y]$$

we start with the trial solution

$$x = 0, \quad y = 0, \quad z = 0$$

I$^{st}$ Iteration

$$x^{(1)} = \frac{17}{20} = 0.85$$

$$y^{(1)} = \frac{1}{20}[-18 - 3(0.85)] = -1.0275$$

$$z^{(1)} = \frac{1}{20}[25 - 2(0.85) + 3(-1.0275)]$$
$$= 1.0109$$

$\text{II}^{nd}$ iteration

$$x^{(2)} = \frac{1}{20}\left[17 - (-1.0275) + 2(1.0109)\right]$$
$$= 1.0025$$

$$y^{(2)} = \frac{1}{20}\left[-18 - 3(1.0025) + 1.0109\right]$$
$$= -0.9998$$

$$z^{(2)} = \frac{1}{20}\left[25 - 2(1.0025) + 3(-0.9998)\right]$$
$$= 0.9998$$

$\text{III}^{rd}$ Iteration

$$x^{(3)} = \frac{1}{20}\left[17 - (-0.9998) + 2(0.9998)\right]$$
$$= 0.99997 \approx 1$$

$$y^{(3)} = \frac{1}{20}\left[-18 - 3(0.99997) + 0.9998\right]$$
$$= -1.0000055 \approx -1$$

$$z^{(3)} = \frac{1}{20}\left[25 - 2(0.99997) + 3(-1.0000055)\right]$$
$$= 1.0000022 \approx 1$$

Thus $\boxed{x = 1,\ y = -1,\ z = 1}$

Q.7. Given

$$x+2y+z=3$$
$$2x+3y+2z=5$$
$$3x-5y+5z=2$$

(Gauss-elimination)

Augmented Matrix

$$[A:B] = \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 5 \\ 3 & -5 & 5 & 2 \end{array}\right]$$

Apply $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$[A:B] \sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & -11 & 2 & -7 \end{array}\right]$$

$R_3 \to R_3 - 11R_2$

$$\sim \left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

$\Rightarrow 2z=4$

$\boxed{z=2}$

$\Rightarrow \quad -y=-1 \quad \Rightarrow \boxed{y=1}$

Thus

$\boxed{x=-1}$