

INTERNAL ASSESSMENT TEST - II
DESIGN OF MACHINE ELEMENTS - I

1. A shaft of circular section is subjected to a turning moment that fluctuates between 800 kNm and 600 kNm and also bending moment that fluctuates between +500 kNm and -300 kNm. The material selected for the shaft has a stress value of 100 MPa at endurance limit and shear stress value of 120 MPa at the yield point. Determine the diameter of the solid circular shaft taking a value of 2.5 for the factor of safety. Surface factor, size factor and load factor can be taken as 0.9, 0.85 and 1.0 respectively. Shear stress concentration factor is 1.8 and the notch sensitivity is 0.95. (Note: Choose standard diameter in mm)

2. A cold drawn steel rod of circular cross section is subjected to a variable bending moment of 565 Nm to 1130 Nm as the axial load varies from 4500N to 13500N. The maximum bending moment occurs at the same instant that the axial load is maximum. Determine the required diameter of the rod (in mm) for a factor of safety =2. Neglect stress concentration effect. Take ultimate stress as 550 MPa., Yield stress as 470MPa and endurance limit as 50% of the ultimate strength and size, load and surface correction coefficients as 0.85, 1 and 0.85 respectively.

3. A weight of 1 kN is dropped from a height of 50 mm at the free end of a cantilever beam of effective length 300 mm. Determine the cross section of the cantilever beam of square cross section (in mm) if the allowable stress in the material of the beam is limited to 80 MPa.

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18ME52 - SOLUTIONS KEY**

10. Combined stress by / &

Given :- $M_t \text{ max} = 800 \text{ kNm} = 8 \times 10^8 \text{ N-mm}$

$M_t \text{ min} = 600 \text{ kNm} = 6 \times 10^8 \text{ N-mm}$

$M_b \text{ max} = 500 \text{ kNm} = 5 \times 10^8 \text{ N-mm}$

$M_b \text{ min} = -300 \text{ kNm} = -3 \times 10^8 \text{ N-mm}$

$\tau_{-1} = 100 \text{ MPa}$; $\tau_y = 120 \text{ MPa}$; $FOS = n = 2.5$

$e_{sz} = 0.9$; $e_{sz} = 0.85$; $e_t = 1$; $k_2 = 1.8$;
 $q = 0.95$

Combined stress by Soderberg reaction

* Considers bending load :-

Bending moment amplitude $M_{ba} = \frac{M_{bmax} - M_{bmin}}{2}$

$$= \frac{5 \times 10^8 - (-3 \times 10^8)}{2}$$

$$= 4 \times 10^8 \text{ N-mm}$$

mean bending moment

$M_{bm} = \frac{M_{bmax} + M_{bmin}}{2}$

$$= \frac{5 \times 10^8 - 3 \times 10^8}{2}$$

$$= 1 \times 10^8 \text{ N-mm}$$

Bending stress amplitude $\sigma_{ba} = \frac{m_{ba}}{Z_b} = \frac{4 \times 10^8}{\frac{\pi}{32} d^3}$

$$= \frac{40.744 \times 10^8}{d^3}$$

mean bending stress $\sigma_{bm} = \frac{m_{bm}}{Z_b} = \frac{1 \times 10^8}{\frac{\pi}{32} d^3}$

$$= \frac{10.186 \times 10^8}{d^3}$$

* consider torsional load:

Torque amplitude = $\frac{m_{t_{max}} - m_{t_{min}}}{2}$

i.e. $m_{ta} = \frac{8 \times 10^8 - 6 \times 10^8}{2} = 1 \times 10^8 \text{ N-mm}$

mean torque = $\frac{m_{t_{max}} + m_{t_{min}}}{2}$

i.e. $m_{tm} = \frac{8 \times 10^8 + 6 \times 10^8}{2}$

$$= 7 \times 10^8 \text{ N-mm}$$

stress amplitude $\tau_a = \frac{m_{ta}}{Z_p} = \frac{1 \times 10^8}{\frac{\pi}{16} d^3}$

$$= \frac{5.093 \times 10^8}{d^3}$$

$$\text{mean stress } \tau_m = \frac{m b_m}{z_p} = \frac{7 \times 10^8}{\frac{\pi}{16} d^3} = \frac{35.65 \times 10^8}{d^2}$$

According to maximum shear stress theory, -

$$\begin{aligned} \text{combined mean shear stress } \tau_m &= \frac{1}{2} \sqrt{\sigma_{bm}^2 + 4\tau_m^2} \\ &= \frac{1}{2} \sqrt{\left(\frac{10.186 \times 10^8}{d^2}\right)^2 + 4\left(\frac{35.65 \times 10^8}{d^2}\right)^2} \\ &= \frac{3.6 \times 10^9}{d^2} \end{aligned}$$

$$\begin{aligned} \text{combined shear stress amplitude } \tau_m &= \frac{1}{2} \sqrt{\sigma_{bm}^2 + 4\tau_m^2} \\ &= \frac{1}{2} \sqrt{\left(\frac{40.743 \times 10^8}{d^2}\right)^2 + 4\left(\frac{35.65 \times 10^8}{d^2}\right)^2} \\ &= \frac{2.4 \times 10^9}{d^2} \end{aligned}$$

Now, according to Soderberg's straight line relation, according to stress

concentration

$$\frac{k_{-1} \tau_a''}{\tau_{-1} d} + \frac{\tau_m''}{\tau_{yd}} = 1$$

i.e., $k_{-1} \tau_a''$

$$\frac{k_{-1} \tau_a''}{\sigma_{-1} e_d \cdot e_{sz} \cdot e_{sz}} + \frac{\tau_m''}{\tau_{cy}} = \frac{1}{n}$$

or

$$\frac{k_{-2} \tau_a''}{\sigma_{-1} ABC} + \frac{\tau_m''}{\tau_{cy}} = \frac{1}{n}$$

Fatigue stress concentration factor

$$k_{-2} = 1 + q (k_{\tau} - 1)$$

$$= 1 + 0.95 (1.8 - 1) = 1.76$$

$$\therefore \frac{1.76 \times 1 \times 10^9}{d^3 \times 100 \times 0.85 \times 90} + \frac{3.6 \times 10^9}{120} = \frac{1}{25}$$

$$(\because \tau_1 = \sigma_{-1} e_d)$$

$$\therefore \text{diameter of shaft } d = 580.665 \text{ mm}$$

$$\boxed{d = 600 \text{ mm}} //$$

Given

Combined stress by Soderberg / Goodman relation.

(i) consider the Bending load:-

$$\text{Amplitude bending moment } M_{ba} = \frac{(1130 - 565) \times 10^3}{2}$$
$$= 282.5 \times 10^3 \text{ Nmm}$$

$$\text{mean bending moment } M_{bm} = \frac{(1130 + 565) \times 10^3}{2}$$
$$= 847.5 \times 10^3 \text{ Nmm}$$

$$\text{Amplitude Bending stress } \sigma_{ba} = \frac{32M_{ba}}{\pi d^3} = 287.75 \times 10^4 \frac{\text{N}}{\text{mm}^2}$$

$$\text{mean Bending stress } \sigma_{bm} = \frac{32M_{bm}}{\pi d^3} = 863.25 \times 10^4 \frac{\text{N}}{\text{mm}^2}$$

(ii) consider the Axial load:-

~~mean~~
Amplitude load, $F_a = 4500 \text{ N}$

$$\text{mean load, } F_m = \frac{13500 + 4500}{2} = 9000 \text{ N}$$

$$\text{Amplitude stress, } \sigma_a = \frac{4F_a}{\pi d^2} = 5.73 \times 10^3 \frac{\text{N}}{\text{mm}^2}$$

$$\text{mean stress, } \sigma_m = \frac{4 \times 9000}{\pi d^2} = 11.45 \times 10^3 \frac{\text{N}}{\text{mm}^2}$$

combined mean stress, $\sigma_m'' = \sigma_{bm} + \sigma_m$

$$= 863.25 \times \frac{10^4}{d^3} + 11.45 \times \frac{10^3}{d^2}$$

Combined Amplitude stress, $\sigma_a'' = \sigma_{ba} + \sigma_a$

$$= 287.75 \times \frac{10^4}{d^3} + 5.73 \times \frac{10^3}{d^2}$$

According to Soderberg straight line equation.
(Eq 5.88g - DPB)

$$\frac{\sigma_a''}{\sigma_{-1d}} + \frac{\sigma_m''}{\sigma_y} = 1$$

$$\sigma_{-1d} = \frac{\sigma_{-1}}{n_{-1d}}$$

$$\sigma_{-1} = 0.5 \times 550 = 275 \text{ MPa}$$

$$n_{-1d} = \frac{1}{e_1} \cdot \frac{1}{e_{s2}} \cdot \frac{1}{e_{s3}} \cdot n$$

$$\sigma_{-1d} = 99.63 \text{ N/mm}^2$$

$$\sigma_{yd} = \frac{\sigma_y}{n} = \frac{470}{2} = 235 \text{ N/mm}^2$$

Sub the above values in equation 5.88g

$$d = 41.23 \approx 45 \text{ mm} \quad (\text{Standard diameter}).$$

$$W = 1000 \text{ N} = 1000; h = 50 \text{ mm}; l = 300 \text{ mm};$$

$$\sigma_b = 50 \text{ MPa}; b = d \text{ (square)}$$

$$\text{Assume } E = 206.8 \times 10^5 \text{ N/mm}^2$$

$$\text{Maximum B.M} = M_b = Wl = 1000 \times 300 = 3 \times 10^5 \text{ Nmm}$$

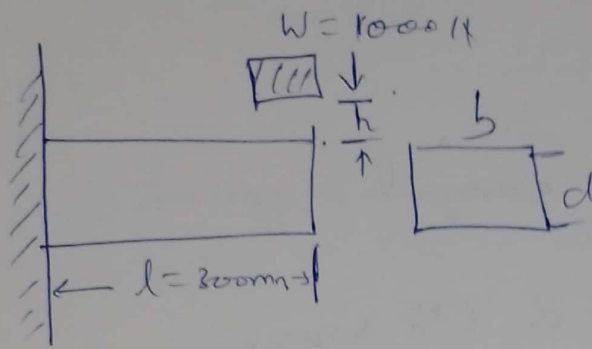
$$\text{From table, maximum static deflection } y = \frac{Wl^2}{3EI}$$

$$\text{From table 1.3.a, } I = \frac{bd^3}{12} = \frac{b^4}{12}$$

$$\begin{aligned} \Delta_m \text{ (or) } y &= \frac{1000 \times 300^3}{3 \times 206.8 \times 10^3 \times \left(\frac{b^4}{12}\right)} \\ &= \frac{522.244 \times 10^3}{b^4} \end{aligned}$$

Bending stress due to static load

$$\begin{aligned} \sigma_n &= \frac{M_b}{I} \times c, \text{ where } c = \frac{d}{2} = \frac{b}{2} \\ &= \frac{3 \times 10^5}{\left(\frac{b^4}{12}\right)} \cdot \frac{b}{2} = \frac{18 \times 10^5}{b^3} \end{aligned}$$



Impact stress due to bending $\sigma_b = \sigma_b \left[1 + \sqrt{1 + \frac{2ah}{y}} \right]$

$$\text{i.e. } , 80 = \frac{18 \times 10^5}{b^3} \left[1 + \sqrt{1 + \frac{2 \times 50}{\left(\frac{522.244 \times 10^3}{b^4} \right)}} \right]$$

$$\text{i.e. } 4.444 \times 10^{-5} b^3 = 1 + \sqrt{1 + 1.9148 \times 10^{-4} b^4}$$

$$\text{i.e. } (4.444 \times 10^{-5} b^3 - 1) = \sqrt{1 + 1.9148 \times 10^{-4} b^4}$$

Squaring on both sides

$$1.9753 \times 10^{-5} b^3 - 8.8889 \times 10^{-5} b^3 + 1 = 1 + 1.9148 \times 10^{-4} b^4$$

$$\text{i.e. } b^3 - 45000 = 96936.756$$

$$\text{i.e. } b^3 - 96936.756 - 45000 = 0$$

By trial and error method.

$$\text{Say } b = 312 \text{ mm}$$

width of beam = depth of beam $b = 311.58 \text{ mm}$