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| CMR INSTITUTE OF TECHNOLOGY | |  | | | | | | | |
| | | | | Internal Assessment Test - III | | | | | |
| Sub: | Principles of Communication Systems | | | Code: | 18EC53 | | | | |
| Date: | 12/12/2020 | Duration: | 90 mins | Max Marks: | 50 | Sem: | 5th | Branch: | ECE |

| No | Question | Marks | CO | RBT |
|----|---|-------|-----|-----|
| 1 | State and prove sampling theorem | 10 | CO3 | L2 |
| 2 | a) A TV signal of bandwidth 4.2MHz is transmitted using binary PCM. The number of quantization level is 512. Calculate a) Code word length b) Transmission BW c) Final bit rate. Assume $f_s = 2f_m$ | 06 | CO4 | L3 |
| | b) Represent Binary data "10011101" in polar NRZ and bipolar RZ format. | 04 | CO4 | L2 |
| 3 | a) For a sinusoidal modulating signal show the signal to quantization error ratio is $1.8 + 6R$. Where R is number of bits per sample. | 05 | CO4 | L2 |
| | b) Draw block diagram of TDM and explain the principle. | 05 | CO3 | L1 |
| 4 | Define PAM. Obtain the expression for Fourier transform of PAM signal. | 10 | CO3 | L2 |
| 5 | Write notes on the following line codes. a) NRZ b) RZ c) Manchester code d) Differential Code | 10 | CO4 | L2 |
| 6 | Find the Nyquist rate and Nyquist interval for a) $m_1(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$ b) $m_2(t) = \frac{\sin 500\pi t}{\pi t}$ | 10 | CO3 | L3 |

- Answer any 5 full Questions.
- Attempt the questions with main points, expressions, derivations if any, wave forms and block diagram if any. Finish the examination in stipulated time
- After completing the write up scan and upload the answer script.

Solution:

1. Sampling Theorem

Statement: Sampling theorem states that any continuous time signal can be completely represented in its samples and recovered back if the sampling frequency is greater than or equal to twice the highest frequency component of base band signal.

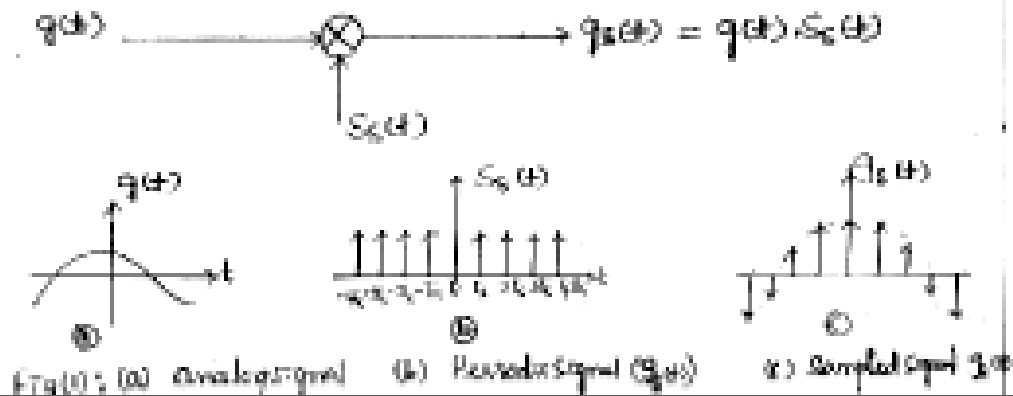
That is Sampling frequency, $f_s \geq 2W$.

Where W = Highest frequency in base band continuous time signal.

This condition is also called Nyquist condition for sampling process.

Explanation and Proof:

* Consider an arbitrary signal $g(t)$ of finite energy which is specified for all time. A segment of the signal $g(t)$ is shown in fig (a). Suppose, that we sample the signal $g(t)$ instantaneously and at a uniform rate, once every T_s seconds. Consequently, we obtain an infinite sequence of samples spaced T_s seconds apart and denoted by $\{g(nT_s)\}$, where n takes on all possible integer values. We refer to T_s as the sampling period, and to its reciprocal $f_s = 1/T_s$ as the sampling rate. This ideal form of sampling is called instantaneous sampling.



* Let $q_s(t)$ denote the signal obtained by individually weighting the elements of a periodic sequence spaced T_s seconds. Therefore, sampled output $q_s(t)$ is given by,

$$q_s(t) = q(t) \cdot S_s(t) \quad \text{--- (1)}$$

* Let $S_s(t)$ denote the periodic impulse train and is represented as,

$$S_s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad \text{--- (2)}$$

Substituting Eqn (2) in Eqn (1) we get

$$q_s(t) = q(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

Using shifting property of impulse function

$$\text{i.e., } q(t) \cdot \delta(t - nT_s) = q(nT_s) \delta(t - nT_s)$$

$$\therefore \boxed{q_s(t) = \sum_{n=-\infty}^{\infty} q(nT_s) \delta(t - nT_s)} \quad \text{--- (3)}$$

For frequency domain consider,

$$q_s(t) = q(t) \cdot S_s(t)$$

Taking Fourier Transform on both sides, we get

$$Q_s(f) = Q(f) * S_s(f) \quad \text{--- (4)}$$

where,

$$S_s(f) = f_s \sum_{n=-\infty}^{\infty} \delta(f - n f_s) \quad \text{--- (5)}$$

Substituting Eqⁿ (5) in Eqⁿ (4) we get.

$$G_s(f) = G(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f - n f_s)$$

From convolution property of impulse function

$$\Delta \delta(t), \quad G(f) * \delta(f - n f_s) = G(f - n f_s)$$

$$\therefore G_s(f) = f_s \sum_{n=-\infty}^{\infty} G(f - n f_s) \quad \text{--- (6)}$$

Eqⁿ (6) can be rewritten as,

$$G_s(f) = f_s G(f) + f_s \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} G(f - n f_s) \quad \text{--- (7)}$$

When the spectrum of $G_s(f)$ is passed through an LPE then the 2nd term in RHS of Eqⁿ (7) is eliminated resulting in

$$G_s(f) = f_s \cdot G(f)$$

$$\therefore G(f) = \frac{1}{f_s} \cdot G_s(f) \quad \text{--- (8)}$$

where $f_s = \omega T$

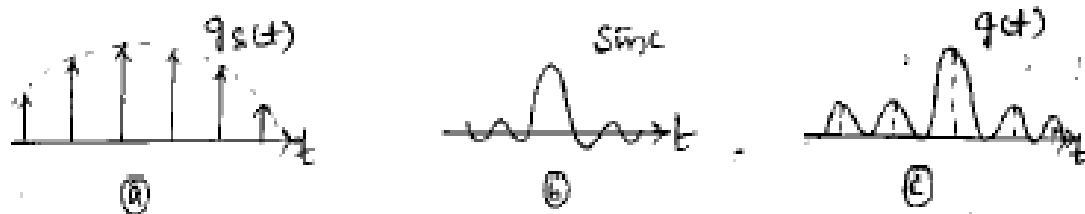


Fig : Recovering $g(t)$ signal from sequence of samples $g_s(t)$.

Now, we may state the sampling theorem for strictly bandlimited signals of finite energy into two equivalent parts :

- 1) A bandlimited signal of finite energy, which only has frequency components less than " ω " Hertz, is completely described by specifying the values of the signal at instants of time separated by $\frac{1}{2\omega}$ seconds.
- 2) A bandlimited signal of finite energy, which only has frequency components less than " ω " Hertz, may be completely recovered from a knowledge of its samples taken at the rate of 2ω samples per second.

The sampling rate of 2ω samples per second, for a signal bandwidth of ' ω ' Hertz, is called the Nyquist rate; its reciprocal $\frac{1}{2\omega}$ (measured in seconds) is called the Nyquist interval.

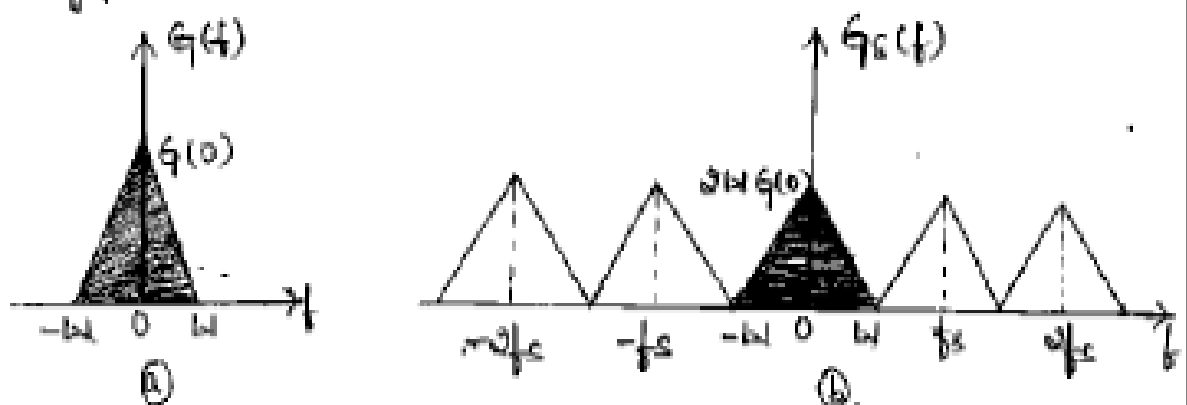
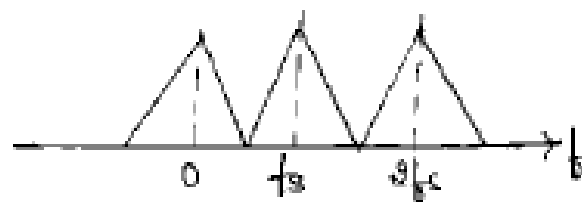


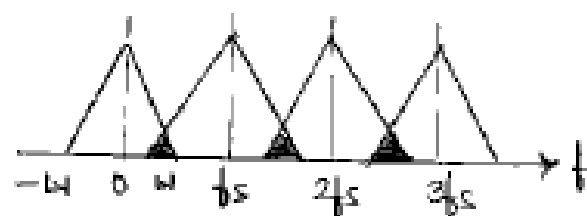
Fig : (a) spectrum of a strictly band limited signal $g(t)$.
 (b) spectrum of a sampled version of $g(t)$ for $T_s = \frac{1}{2\omega}$.

NOTE: the concept of undersampling and over sampling is explained below.

- 1) When sampling frequency $f_s = 2W$ then this type of sampling is called correct sampling and here there is no aliasing effect seen in this mechanism. i.e. when $f_s = 2W$.



- 2) When $f_s < 2W$ then it is undersampling and there will be aliasing effect induced here.



- 3) When $f_s > 2W$ then it is over sampling and there will be no aliasing effect.



2. A TV signal of bandwidth 4.2MHz is transmitted using binary PCM. The number of quantization level is 512. Calculate a) Code word length b) Transmission BW c) Final bit rate. Assume $f_s = 2f_m$

Note:

1) Levels = 2^R

Word length = R

2) Transmission bandwidth of PCM $\geq R * W$, R bit per sample, W bandwidth of message signal.

3) Bit Rate = $R * 2W$ {Nyquist Rate = $2W$ }

Given: $W = 4.2\text{MHz}$, Levels = 512

i) $2^R = 512$, $R = 9$: Code length = 9bits

ii) Final Bit Rate: $R * 2W = 9 * 2 * 4.2\text{M bits/Sec} = 75.6\text{Mbps}$

iii) Min Transmission bandwidth of PCM = $R * W = 9 * (4.2\text{ M})\text{Hz} = 37.8\text{ MHz}$

3 a) For a sinusoidal modulating signal show the signal to quantization error ratio is $1.8+6R$. Where R is number of bits per sample.

$$\textcircled{a} \sigma_q^2 = \frac{M_{\max}^2 2^{-2R}}{3}$$

here $M_{\max} = A_m$

$$\sigma_q^2 = \frac{A_m^2 2^{-2R}}{3}$$

→ Note:
 If number of bits ↑
 by one, the SNR
 ↑ by 6dB

$$\textcircled{b} P = \frac{A_m^2}{2}$$

$$\begin{aligned} (\text{SNR})_0 &= \frac{P}{\sigma_q^2} = \frac{A_m^2}{2} \times \frac{3 \times 2^{2R}}{A_m^2} \\ &= 3 \times 2^{2R-1} = \underline{\underline{\frac{3}{2} 2^{2R}}} \end{aligned}$$

Indecibel

$$\begin{aligned} \log_{10}(\text{SNR})_0 &= 10 \left[\log_{10}(1.5) + 2R \log_{10}(2) \right] \\ &= 10 \left[0.176 + 2R \times 0.3 \right] = \underline{\underline{1.8 + 6R}} \end{aligned}$$

b) Draw block diagram of TDM and explain the principle.

* TIME DIVISION MULTIPLEXING : [TDM]

Time Division Multiplexing is a method of transmitting and receiving independent signals over a common channel by means of synchronised switches at each end of transmission line so that each signal appears on the line only a fraction of time in an alternating pattern.

* Fig(5) shows the block diagram of TDM system.

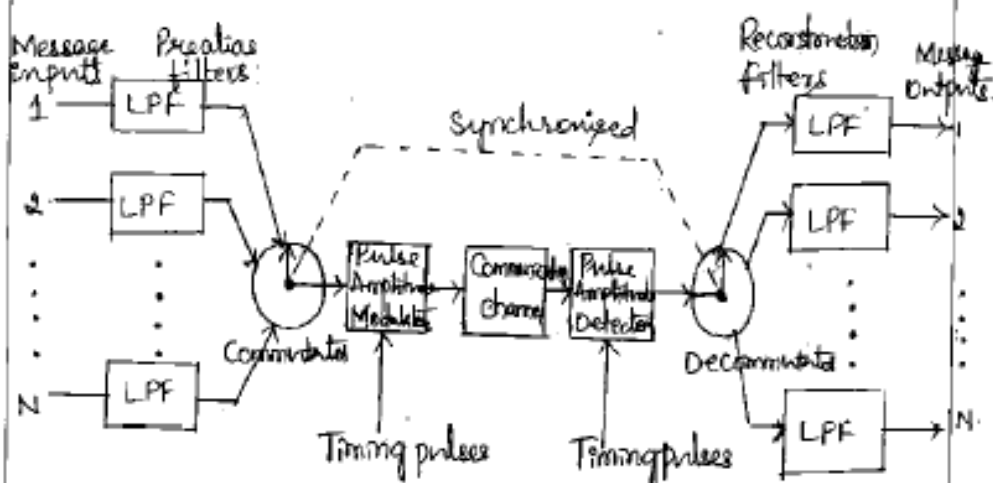


Fig 5 : Block Diagram of TDM system.

* The concept of TDM is illustrated in the fig(5). The Lowpass filters are used to remove high frequency components present in the message signal. The output of the pre-alias filters are then fed to a commutator, which is usually implemented using electronic switching circuitry.

* The function of commutator is as follows:

- To take a narrow sample of each of the 'N' samples of input at a rate of $f_s \geq N \cdot B$.
- To sequentially interleave (multiplex) these 'N' samples inside a sampling interval $T_s = 1/f_s$.
- * The multiplexed signal is then applied to a pulse amplitude modulator whose purpose is to transform the multiplexed signal into a form suitable for transmission over a common channel.
- * At the receiving end, the pulse amplitude demodulator performs the reverse operation of PAM and the demultiplexer distributes the signals to the appropriate low pass reconstruction filters. The demultiplexer operates in synchronisation with the commutator.

4. Define PAM. Obtain the expression for Fourier transform of PAM signal

* PULSE AMPLITUDE MODULATION :

* It is an analog pulse Modulation scheme in which the amplitudes of a train of rectangular carrier pulses are varied in accordance with the sample values of the modulating signal.

* In PAM, the top of each modulated rectangular pulse is maintained flat. So PAM is same as flat-top sampling.

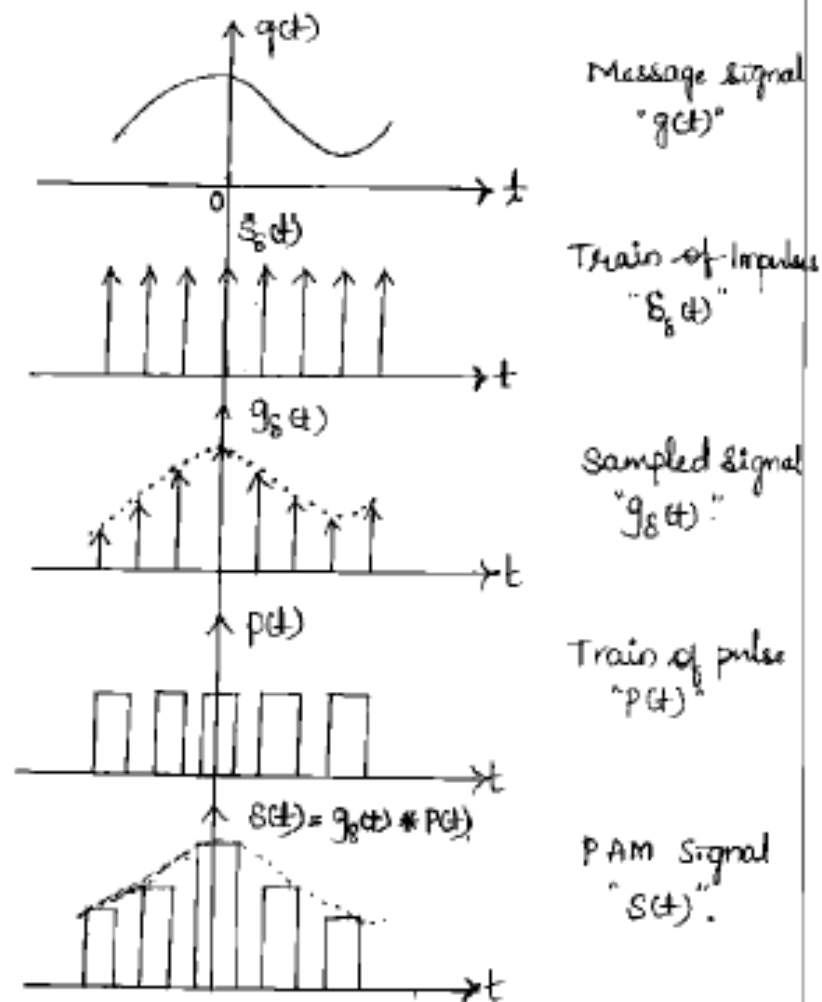


Fig 4: Pulse Amplitude Modulation

The waveform of a PAM signal is illustrated in fig(4).

* Let $s(t)$ denote the sequence of flat-top samples of PAM signal, and it is expressed as

$$s(t) = \sum_{n=-\infty}^{\infty} q(nT_s) p(t - nT_s) \quad \text{--- (1)}$$

where,

$q(nT_s)$ is the sample value of $q(t)$ obtained at time $t = nT_s$.

T_s is sampling period.

$p(t)$ is standard rectangular pulse train of duration T .

Advantages of PAM :

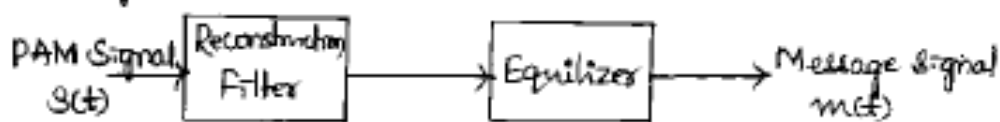
It is a base for all the digital modulation technique.

Disadvantages of PAM :

- 1) Due to Nyquist Criteria, it requires high bandwidth for transmission.
- 2) Since, amplitude keeps varying, so there is noise associated with it.

* Detection of PAM signal

The original message signal $m(t)$ is obtained by passing PAM signal to the reconstruction filter followed by equalizer.



Eq(4b) - Recovering $m(t)$ from PAM signal

$$m_\delta(t) * h(t) = \sum_{n=-\infty}^{\infty} m(nT_s) h(t - nT_s)$$

$$s(t) = m_\delta(t) * h(t)$$

$$S(f) = M_\delta(f) H(f)$$

5. Line codes

Binary: 01101001

1) Unipolar Nonreturn to zero (NRZ) Signalling
 In this line code symbol '1' is represented by transmitting a pulse of amplitude 'A' and symbol '0' is represented by switching off the pulse.

2) Polar Non-return to zero (NRZ) Signalling
 In this line code, symbol 1 and 0 are represented by transmitting pulses of amplitudes $+A$ and $-A$ respectively.

3) Unipolar Return to zero (RZ) Signalling
 Here, symbol 1 is represented by a rectangular pulse of amplitude A and half-symbol width and symbol 0 is represented by transmitting no pulse.

4) Bipolar Return to zero (BRZ) Signalling
 This line code uses three amplitude levels as shown in fig. Specifically, +ve & -ve pulses of equal amplitude are used alternatively for symbol '1', '0' for no pulse.

5) Split-phase (Manchester code)
 Symbol 1 is represented a positive pulse of

(a) → Unipolar NRZ
 (b) → Polar NRZ
 (c) → Unipolar RZ
 (d) → Bipolar RZ
 (e) → Manchester code

amplitude 'A' followed by a negative pulse of amplitude -A with both pulses being a half-symbol wide. For symbol '0', the polarities of these two pulses are reversed.

* Differential Encoding :

this method is used to encode information in terms of signal transitions. In particular, a transition is used to designate symbol 0 in the incoming binary data stream, while no transition is used to designate symbol 1 as shown in fig.

(a) Original binary data 0 1 1 0 1 0 0 1

(b) Differentially encoded data "1" 0 0 0 1 1 0 1 1

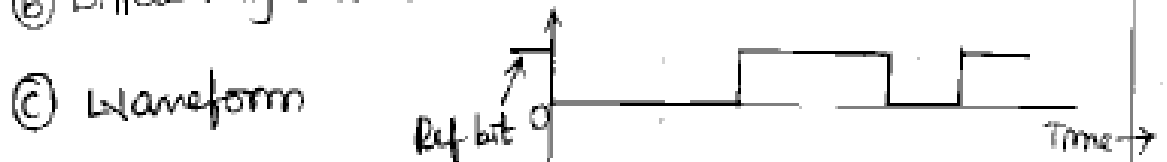


Fig : Differential encoding.

6. Find the Nyquist rate and Nyquist interval for

a) $m_1(t) = \frac{1}{2\pi} \cos(4000\pi t) \cos(1000\pi t)$

Rate: $f_{s1} + f_{s2} = 2500\text{Hz}$

$$\text{b) } m_2(t) = \frac{\sin 500\pi t}{\pi t}$$

b)

$$\frac{1}{\pi} \text{sinc}(\omega t) \iff \text{rect}\left(\frac{\omega}{2W}\right)$$

Rate=250Hz