CMR
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Internal Assesment Test-I									
Sub:	Engineering Electromagnetics							Code:	18EC55
Date:	12/09 /2020	Duration:	60 mins	Max Marks:	50	Sem:	5th	Branch:	ECE(B,C)
Answer All FULL Questions									

	Marks	OBE CO	OBE RBT
1. State and explain Coulomb's law in vector form.	[05]	CO1	L1

Soln. The force b/w two very small charged objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the dist. b/w them.

Q_1 —— R —— Q_2

i.e. $F = \dfrac{k Q_1 Q_2}{R^2}$

Q_1 & Q_2 → +ve or −ve quantities of charge
unit Coulomb (C)

R → separation in m.
k → constant of proportionality.

$k = \dfrac{1}{4\pi\epsilon_0}$

where, ϵ_0 → permittivity of free space.

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m
$= \dfrac{1}{36\pi} \times 10^{-9}$ F/m.

F → Force in Newton.

1.(a) $\therefore F_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2} (-\hat{a}_{21}) = -\vec{F_2}$

$\Rightarrow$ Coulomb's law is a mutual force.

Important observations:

i) charges should be point charges and stationary in nature.

ii) Should consider the signs of the charges to decide whether the force will be attractive or repulsive.

iii) Coulomb's law is linear.

i.e. if $\vec{F_2} = -\vec{F_1}$

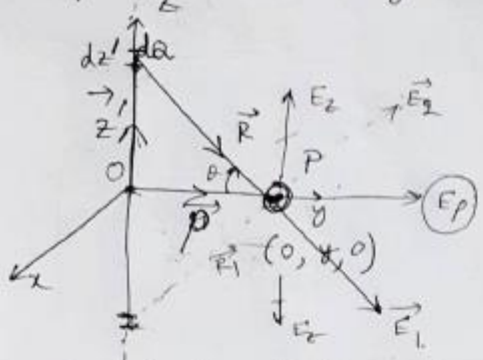
then, $n\vec{F_2} = -n\vec{F_1}$

where n is a scalar.

iv) Force on a charge in the presence of several other charges is the sum of the forces on that charge due to each of the other charges acting alone.

2. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution. [10] CO1 L1

Find the electric field intensity due to a line charge distribution of ρ_L C/m of infinite length where charge is uniformly distributed along the length.



$\frac{z'}{\rho} = \tan\theta$

$z' = \rho \tan\theta$

ρ_L C/m

Find $\vec{E}$ at P (0, y, 0) because of the line charge along z-axis.

We consider incremental charge

$dQ = \rho_L dz'$ . . . (1) [charge on length dz']

$\vec{R} = \vec{\rho} - \vec{z}'$ $\quad (\vec{z'} + \vec{R} = \vec{\rho})$

$\vec{R} = (\rho \hat{a}_\rho - z' \hat{a}_z)$ . . (2) $\quad |\vec{R}| = \sqrt{\rho^2 + z'^2}$

$\therefore$ The field at P, because of dQ,

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 |\vec{R}|^2} \hat{a}_R = \frac{\rho_L dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)} \cdot \frac{(\rho\hat{a}_\rho - z'\hat{a}_z)}{(\rho^2 + z'^2)^{1/2}}$$

$[\because |\vec{R}| = \sqrt{\rho^2 + z'^2}]$

$$= \frac{\rho_L dz' \cdot (\rho \hat{a}_\rho - z' \hat{a}_z)}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

We know from the symmetry of the problem - (3)

$$\vec{dE} = \frac{\rho_L dz' \rho \hat{a}_\rho}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

∴ The field at P

$$\vec{E} = \int d\vec{E} = \int_{z'=-\infty}^{\infty} \frac{\rho_L \, dz' \rho \, \hat{a}_\rho}{4\pi\epsilon_0(\rho^2+z'^2)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{dz'}{(\rho^2+z'^2)^{3/2}} \hat{a}_\rho$$

note: $\tan\theta = \frac{z'}{\rho}$

$z' = \rho \tan\theta$ | $z' \to \infty, \ \theta = \pi/2$

$dz' = \rho \sec^2\theta \, d\theta$ | $z' \to -\infty, \ \theta = -\pi/2$

$$\therefore \vec{E} = \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2+\rho^2\tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2)^{3/2}(1+\tan^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \cancel{\rho}}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\cancel{\rho} \sec^2\theta \, d\theta}{\rho^3 (\sec^2\theta)^{3/2}} \hat{a}_\rho$$

$$= \frac{\rho_L \cancel{\rho}}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{\sec^3\theta} \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta} \, d\theta \, \hat{a}_\rho$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \hat{a}_\rho = \frac{\rho_L}{4\pi\epsilon_0} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} \hat{a}_\rho$$

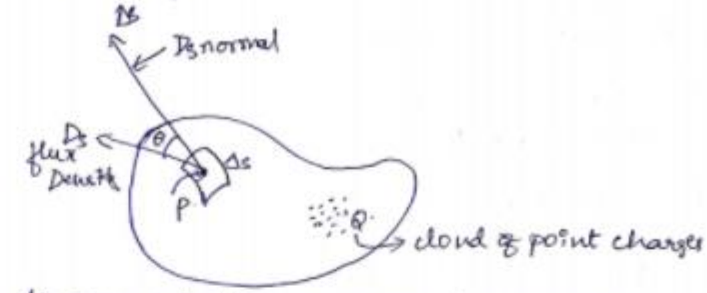
$$= \frac{\rho_L}{4\pi\epsilon_0} [1+1] \hat{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho$$

∴ The electric field intensity due to infinite static line charge along z-axis,

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \hat{a}_\rho \ \text{V/m}}$$

3. Write and explain the mathematical form of Gauss's law. [05] CO2 L2

Gauss' law - The electric flux passing through any closed surface is equal to the total charge enclosed by the surface.



At any point P consider an incremental surface Δs. $\vec{D_s}$ makes an angle θ with $\vec{\Delta S}$

Then flux crossing $\vec{\Delta S}$ is then,

$$\Delta\psi = \text{flux crossing } \Delta s$$

$$\Delta\psi = \vec{D_s}\cdot\vec{\Delta S}$$

$$\boxed{D_{s,normal}\,\Delta s = D_s\cos\theta\,\Delta s = D_s\cdot\Delta s}$$

∴ Total flux passing through entire closed surface,

$$\psi = \int d\psi = \oint_{\substack{closed\\surface}} \vec{D_s}\cdot\vec{dS} = \text{charge enclosed} = Q.$$

Mathematical form of Gauss' law $\Rightarrow$ $\boxed{\psi = \oint_S \vec{D_s}\cdot\vec{ds} = Q = \text{charge enclosed}}$

Q can be → n' point charges $= \Sigma Q_n$.

→ line charge $Q = \int \rho_L dL$

→ surface (not closed) $Q = \int_S \rho_S ds$

→ Volume charge $Q = \int_{vol} \rho_v dV$

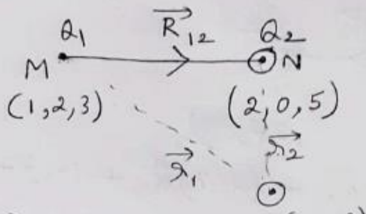
$$\boxed{\oint_S \vec{D_s}\cdot\vec{ds} = \int_{vol} \rho_v dV}$$

4. A charge $Q_1 = 3 \times 10^{-4}$ C is located at (1, 2, 3) and a second charge $Q_2 = 10^{-4}$ C is located at (2,0,5) in vacuum. Find the force exerted on Q_2 by Q_1. [05] CO1 L3

Consider two charges, $Q_1 = 3\times10^{-4}$ C at $M(1,2,3)$ and a charge $Q_2 = -10^{-4}$ C at $N(2,0,5)$ in vacuum. Find the force on Q_2 due to Q_1.

$Q_1 = 3\times10^{-4}C$ | $M(1,2,3) \rightarrow Q_1$

$Q_2 = -10^{-4}C$ | $N(2,0,5) \rightarrow Q_2$



$$\vec{R}_{12} = (2-1)\hat{a}_x + (0-2)\hat{a}_y + (5-3)\hat{a}_z$$

$$\text{or } \vec{R}_{12} = \hat{a}_x - 2\hat{a}_y + 2\hat{a}_z \qquad \epsilon_0 = 8.854\times10^{-12} \text{ F/m}$$

$$\boxed{\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{R}_{12}|^2}\hat{a}_{12} \text{ N}}$$

$$|\vec{R}_{12}| = \sqrt{1^2+(-2)^2+(2)^2} = \underline{\underline{3}}$$

$$\hat{a}_{12} = \frac{\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z}{\pm 3}$$

$$\vec{F}_2 = \frac{(3\times10^{-4})\times(-10^{-4})}{4\pi\times8.854\times10^{-12}}\cdot\frac{1}{3^2}\cdot\frac{(\hat{a}_x - 2\hat{a}_y + 2\hat{a}_z)}{3}$$

$$= (-9.9863\,\hat{a}_x + 19.97\,\hat{a}_y - 19.97\,\hat{a}_z) \text{ N}$$

5. A uniform line charge of infinite length with $\rho_L = 40$ nC/m lies along z-axis. Find E at (-2, -2,8) in air. [05] CO2 L1

A uniform line charge of infinite length with $\rho_L = 40$ nC/m along z-axis, Find $\vec{E}$ $(-2,2,8)$ in air.

(ρ, ϕ, z)

(r, θ, ϕ)

$(10, 20°, 30)$

Solution

$$\vec{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\hat{a}_\rho \text{ V/m}$$

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$$\boxed{\rho = \sqrt{x^2+y^2}}$$

$$\rho = \sqrt{(-2)^2+2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\therefore \vec{E} = \frac{40\times10^{-9}}{2\times3.14\times8.854\times10^{-12}\times2\sqrt{2}}\hat{a}_\rho$$

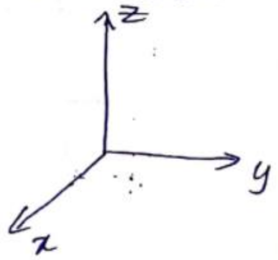
$$= 254.2\,\hat{a}_\rho \text{ V/m}$$

$$E = 254.2 \text{ V/m}$$

$$\therefore \boxed{\vec{E} = 254.2\,\hat{a}_\rho \text{ V/m}}$$

6. Find **E**, at origin due to a point charge 12 nC at (2, 0, 6). [05] CO1 L3

E at (1,1,1) due to a 3nC point charge at (1,1,0).



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 |\vec{R}|^2} \cdot \hat{a}$$

$$\vec{R} = (1-1)\hat{a}_x + (1-1)\hat{a}_y + (1-0)\hat{a}_z$$

$$\vec{R} = \hat{a}_z \quad |\vec{R}| = 1 \quad \hat{a} = \hat{a}_z$$

$$\vec{E} = \frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}} \times \hat{a}_z = \underline{\underline{26.96\ \hat{a}_z\ V/m}}$$

7. An infinite surface charge with uniform surface charge density ρs = 120 μC/m² lies on the plane z = -5 m. Find **D** at (2, -3, 6) in air. [05]

$\rho_s = 120\,\mu C/m^2$ on plane $z = -5\,m$.

D at (2,-3,6) in air

$$\vec{D} = \frac{\rho_s}{2}\hat{a}_z = \frac{120\mu}{2}\hat{a}_z$$

$$= 60\mu\,\hat{a}_z$$

$$\boxed{\vec{D} = 60\,\hat{a}_z\ \mu C/m^2}$$

<table>
<tr><td>MCQ</td><td>(2X5)
1. Electric flux density and electric field intensity are related as,
(a) Electric flux density = permittivity * Electric field intensity
(b) Electric flux density = permeability * Electric field intensity
(c) Electric field intensity = permittivity * Electric flux density
(d) Electric field intensity = permeability * Electric flux density

ANSWER : I
2. The electric field intensity at a point P above an infinite surface charge is not dependent on the height of point P above the infinite charged sheet:

(a) TRUE
(b) FALSE
ANSWER : I

3.Unit of E is:
(a) V/m
(b) C/m^2
(c) N
(d) C/m
ANSWER : I

4. The Gaussian surface of an infinite line charge will be:
a) Sphere
b) Cyinder
c) Cube
d) Cuboid
ANSWER : II

5.
Gauss's law of electrostatics is not related to which of the following:
D
E
Q
M
ANSWER : IV</td><td></td><td></td><td></td></tr>
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