IAT - 2 (Solutions)
(Chemistry Cycle)
Calculus and Linear Algebra
(18MAT11)

Q1. Find the evolute of the parabola.

Sol

Sol^n given $y^2 = 4ax$

Let $y = 2at, \quad x = at^2$

$$y_1 = \frac{dy}{dx}, \quad \frac{dy}{dt} = 2a, \quad \frac{dx}{dt} = 2at$$

$$y_1 = \frac{1}{t}, \quad y_2 = \frac{-1}{t^2}\frac{dt}{dx} = \frac{-1}{2at^3}$$

$$\bar{x} = x - \frac{y_1}{y_2}(1+y_1^2) = at^2 - \frac{1/t}{(-1/2at^3)}\left(1+\frac{1}{t^2}\right)$$

$$\bar{x} = 3at^2 + 2a \quad -(1)$$

$$\bar{y} = y + \frac{(1+y_1^2)}{y_2} = 2at + \frac{\left(1+\frac{1}{t^2}\right)}{(-1/2at^3)}$$

$$= -2at^3 \quad \Rightarrow (2)$$

from (1) $\quad t^2 = \dfrac{\bar{x}-2a}{3a} \quad -(3)$

from (2) & (3) $\quad (\bar{y})^2 = 4a^2\left(\dfrac{\bar{x}-2a}{3a}\right)^3$

$$\Rightarrow 27(\bar{y})^2 a = 4(\bar{x}-2a)^3$$

Thus the evolute is

$$27ay^2 = 4(x-2a)^3$$

Q2. given that $r^m \cos m\theta = a^m$ — (1)

$$m\log r + \log\cos m\theta = m\log a$$

$$\frac{m}{r}\frac{dr}{d\theta} - \frac{m\sin m\theta}{\cos m\theta} = 0$$

$$\cot\phi = \tan m\theta = \cot\left(\frac{\pi}{2} - m\theta\right)$$

$$\phi = \frac{\pi}{2} - m\theta$$

Pedal equation $p = r\sin\phi$

$$p = r\sin\left(\frac{\pi}{2} - m\theta\right)$$

$p = r\cos m\theta$ — (2)

from (1) $\cos m\theta = \dfrac{a^m}{r^m}$

$$\Rightarrow p = r\cdot\frac{a^m}{r^m} \Rightarrow \boxed{p = \frac{a^m}{r^{m-1}}}$$

3. $r = a(1+\cos\theta)$ —①

$\log r = \log a + \log(1+\cos\theta)$

$$\frac{1}{r}\frac{dr}{d\theta} = 0 - \frac{\sin\theta}{1+\cos\theta} \Rightarrow r_1 = -r\tan\theta/2$$

$$r_2 = -r_1\tan\theta/2 - \frac{r}{2}\sec^2\theta/2$$

$$= r\tan^2\theta/2 - \frac{r}{2}\sec^2\theta/2 = \frac{r}{2}\sec^2\theta/2 - r$$

$$\rho = \frac{[r^2+r_1^2]^{3/2}}{r^2+2r_1^2-rr_2}$$

$$\rho = \frac{[r^2+r^2\tan^2\theta/2]^{3/2}}{r^2+2r^2\tan^2\theta/2 - \frac{r^2}{2}\sec^2\theta/2 + r^2}$$

$$= \frac{r^3 \cdot \sec^3\theta/2}{r^2\{2+2\tan^2\theta/2 - \frac{1}{2}\sec^2\theta/2\}} = \frac{r\sec^3\theta/2}{3/2\sec^2\theta/2}$$

$$= \frac{2r}{3}\sec\theta/2 \quad —②$$

from ① $\cancel{\&\ ②}$ $r = 2a\cos^2\theta/2 \Rightarrow \sec^2\theta/2 = \frac{2a}{r}$

from ② $\rho^2 = \frac{2r^2}{3}\sec^2\theta/2 \Rightarrow \rho^2 = \frac{4a}{3}r.$

$\therefore \rho^2 \propto r$

Ques 4 $r(1+\cos\theta) = a \quad ; \quad r(1-\cos\theta) = b$

$\log r + \log(1+\cos\theta) = \log a \; ; \; \log r + \log(1-\cos\theta) = \log b$

$\frac{1}{r}\frac{dr}{d\theta} - \frac{\sin\theta}{1+\cos\theta} = 0 \; ; \; \frac{1}{r}\frac{dr}{d\theta} + \frac{\sin\theta}{1-\cos\theta} = 0$

$\cot\phi_1 = \frac{\sin\theta}{1+\cos\theta} \quad ; \quad \cot\phi_2 = \frac{-\sin\theta}{1-\cos\theta}$

$\tan\phi_1 = \frac{1+\cos\theta}{\sin\theta} \quad ; \quad \tan\phi_2 = \frac{-(1-\cos\theta)}{\sin\theta}$

$$\tan\phi_1 \cdot \tan\phi_2 = \left(\frac{1+\cos\theta}{\sin\theta}\right)\left\{\frac{-(1-\cos\theta)}{\sin\theta}\right\}$$

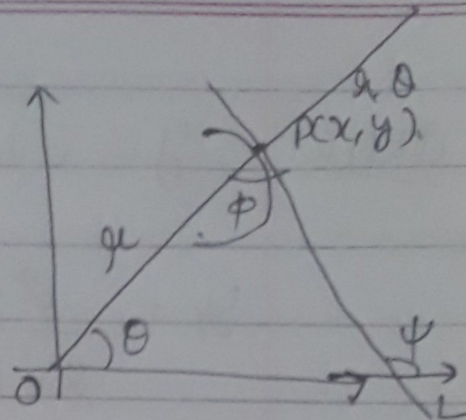
$$= -\frac{(1-\cos^2\theta)}{\sin^2\theta} = -\frac{\sin^2\theta}{\sin^2\theta}$$

$$= -1$$

$\therefore$ Angle between these curves is $\pi/2$

Ques 5 Let $P(r, \theta)$ be any point on the curve $r = f(\theta)$.

$\therefore x\hat{O}P = \theta$ and $OP = r$



Let PL be the tangent at P.
such that

$\angle PLX = \psi$, $\angle POL = \theta$, $\angle OPL = \phi$

from figure

$$\psi = \phi + \theta \Rightarrow \tan\psi = \tan(\phi + \theta)$$

$$\tan\psi = \frac{\tan\phi + \tan\theta}{1 - \tan\phi\tan\theta} \quad \text{---(1)}$$

Let (x, y) be cartesian co-ordinates of P, so that

$$x = r\cos\theta, \quad y = r\sin\theta$$

$$\tan\psi = \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$\tan\psi = \frac{r\cos\theta + r_1\sin\theta}{-r\sin\theta + r_1\cos\theta}, \quad r_1 = \frac{dr}{d\theta}$$

$$= \frac{\frac{r}{r_1} + \tan\theta}{1 - \frac{r}{r_1}\tan\theta} \quad \text{---(2)}$$

comparing (1) & (2)

$$\tan\phi = \frac{r}{r_1} = r\frac{d\theta}{dr}$$

Ques 6

$$y = \sqrt{1+\sin 2x} \qquad y(0) = 1$$

$$y_1 = \frac{2\cos 2x}{2\sqrt{1+\sin 2x}} \qquad y_1(0) = 1.$$

$$y y_1 = \cos 2x$$

$$y y_2 + y_1^2 = -2\sin 2x. \quad \begin{cases} y_2(0)\cdot 1 + (1)^2 = 0 \\ y_2(0) = -1 \end{cases}$$

$$y_1 y_2 + y y_3 + 2y_1 y_2 = -2\cos 2x \quad \text{--- (1)}$$

$$1(-1) + (1)\, y_3(0) + 2(1)(-1) = -2$$

$$-1 + y_3(0) - 2 = -2 \Rightarrow y_3(0) = 1$$

Maclaurin's expansion is

$$y = y(0) + x y_1(0) + \frac{x^2}{2!} y_2(0) + \frac{x^3}{3!} y_3(0) + \frac{x^4}{4!} y_4(0) + \dots \quad \text{--- (2)}$$

from (1) again

$$3\{ y_1 y_3 + y_2^2 + y_1 y_3 + y y_4 \} = 4\sin 2x$$

$$3\{1+1+1+y_4(0)\}=0$$

$$y_4(0)=-3$$

$\therefore$ from (2)

$$\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2!} + \frac{x^3}{3!}$$

Que-7 Let $K = \lim_{x\to 0}\left\{\frac{a^x+b^x+c^x+d^x}{4}\right\}^{1/x}$

$$\log K = \lim_{x\to 0} \frac{1}{x}\log\left(\frac{a^x+b^x+c^x+d^x}{4}\right)$$

$$= \lim_{x\to 0} \frac{\log\left(\frac{a^x+b^x+c^x+d^x}{4}\right)}{x} \quad \left[\frac{0}{0}\right]$$

By L'Hospital rule.

$$= \lim_{x\to 0} \frac{\left[\frac{\cancel{4}}{a^x+b^x+c^x+d^x}\right]\frac{1}{4}\left\{\begin{array}{c}a^x\log a + b^x\log b + c^x\log c\\ +d^x\log d\end{array}\right\}}{1}$$

$$= \frac{\log a + \log b + \log c + \log d}{4} = \frac{1}{4}\log(abcd)$$

$$\log K = \log(abcd)^{1/4} \Rightarrow \boxed{K=(abcd)^{1/4}}$$

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Internal Assessment Test II–February 2021

Sub:	Calculus and Linear Algebra				Sub Code:	18MAT11	
Date:	26/02/2021	Duration:	90 mins	Max Marks: 50	Sem / Sec:	I / A to G (PHY CYCLE)	OBE

		MARKS	CO	RBT
1.	Find the evolute of the parabola $y^2 = 4ax$.	[08]	CO6	L3
2.	Find the pedal equation of the curve $r^m = a^m(cosm\theta + sinm\theta)$.	[07]	CO6	L3
3.	Show that for the curve $r(1-\cos\theta) = 2a$, ρ^2 varies as r^3.	[07]	CO6	L3
4.	Find the angle between the curves $r = 6cos\theta$, $r = 2(1 + cos\theta)$	[07]	CO6	L3
5.	Derive the expression for angle between the radius vector and the tangent.	[07]	CO6	L3
6.	Obtain the Maclaurin's series expansion of $\log(1 + cos\theta)$ upto term containing x^4.	[07]	CO6	L3
7.	Evaluate $\lim_{x\to 0}\left[\frac{a^x+b^x+c^x}{3}\right]^{1/x}$	[07]	CO6	L3