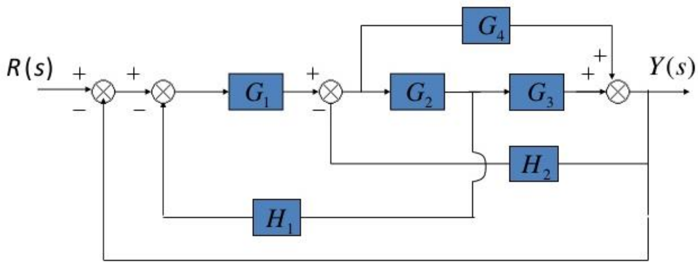
CONTROL ENGINEERING - 17ME73/15ME73

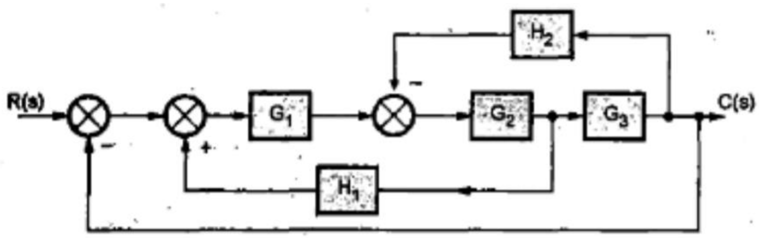
IAT-3

50 MARKS
1.5 HRS
EACH QUESTION CARRIES 10 MARKS

1. Reduce the block diagram and obtain the transfer function Y(s)/R(s).



2. Use Mason's Gain Formula and obtain the transfer function C(s)/R(s).



3. A unity feedback system has G(s) = $\frac{30(s+3)(s-4)}{s(s^2-16)(s+1)}$

   Determine:

   a) Type of system
   b) All error coefficients
   c) Error for ramp input of magnitude 4

4. Obtain unit step response of a first order system and explain its features. Explain the relationship between closed loop poles and transient response.

5. A system has the following transfer function:

$$\frac{C(s)}{R(s)} = \frac{10}{s-20}$$

   Determine its unit step and unit ramp response for zero initial conditions. Sketch the responses.

Control Engineering

IAT-3

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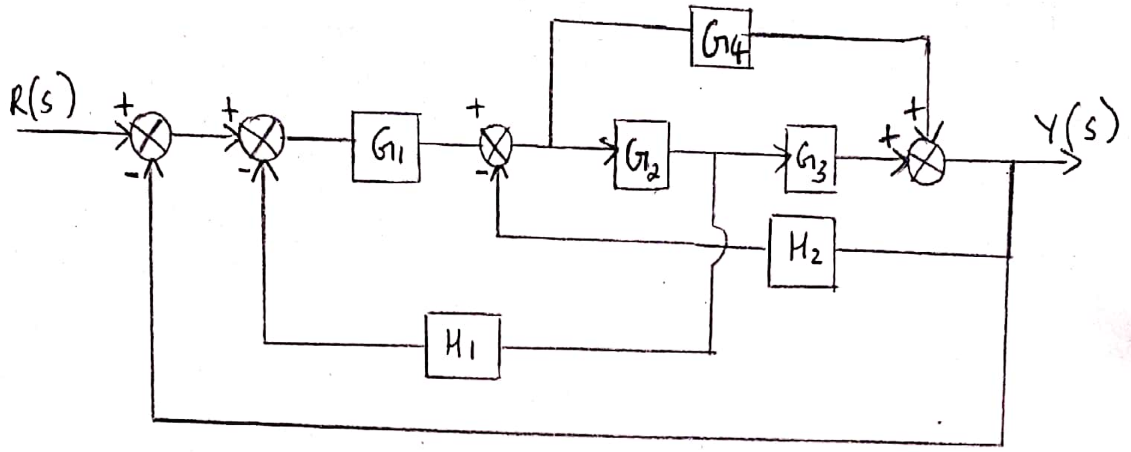
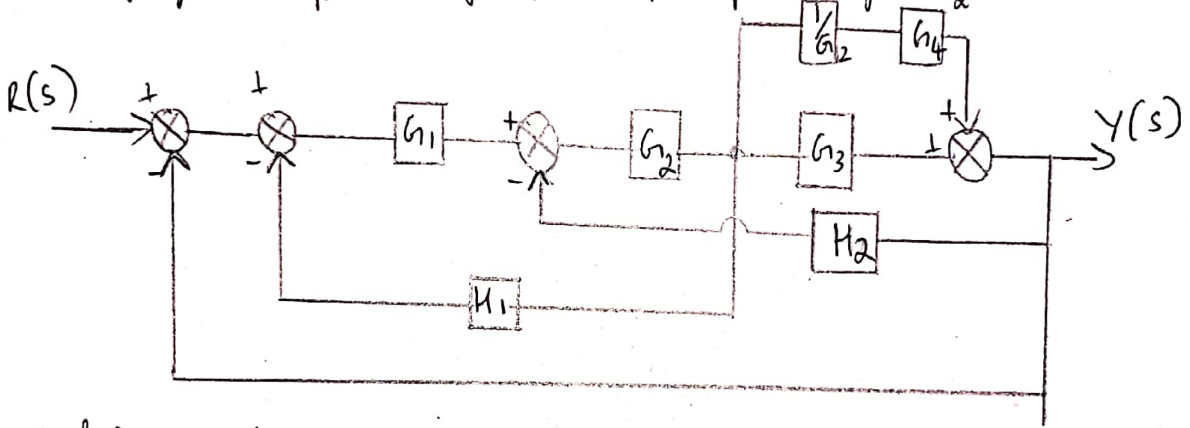
1CR17ME077

1.



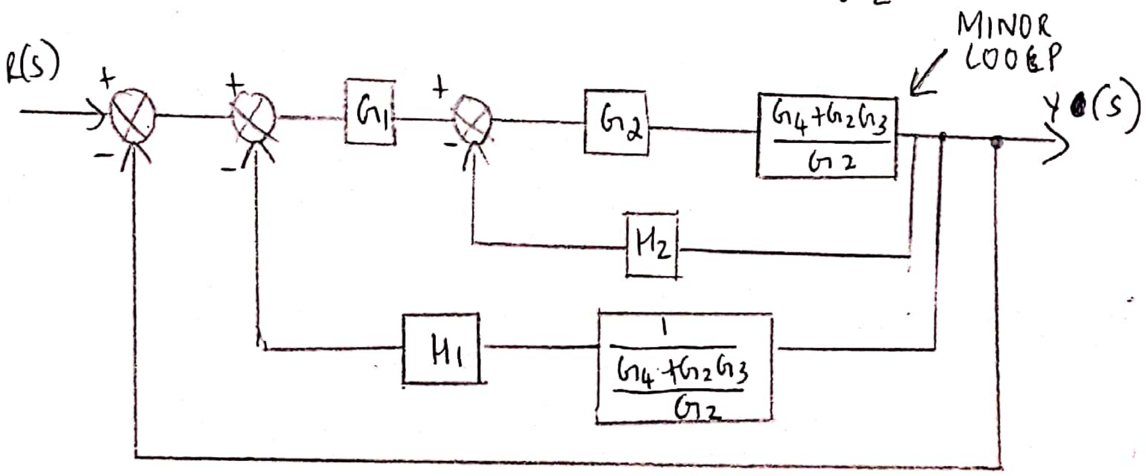
Step 1 : Shifting take off point of G_4 ~~and~~ by putting it after G_2.



$\frac{1}{G_2}$ and G_4 are in series which are together parallel with G_3

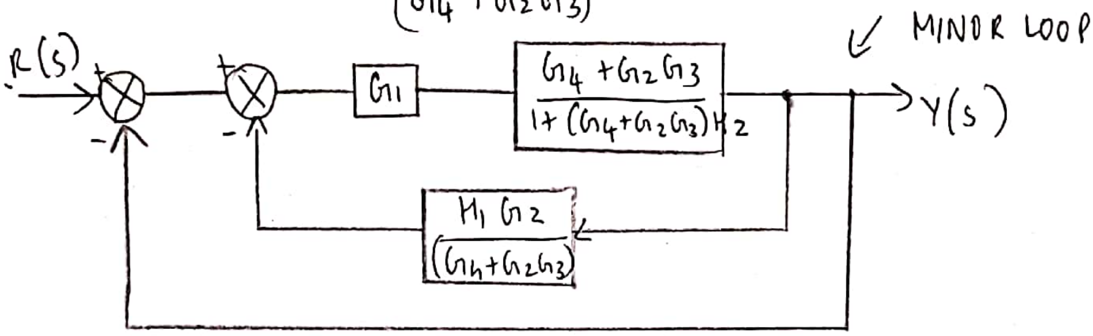
$$\therefore \quad \frac{G_4}{G_2} + G_3 = \frac{G_4 + G_2 G_3}{G_2}$$

Step 2 : Shifting take off point after G_2 ~~to~~ beyond $\frac{G_4 + G_2 G_3}{G_2}$



Solving the Minor Loop: $\cancel{\dfrac{G_4+G_2G_3}{1+(G_4+G_2G_3)H_2}}$ $\qquad \dfrac{G_4+G_2G_3}{1+(G_4+G_2G_3)H_2}$

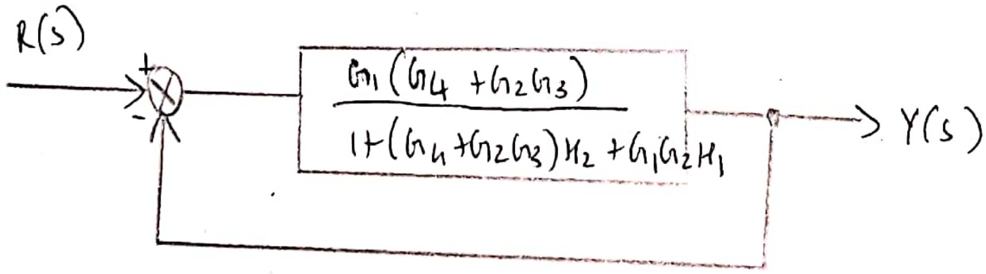
Solving the blocks in series: $\dfrac{H_1G_2}{(G_4+G_2G_3)}$



Solving the minor loop

$$\frac{\dfrac{G_1(G_4+G_2G_3)}{1+(G_2G_3+G_4)H_2}}{\dfrac{G_1(G_4+G_2G_3)H_1G_2}{(1+(G_4+G_2G_3)H_2)(G_4+G_2G_3)}} =$$

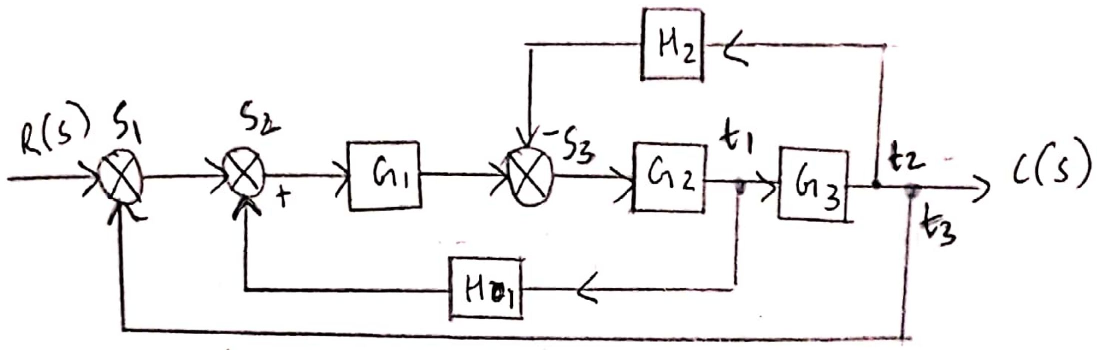
$$= \frac{G_1(G_4+G_2G_3)}{1+(G_4+G_2G_3)H_2+G_1G_2H_1}$$



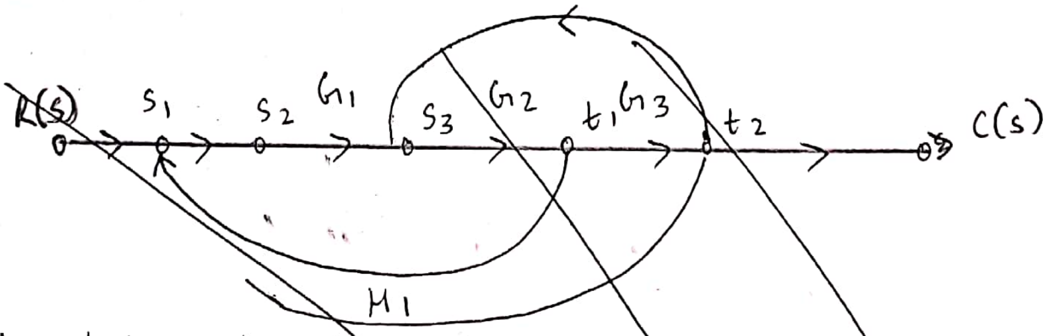
$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1(G_4+G_2G_3)}{1+(G_4+G_2G_3)H_2+G_1G_2H_1}}{1+\dfrac{G_1(G_4+G_2G_3)}{1+(G_4+G_2G_3)H_2+G_1G_2H_1}}$$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3+G_1G_4}{1+G_2G_3H_2+G_4H_2+G_1G_2H_1+G_1G_2G_3+G_1G_4}}$$

2.

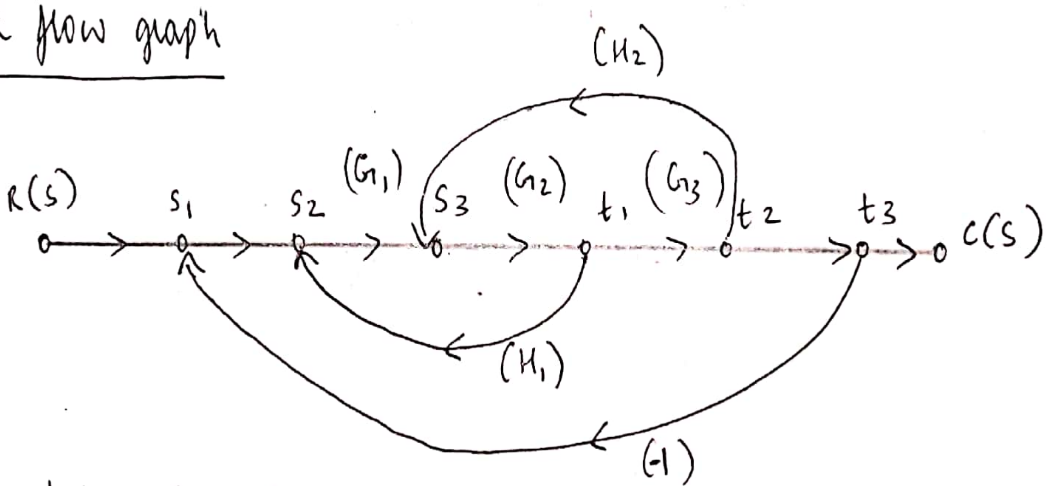


Signal flow graph



~~No. of forward paths = 1~~

Signal flow graph

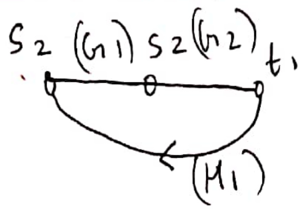


No. of forward paths = 1

$T_1 = 1 \times 1 \times G_1 \times G_2 \times G_3 \times 1 \times 1 \times 1 = G_1 G_2 G_3$

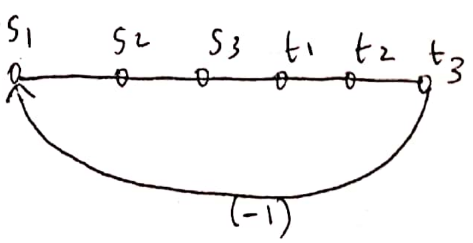
Feedback loops

1



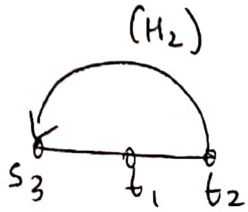
$L_1 = + G_1 G_2 H_1$

2



$L_2 = -G_1 G_2 G_3$

3



$L_3 = - G_2 G_3 H_2$

Two non touching loops : 0

$$\Delta = 1 - [L_1 + L_2 + L_3] + [0]$$

$$\Delta = 1 - [G_1 G_2 H_1 - G_1 G_2 G_3 - G_2 G_3 H_2]$$

by Mason gains formula :

$$T.F = \frac{\Sigma T_K \Delta_K}{\Delta}$$

for T_1 : [all the loops are touching forward path 1] $\therefore \Delta_1 = 1$

$$\frac{C(s)}{R(s)} = \frac{T_1 \Delta_1}{\Delta} \quad \frac{G_1 G_2 G_3 \times 1}{1 - [G_1 G_2 H_1 - G_1 G_2 G_3 - G_2 G_3 H_2]}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1 + G_1 G_2 G_3 + G_2 H_2 G_3}}$$

3) unity feedback system has $G(s) = \dfrac{30(s+3)(s-4)}{s(s^2-16)(s+1)}$

$$G(s)H(s) = \frac{30(s+3)(s-4)}{s(s^2-16)(s+1)} = \frac{30(s+3)(s-4)}{s(s^2-4^2)(s+1)}$$

$$= \frac{30(s+3)\cancel{(s-4)}}{s(s+4)\cancel{(s-4)}(s+1)} = \frac{30(s+3)}{s(s+4)(s+1)}$$

$$\text{now, } G(s)H(s) = \frac{\overset{15}{\cancel{30}} \times 3(1+0.33s)}{s \times \underset{2}{\cancel{4}}(1+0.25s)(1+s)}$$

$$G(s)H(s) = \frac{22.5(1+0.33s)}{s(1+0.25s)(1+s)}$$

Comparing it with the standard form : $G(s)H(s) = \dfrac{K(1+T_1 s)(1+T_2 s)\ldots}{s^j (1+T_a s)(1+T_b s)\ldots}$

$j = 1$

a) $\therefore$ Type of the system = Type I

b) <u>Error Coefficients</u>

1) $K_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \dfrac{22.5(1+0.3s)}{s(1+0.25s)(1+s)} = \infty$

2) $K_v = \lim_{s \to 0} s.G(s)H(s) = \lim_{s \to 0} \dfrac{22.5(1+0.3s)}{(1+0.25s)(1+s)} = 22.5$

3) $K_a = \lim_{s \to 0} s^2(G(s)H(s)) = \lim_{s \to 0} \dfrac{s.22.5(1+0.3s)}{(1+0.25s)(1+s)} = 0$

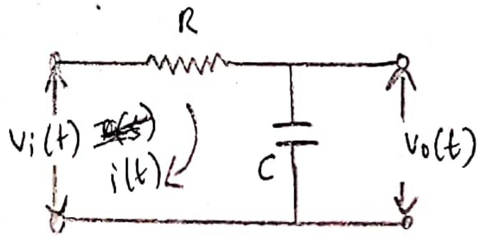
c) <u>Error for ramp input of magnitude 4</u>

Steady state error for ramp input, $e_{ss} = \dfrac{A}{K_v}$

$A = 4$; $K_v = 22.5$

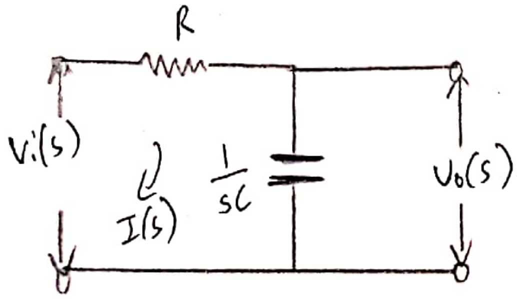
$\therefore e_{ss} = \dfrac{4}{22.5} = \underline{0.17}$

4. <u>Unit step Response of a first Order System</u>



$V_i(s) = I(s)R + \dfrac{1}{sC} I(s)$

and $V_o(s) = \dfrac{1}{sC}(I(s))$



$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC} I(s)}{I(s)R + \frac{1}{sC} I(s)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{1+sRC} = \frac{1}{1+Ts}$$

$[\because T = RC]$

$$\frac{V_o(s)}{V_i(s)} = \frac{C(s)}{R(s)} = \frac{1}{1+sRC}$$

$$C(s) = R(s) \cdot \frac{1}{1+sRC}$$

For unit step response, $r(t) = 1$ for $t \geq 0$

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{s(1+sRC)}$$

Taking partial fractions : $C(s) = \frac{A}{s} + \frac{B}{1+sRC}$

$$A(1+sRC) + Bs = 1$$

Put $s = 0 \Rightarrow A = 1$

Put $s = \frac{-1}{RC}$ $\quad \therefore B = -RC$

Now substituting values of A and B

$$C(s) = \frac{1}{s} - \frac{RC}{1+sRC}$$

Can be written as : $C(s) = \frac{1}{s} - \frac{1}{\frac{1}{RC}+s}$

Taking Inverse Laplace : $\boxed{c(t) = 1 - e^{-t/RC}}$

~~With~~ As $c(t) = C_{ss} + C_t(t)$

$$C_{ss} = 1$$

$$C_t(t) = e^{-\frac{t}{RC}}$$

$\therefore$ The transient term is dependent on R and C and its rate of exponential decay is controlled by $\left(-\frac{1}{RC}\right)$ (which is also the pole of the system.

Relationship between closed loop poles and transient response

In the transient response, output $c(t) = 1 - e^{-t/RC}$, the exponential index is $\left(-\frac{1}{RC}\right)$ which is the closed loop pole.

5. $$\frac{C(s)}{R(s)} = \frac{10}{s-20}$$

a) Unit Step ~~Response~~ Response

$$\frac{C(s)}{R(s)} = \frac{10}{s-20}$$ [for unit step input, $R(s) = \frac{1}{s}$

$$\therefore C(s) = \frac{10}{s(s-20)} \Rightarrow \frac{A}{s} + \frac{B}{s-20} = 10$$

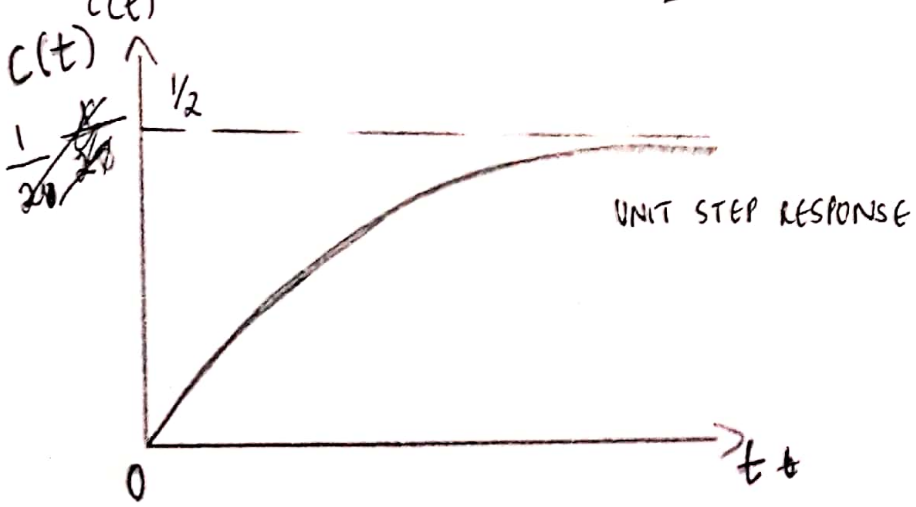
$$A(s-20) + Bs = 10$$

Put $s = 0 \Rightarrow -20A = 1 \quad A = \frac{-10}{20} = \frac{-1}{2}$

Put $s = 20 \Rightarrow 20B = 1 \quad B = \frac{10}{20} = \frac{1}{2}$

$$\therefore C(s) = \cancel{\frac{-1}{20s}} + \cancel{\frac{1}{20(s-20)}} = \frac{-1}{2s} + \frac{1}{2(s-20)}$$

$$c(t) = L^{-1}\left[\frac{-1}{2s} + \frac{1}{2(s-20)}\right] = \left[\frac{-1}{2} + \frac{e^{+20t}}{2}\right]$$



b) <u>Unit Ramp Response</u>

$$R(s) = \frac{1}{s^2}$$

$$C(s) = \frac{10}{s^2(s-20)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s-20)}$$

$$A(s-20) + Bs(s-20) + Cs^2 = 10$$

Put $s = 20$

$$C = \frac{\cancel{0}\cdot 10}{20^2} = 0.025$$

Put $s = 0$

$$-20A = 10$$

$$A = \frac{-10}{20} = -\frac{1}{2}$$

Put $s = 1$

$$10 = -\frac{1}{2}(1-20) + B(1-20) + 0.025$$

$$10 = 9.5 \cancel{+} -19 \cancel{0} B + 0.025$$

$\therefore \cancel{B = -0.025}$ $\quad \therefore B = -\frac{1}{40}$

<u>Output.</u>

$$C(s) = -\frac{1}{2s^2} - \frac{1}{40s} + \frac{0.025}{s-20}$$

$$c(t) = L^{-1}(C(s))$$

$$\boxed{c(t) = -\frac{1}{2}t - \frac{1}{40} + 0.025 \times e^{+20t}}$$