IAT_3 - Dynamics of Machines

5 questions - 8 marks each
1 question - 10 marks

Questions

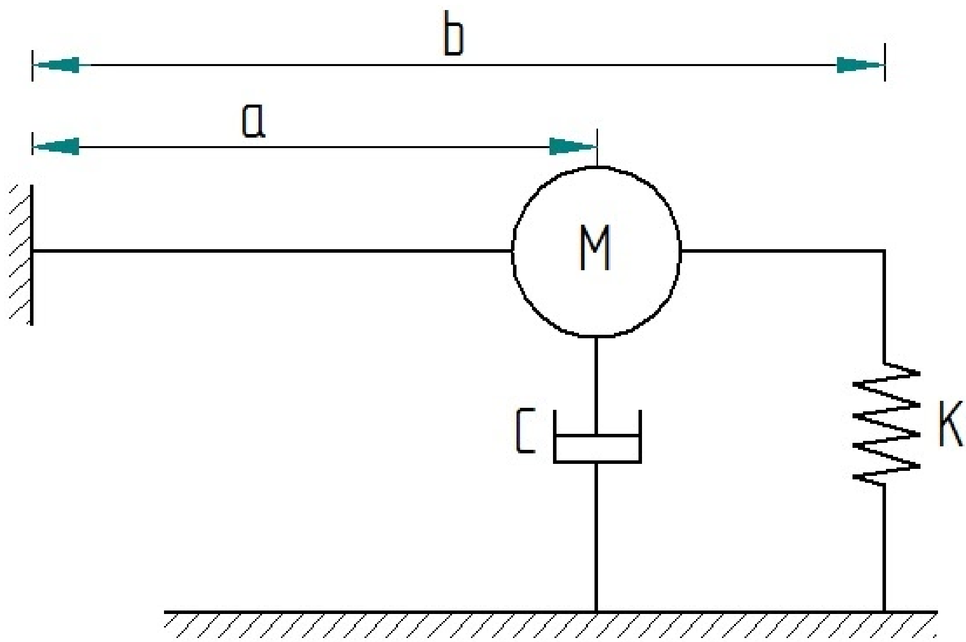
The barrel of a large gun weighing 500 kg recoils 1.2 m on the firing. The gun fires shots weighing 10 kg at 2 km/s. At the end of the recoil dampers are engaged to bring the barrel to its original position within minimum possible time. Determine the spring and damping coefficient of the dashpot? *

- ○ K = 5555.56 N/m, Cc = 3002.1 N-s/m
- ○ K= 4999.34 N/m, Cc = 3333.33 N-s/m
- ◉ K = 5555.56 N/m, Cc = 3333.33 N-s/m
- ○ K= 4999.34 N/m, Cc = 3002.1 N-s/m

A machine of mass 100 kg is mounted on springs and is fitted with a dashpot to damp out vibrations. There are 4 springs each of stiffness 7.5 N/mm and it is found that the amplitude of vibration diminishes from 3.84 cm to 0.64 cm in 2 complete oscillations. Assuming that the damping force varies as the velocity, determine the resistance of dashpot at unit velocity. *

- ◉ 489 N-sec/m
- ○ 475 N-sec/m
- ○ 499 N-sec/m
- ○ 470 N-sec/m

Determine the natural frequency of the damped oscillation *



$$f_d = \frac{1}{2\pi}\sqrt{\frac{K}{M}\left(\frac{b}{a}\right)^2 - \left(\frac{C}{2M}\right)^2}$$

Option 1

$$f_d = \frac{1}{2\pi}\sqrt{\frac{K}{M}\left(\frac{b}{2a}\right)^2 - \left(\frac{C}{M}\right)^2}$$

Option 2

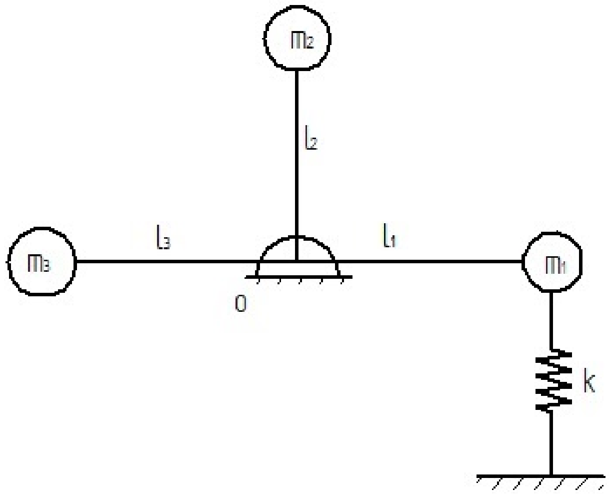
$$f_d = \frac{1}{2\pi}\sqrt{\frac{K}{M}\left(\frac{2b}{a}\right)^2 - \left(\frac{C}{M}\right)^2}$$

Option 3

$$f_d = \frac{1}{2\pi}\sqrt{\frac{K}{M}\left(\frac{b}{a}\right)^2 - \left(\frac{2C}{M}\right)^2}$$

Option 4

Determine the natural frequency of the system shown in figure *



$$f_d = \frac{1}{2\pi}\sqrt{\frac{k\, l_1^2 - m_2\, g\, l_2}{(m_3\, l_3^2 + m_2\, l_2^2 + m_1\, l_1^2)}}\ \text{Hz}$$

Option 1

$$f_d = \frac{1}{2\pi}\sqrt{\frac{k l_2^2 - m_2 g l_1}{(m_3 l_3^2 + m_2 l_2^2 + m_1 l_1^2)}}\ \text{Hz}$$

Option 2

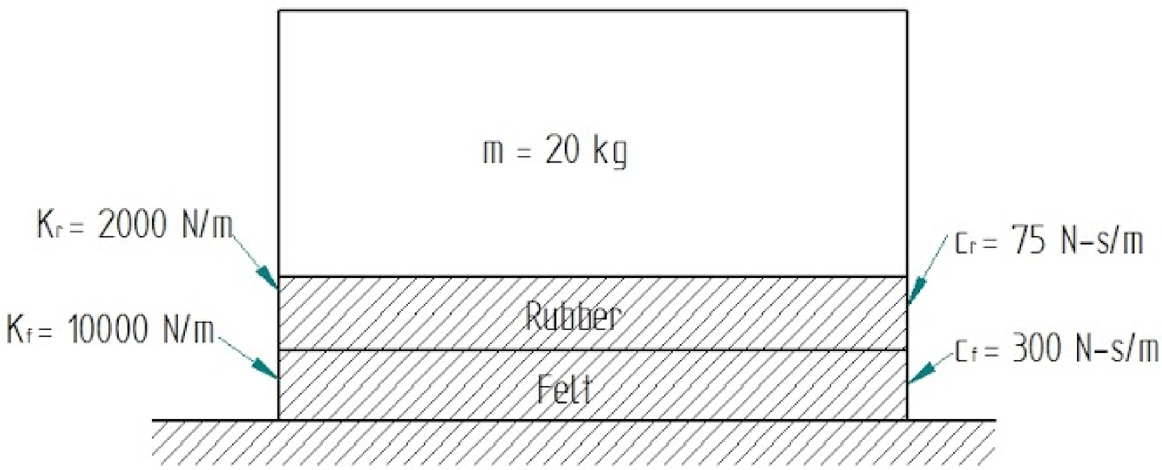
$$f_d = \frac{1}{2\pi}\sqrt{\frac{k l_1^2 - m_2 g l_2}{(m_2 l_3^2 + m_2 l_2^2 + m_1 l_1^2)}}\ \text{Hz}$$

Option 3

$$f_d = \frac{1}{2\pi}\sqrt{\frac{k l_1^2 - m_2 g l_2}{(m_3 l_3^2 + m_1 l_2^2 + m_1 l_1^2)}}\ \text{Hz}$$

Option 4

Determine the undamped and damped natural frequencies of the system shown in vertical direction. Mass of isolators are neglected. (10 Marks) *



- (selected) fn = 1.453 Hz , fd = 1.433 Hz
- fn = 1.783 Hz, fd = 1.433 Hz
- fn = 1.453 Hz, fd = 1.783 Hz
- fn = 1.783 Hz , fd = 1.783 Hz

A body of mass of 10 kg is suspended from a helical spring having a stiffness of 2 N/mm. A damper having a resistance of 5 N at a velocity of 0.1 m/sec is connected between the mass and the fixed end of the spring. Determine (i) ratio of successive amplitude. (ii) Amplitude of body after 10 cycles if the initial amplitude is 15 mm. *

- (selected) Ratio of successive amplitude = 3.09, Amplitude after 10 cycles = 0.0001812
- Ratio of successive amplitude = 0.000182, Amplitude after 10 cycles = 3.09
- Ratio of successive amplitude = 2.01, Amplitude after 10 cycles = 0.0001812
- Ratio of successive amplitude = 2.02, Amplitude after 10 cycles = 0.01812

CMR

Solutions for Numericals

Q2.

Given. $k = 2\ N/mm = 2000\ N/m$

$F = 5\ N$; $\dot{x} = 0.1\ m/sec$

$m = 10\ kg$; $n = 10$; $x_0 = 15\ mm$

Damping co-efficient $F = c\dot{x}$

$5 = c \times 0.1$

$c = 50\ N\text{-}sec/m$

Critical damping coefficient

$C_c = 2m\omega_n = 2\sqrt{km}$

$= 2\sqrt{2000 \times 10}$

$C_c = 282.84\ N\text{-}sec/m$

Damping factor $\xi = \frac{c}{C_c} \Rightarrow \frac{50}{282.84}$

$\xi = 0.1768$

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$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \Rightarrow \frac{2\pi(0.1788)}{\sqrt{1-0.178^2}}$$

$\delta = 2\pi($

$$\delta = 1.129$$

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) \Rightarrow 1.129 = \ln\left(\frac{x_n}{x_{n+1}}\right)$$

$$\Rightarrow \frac{x_n}{x_{n+1}} = e^{1.129}$$

$$\Rightarrow \frac{x_n}{x_{n+1}} = 3.1$$

ii) Amplitude after the cycle

$$\frac{x_0}{x_{10}} = \left(\frac{x_0}{x_1}\right)^{10} \Rightarrow (3.1)^{10}$$

$$\frac{x_0}{x_{10}} = 79735.4$$

$$x_{10} = \frac{x_0}{79735.4} \Rightarrow \frac{15}{79735.4}$$

$$x_{10} = 1.88\times10^{-4}\ mm$$

Amplitude after | $x_{10} = 1.88\times10^{-4}$ mm

Q
Q
6.

Sol^n

m = 100 kg ; No of springs = 4

x_0 = 3.84 cm ; x_2 = 0.64 cm

k = 7.5 N/mm

k = 7500 N/m

combined stiffness = 4 × 7500 = 30,000 N/m

$$\frac{x_0}{x_2} = \frac{3.84}{0.64} = 6$$

$$\frac{x_0}{x_2} = \left(\frac{x_0}{x_1}\right)^2 \quad \Big| \quad 6 = \left(\frac{x_0}{x_1}\right)^2$$

$$\frac{x_0}{x_1} = 6^{1/2}$$

$$\boxed{\frac{x_0}{x_1} = 2.45}$$

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$$\delta = \ln\frac{x_0}{x_1} = \ln(2.45) = 0.896$$

$$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

$$\boxed{\xi = 0.1412}$$

Damping resistay

$$c = 2m\omega_n\xi$$

$= 2 \times 100 \times \sqrt{\frac{k}{m}} \times 0.1412$ | ~~2~~ $\times 10$

$= 2 \times 100 \times \sqrt{\frac{~~7~~ 500 \times 4}{100}} \times 0.1412$ $\Rightarrow$ 489 N-sec/m

16) Damped freq of free oscillation

Natural freq of free oscillation

$$\frac{f_d}{f_n} = \frac{\frac{1}{2\pi}\omega_d}{\frac{1}{2\pi}\omega_n} = \frac{\omega_d}{\omega_n}$$

$$= \frac{\omega_n\left(\sqrt{1-\xi^2}\right)}{\omega_n}$$

$$= \sqrt{1-0.1412^2}$$

$$\frac{f_d}{f_n} = 0.99$$

SD

Q.
4.

spring force = $kb\theta$
Damping force = $ca\theta$
equation of motion =

$$I\theta'' + kb\theta . b + ca\theta' a = 0$$

$$ma^2\theta'' + kb^2\theta + ca^2\theta = 0$$

$$\theta'' + \frac{ca^2}{ma^2}\theta'' + \frac{kb^2}{ma^2}\theta = 0$$

$$\theta'' + \frac{c}{m}\theta' + \frac{kb^2}{mc^2}\theta = 0$$

Rate of equation are

$$\alpha_{1,2} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\left(\frac{kb^2}{ma^2}\right)}}{}$$

$$= \frac{-c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{kb^2}{ma^2}}$$

$$\boxed{\alpha_{1,2} = \frac{-c}{2m} \pm \sqrt{\frac{kb^2}{ma^2} - \left(\frac{c}{2m}\right)^2}}$$

$$\boxed{c = \frac{1}{2\zeta}}$$

$$F_d = \frac{1}{2\pi}\sqrt{\frac{k}{m}\left(\frac{b}{a}\right)^2 - \left(\frac{c}{2m}\right)^2}$$

Q. **Solution**

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{1\times10^4} + \frac{1}{2\times10^3} = \frac{3}{5000}$$

$$\frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} \Rightarrow \frac{1}{300} + \frac{1}{75} = \frac{1}{60}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{5000/3}{20}}$$

$$\boxed{\omega_n = 9.128709}$$

$$f_n = \frac{9.128707}{2\pi}$$

$$\boxed{f_n = 1.452879}$$

$$\omega_d = \sqrt{1-\xi^2}\,\omega_n$$

$$\xi = \frac{c_{eq}}{2\sqrt{k_{eq} m}} = \frac{60}{2\times\sqrt{5000/3\times20}}$$

$$\xi = 0.164316$$

$$\omega_d = \sqrt{1-0.164316^2} \times 9.128704$$

$$f_d = \frac{9.128709}{2\times\pi}$$

$$\boxed{f_d = 1.4333137}$$