ITC IAT 2

Answer all 5 full questions. Each question carries 10 marks.
Make use of workbook for solving all the questions.
Write your USN in each page of the workbook in the left top corner.
Questions are MCQs and Fill in the blanks.
At the end of the test upload the solutions workbook.

*Required

1. Email address *

2. Name *

3. USN *

4. Section *

Mark only one oval.

- A
- B
- C
- D
- E

Q1. Apply Shannon's encoding algorithm to the following symbols and obtain the Code efficiency and redundancy of the so formed code. If S = {a,b,c} and P = {0.5, 0.3, 0.2}.

5. Average Length, L = __________ * 4 points

Mark only one oval.

- ( ) 1.7 bits/symbol
- ( ) 1.6 bits/symbol
- ( ) 1.8 bits/symbol
- ( ) 1.4 bits/symbol

6. Entropy, H(S) = _____ * 4 points

Mark only one oval.

- ( ) 1.5855 bits/symbol
- ( ) 1.4855 bits/symbol
- ( ) 1.7855 bits/symbol
- ( ) 1.2855 bits/symbol

7. Code efficiency = _____ * 1 point

Mark only one oval.

- ( ) 83.18
- ( ) 84.18%
- ( ) 87.38%
- ( ) 86.38%

8. Code Redundancy = ________ * 1 point

Mark only one oval.

- 13.62 %
- 15.82 %
- 16.82 %
- 12.62 %

Q2.

Given the symbols S= {a,b,c,d,e,f } with respective probabilities P= {0.4, 0.2, 0.2, 0.1, 0.07, 0.03}, construct a binary code by applying Shannon-Fano encoding procedure. Determine the code efficiency and redundancy of the so formed code. Draw the code tree for the same.

9. Average Length, L = ______ 4 points

Mark only one oval.

- 1.8 bits/symbol
- 2.8 bits/symbol
- 1.3 bits/symbol
- 2.3 bits/symbol

10. Entropy, H(S) = ______ * 4 points

Mark only one oval.

- 2.209 bits/symbol
- 2.109 bits/symbol
- 1.809 bits/symbol
- 1.909 bits/symbol

11. Code efficiency = ______ * 1 point

Mark only one oval.

- 96.04%
- 95.04%
- 97.04%
- 98.04%

12. Code Redundancy = _________ * 1 point

Mark only one oval.

- 4.96%
- 1.96%
- 2.96%
- 3.96%

Q3. Consider a source with S= {a,b,c,d,e,f,g,h} with respective probabilities P= {0.22, 0.2, 0.18, 0.15, 0.1, 0.08, 0.05, 0.02}, construct a binary Huffman code by placing the composite symbol as low as you can.. Determine the code efficiency and redundancy of the so formed code.

13. Average Length, L = ______ * 4 points

Mark only one oval.

- 2.7 bits/symbol
- 2.8 bits/symbol
- 2.9 bits/symbol
- 2.6 bits/symbol

14. Entropy, H(S) = ______ *

4 points

Mark only one oval.

- 2.535 bits/symbol
- 2.7535 bits/symbol
- 2.635 bits/symbol
- 2.835 bits/symbol

15. Code efficiency = ______ *

1 point

Mark only one oval.

- 96.34 %
- 98.34 %
- 97.34 %
- 94.34 %

16. Code Redundancy = ________ *

1 point

Mark only one oval.

- 2.66 %
- 1.66 %
- 3.66 %
- 5.66 %

Q4.

In a communication system, a transmitter has 3 input symbols A = {a1, a2, a3} and receiver also has 3 output symbols B = {b1,b2,b3}. The Matrix given in figure 4 shows JPM with some marginal probabilities. Find the missing probabilities (*) in the table, P(b3/a1), P(a1/b3). Also check whether the events a1 and b1 are statistically independent or not.

JPM figure 4.

a_i \ b_j	b_1	b_2	b_3
a_1	$\frac{1}{12}$	*	$\frac{5}{36}$
a_2	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{5}{36}$
a_3	*	$\frac{1}{6}$	*
$P(b_j)$	$\frac{1}{3}$	$\frac{14}{36}$	*

17. P(a3, b1) = _____ * 2 points

Mark only one oval.

- 1/3
- 1/9
- 1/12

18. P(a1, b2) = ________ * 2 points

Mark only one oval.

- 1/6
- 1/9
- 1/12

19. P(a3,b3) = ________ * 2 points

Mark only one oval.

- ( ) 1
- ( ) 0
- ( ) 1/2

20. P(b3/a1)______ * 2 points

Mark only one oval.

- ( ) 5/12
- ( ) 5/6
- ( ) 5/36
- ( ) 1/4

21. P(a1/b3)______ * 1 point

Mark only one oval.

- ( ) 1/2
- ( ) 1
- ( ) 1/4
- ( ) 1/12

22. P(a1).P(b1) =_____. Therefore a1 & b1 are ___________ * 1 point

Mark only one oval.

- 1/2, not statistically independent
- 1/6, not statistically independent
- 1/12, statistically independent
- 1/9, not statistically independent

Q5.

A transmitter has an alphabet consisting of 5 letters {a1, a2, a3, a4, a5} and the receiver has an alphabet of four letters {b1,b2,b3,b4}. The joint probabilities of the system are given in the figure 5. Compute the individually the following entropies of this channel H(A), H(B), H(A,B), H(B/A) and H(A/B).

Markov model figure 5.

$$P(A, B) = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{array} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \\ \end{array}$$

$$P(A, B) = \begin{bmatrix} 0.25 & 0 & 0 & 0 \\ 0.10 & 0.30 & 0 & 0 \\ 0 & 0.05 & 0.10 & 0 \\ 0 & 0 & 0.05 & 0.1 \\ 0 & 0 & 0.05 & 0 \end{bmatrix}$$

	b_1	b_2	b_3	b_4
a_1	0.25	0	0	0
a_2	0.10	0.30	0	0
a_3	0	0.05	0.10	0
a_4	0	0	0.05	0.1
a_5	0	0	0.05	0

23. H(A) = _____ bits/symbol 2 points

Mark only one oval.

- 1.857
- 2.066
- 1.809
- 1.257

24. H(B) = ______ bits/symbol * 2 points

Mark only one oval.

- 1.857
- 2.066
- 1.809
- 1.257

25. H(A,B) = ______ bits/symbol * 2 points

Mark only one oval.

- 2.066
- 2.666
- 2.166
- 1.666

26. H (B/A)= ______ bits/symbol * 2 points

Mark only one oval.

- 0.4
- 0.6
- 1.766
- 1.666

27. H (A/B)= ______ bits/symbol *

2 points

Mark only one oval.

- 0.908
- 0.809
- 1.809
- 1.908

Solution Upload

Upload the solution document of IAT 2 as a pdf file

28. Files submitted:

Q.1

$S = \{a, b, c\}$ and $P = \{0.5, 0.3, 0.2\}$

a	b	c
0.5	0.3	0.2
P_1	P_2	P_3

$\alpha_1 = 0$

$\alpha_2 = 0.5$

$\alpha_3 = 0.5 + 0.3 = 0.8$

$\alpha_4 = 0.8 + 0.2 = 1$

The smallest integer value of l_i is found by

$$2^{l_i} \geq \frac{1}{P_i}$$

$2^{l_1} \geq \frac{1}{0.5} \quad \therefore l_1 = 1$

$2^{l_2} \geq \frac{1}{0.3} \quad \therefore 2^{l_2} \geq 3.33 \Rightarrow l_2 = 2$

$2^{l_3} \geq \frac{1}{0.2} \quad \therefore 2^{l_3} \geq 5 \Rightarrow l_3 = 3$.

The decimal numbers α_i are expanded in binary form.

$\alpha_1 = 0$

$\alpha_2 = (0.5)_{10} = (.1)_2$

$0.5 \times 2 = 0$ with carry 1

$\alpha_3 = (0.8)_{10} = (.110\ldots)_2$

$0.8 \times 2 = 0.6$ with carry 1

$0.6 \times 2 = 0.2$ with carry 1

$0.2 \times 2 = 0.4$ with carry 0

Therefore

$\alpha_1 = 0$ and $l_1 = 1$ $\therefore$ code of $a = 0$

$\alpha_2 = (.1)_2$ and $l_2 = 2$ $\therefore$ code of $b = 10$

$\alpha_3 = (.110\ldots)_2$ and $l_3 = 3$ $\therefore$ code for $c = 110$

$L = \sum_{i=1}^{3} P_i l_i$

$= 0.5 \times 1 + 0.3 \times 2 + 0.2 \times 3$

$= 1.7$ ~~bits/symbol~~ binits/symbol

$H(S) = \sum_{i=1}^{3} P_i \log \frac{1}{P_i}$

$= 0.5 \log \frac{1}{0.5} + 0.3 \log \frac{1}{0.3} + 0.2 \log \frac{1}{0.2}$

$= 1.4855$ bits/symbol

$\eta = \frac{H(S)}{L} = \frac{1.4855}{1.7} \times 100\%$

$= 87.38\%$

$R_{\eta_c} = 100 - 87.38$

$= 12.62\%$

Q.2.

$S = \{a, b, c, d, e, f\}$

$P = \{0.4, 0.2, 0.2, 0.1, 0.07, 0.03\}$

Symbols	P_i		P_i		P_i		P_i	
a	0.4	1	0.4	1				
b	0.2	1	0.2	0				
c	0.2	0	0.2	1				
d	0.1	0	0.1	0	0.1	1		
e	0.07	0	0.07	0	0.07	0	0.07	1
f	0.03	0	0.03	0	0.03	0	0.03	0

Symbol	Code	P_i	l_i
a	11	0.4	2
b	10	0.2	2
c	01	0.2	2
d	001	0.1	3
e	0001	0.07	4
f	0000	0.03	4

$$L = \sum_{i=1}^{6} P_i l_i$$

$$= 0.4 \times 2 + (0.2 \times 2) \times 2 + 0.1 \times 3 + 0.07 \times 4 + 0.03 \times 4$$

$= 2.3$ binits/symbol.

$$H(S) = \sum_{i=1}^{6} P_i \log \frac{1}{P_i}$$

$$= 0.4 \log \frac{1}{0.4} + 2 \times 0.2 \log \frac{1}{0.2} + 0.1 \log \frac{1}{0.1} + 0.07 \log \frac{1}{0.07}$$

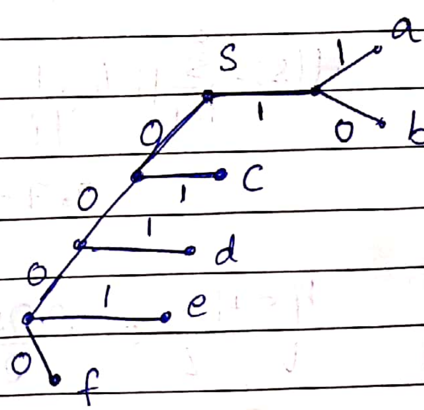
$$+ 0.03 \log \frac{1}{0.03}$$

$$= 0.529 + 0.929 + 0.332 + 0.269 + 0.152$$

$= 2.21$ bits/symbol

$$\eta = \frac{H(S)}{L} = 96.08\%$$

$R_{\eta c} = 3.92\%$



Q.3. $S = \{a, b, c, d, e, f, g, h\}$

$P = \{0.22, 0.2, 0.18, 0.15, 0.1, 0.08, 0.05, 0.02\}$

l_i	symbol	P_i	code	P_i	code	P_i	code
2	a	0.22	10	0.22	10	0.22	10
2	b	0.2	11	0.2	11	0.2	11
3	c	0.18	000	0.18	000	0.18	000
3	d	0.15	001	0.15	001	0.15	001
3	e	0.1	011	0.1	011	→ 0.15	010
4	f	0.08	0100	0.08	0100	0.1	011
5	g	0.05	01010	→ 0.07	0101		
5	h	0.02	01011				

P_i	code	P_i	code	P_i	code	P_i	code
0.25	01	→ 0.33	00	→ 0.42	1	→ 0.58	0
0.22	10	0.25	01	0.33	00	0.42	1
0.2	11	0.22	10	0.25	01		
0.18	000	0.2	11				
0.15	001						

$$L = \sum_{i=1}^{8} P_i l_i = 0.22\times2 + 2\times0.2 + 3\times(0.18+0.15+0.1) + 0.08\times4 + 5(0.05+0.02)$$

$$= 2.8 \text{ binits/symbol}$$

$$H(S) = \sum_{i=1}^{8} P_i \log\frac{1}{P_i} = 0.22\log 1/0.22 + 0.2\log 1/0.2 + 0.18\log 1/0.18 + 0.15\log 1/0.15 + 0.1\log 1/0.1 + 0.08\log 1/0.08 + 0.05\log 1/0.05 + 0.02\log 1/0.02$$

$$= 2.7535 \text{ bits/symbol}$$

$$\eta = \frac{H(S)}{L} = \frac{2.7535}{2.8}\times100\% = 98.34\%$$

$$R_{\eta_c} = 100 - 98.34 = 1.66\%$$

Q.4

$a_i \backslash b_j$	b_1	b_2	b_3
a_1	1/12	?	5/36
a_2	5/36	1/9	5/36
a_3	?	1/6	?
$P(b_j)$	1/3	14/36	?

$\rightarrow P(a_3, b_1) = \frac{1}{3} - \left(\frac{1}{12} + \frac{5}{36}\right) = 1/9$ //

$\rightarrow P(a_1, b_2) = \frac{14}{36} - \left(\frac{1}{9} + \frac{1}{6}\right) = 1/9$ //

$\rightarrow P(a_3, b_3) =$

$P(b) = 1$

$P(b_3) = 1 - \left(\frac{1}{3} + \frac{14}{36}\right) = \frac{5}{18}$

$P(a_3, b_3) = \frac{5}{18} - \left[\frac{5}{36} + \frac{5}{36}\right] = 0$ //

$\rightarrow P(b3/a_1) = \frac{P(b_3, a_1)}{P(a_1)}$

$P(a_1) = \frac{1}{12} + \frac{1}{9} + \frac{5}{36} = \frac{1}{3}$

$\therefore P(b3/a_1) = \frac{5}{36} \times \frac{3}{1} = \frac{5}{12}$ //

$\rightarrow P(a_1 / b_3) = \frac{P(a_1, b_3)}{P(b_3)}$

$P(b_3) = 5/18 \quad \therefore \quad P(a_1/b_3) = \frac{5}{36} \times \frac{18}{5} = 1/2$ //.

$\rightarrow P(a_1) . P(b1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$P(a_1, b_1) = 1/12$

Not statistically independent //

Q.5

$$P(A,B) = \begin{array}{c|cccc} & b_1 & b_2 & b_3 & b_4 \\ \hline a_1 & 0.25 & 0 & 0 & 0 \\ a_2 & 0.10 & 0.30 & 0 & 0 \\ a_3 & 0 & 0.05 & 0.10 & 0 \\ a_4 & 0 & 0 & 0.05 & 0.1 \\ a_5 & 0 & 0 & 0.05 & 0 \end{array}$$

$$H(A) = \sum_{i=1}^{5} P_{a_i} \log \frac{1}{P_{a_i}}$$

$= 0.25 \log 1/0.25 + 0.40 \log 1/0.40$
$+ 0.15 \log 1/0.15 + 0.15 \log 1/0.15 + 0.05 \log 1/0.05$
$= 2.066$ bits/symbol

$$H(B) = \sum_{i=1}^{4} P(b_i) \log 1/P(b_i)$$

$= 0.35 \log 1/0.35 + 0.35 \log 1/0.35 + 0.2 \log 1/0.2$
$+ 0.1 \log 1/0.1$
$= 1.857$ bits/symbol //

$$H(A,B) = \sum_{i=1}^{5} \sum_{j=1}^{4} P(a_i, b_j) \log 1/P(a_i, b_j)$$

$= 0.25 \log 1/0.25 + 3(0.10 \log 1/0.10) + 0.30 \log 1/0.30$
$+ 0.05 \times 3 \times \log 1/0.05$
$= 2.666$ bits/symbol //

$H(B/A) = H(A,B) - H(A)$
$= 2.666 - 2.066 = 0.6$ bits/symbol //

$H(A/B) = H(A,B) - H(B)$
$= 2.666 - 1.857 = 0.809$ bits/symbol //