USN | | | | | | | | | | |

Second Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. ***Answer all objective type questions on any single page of the answer booklet.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) Solution of one of the factors of $p^2 - p - 1 = 0$ with $p = \frac{dy}{dx}$ is y = ________.

A) $\left(\frac{1+\sqrt{5}}{2}\right)x^4 + C$, B) $\left(\frac{2+\sqrt{5}}{2}\right)x^3 + C$ C) $\left(\frac{2-\sqrt{5}}{2}\right)x^2 + C$ D) $\left(\frac{1-\sqrt{5}}{2}\right)x + C$

ii) On solving for x in $P = \tan\left(x - \frac{P}{1+P^2}\right)$ the solution for y = ______.

A) $C + \frac{1}{1+P^2}$ B) $C - \frac{2}{1+P^2}$ C) $C - \frac{3}{1+P^2}$ D) $C - \frac{1}{1+P^2}$

iii) The solution for Clairut's form of the differential equation, $(y - Px)(P-1) = P$ is y = ______.

A) $Cx - \frac{C}{C-1}$ B) $Cx + \frac{C}{C-1}$ C) $C^2 - \frac{Cx}{C-1}$ D) $C^2 + \frac{Cx}{C-1}$

iv) If the given equation is solvable for y then it is of the form,

A) $y = f(x,p)$ B) $x = f(y,p)$ C) $x = f(y/p)$ D) $x = f(p/y)$

b. Solve $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$ with solvable for P. **(05 Marks)**

c. Solve $(px - y)(py + x) = \alpha^2 p$ with $x^2 = u$ and $y^2 = v$ using Clairut's form. **(05 Marks)**

d. Solve $y = x + a\tan^{-1} p$. **(06 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) Solution of $(D^3 - 2D + 4)y = 0$ is y = ______.

A) $C_1e^{2x} + C_2e^{-x}\cos x + C_3e^{-x}\sin x$ B) $C_1e^{2x} + C_2e^{-x}\cos x$

C) $C_1e^{-2x} + C_2e^{x}\cos x + C_3e^{x}\sin x$ D) $C_1\cos x + C_2\sin x$

ii) Particular integral of $(D^2 + 1)y = \sin 2x$ is y_p = _________.

A) $\frac{1}{3}\sin 2x$ B) $\sin 2x$ C) $\cos 2x$ D) $-\frac{1}{3}\sin 2x$

iii) Particular integral of $(D-1)y = \sinh x$ is y = ________.

A) $\frac{1}{2}(xe^x + e^{-x})$ B) $\frac{1}{2}xe^{-x}$ C) $\frac{1}{2}(e^{-x} + e^x)$ D) $\frac{1}{2}$

iv) The displacement in the simple harmonic motion $\frac{d^2x}{dt^2} = -\mu^2 x$ is ________.

A) $C_1\cos\mu t - C_2\sin\mu t$ B) $C_1\cos\mu t + C_2\sin\mu t$

C) $C_1\cos t + C_2\sin t$ D) $C_1\cos t - C_2\sin t$

b. Solve $(D^2 - 6D + 13)y = 8e^{3x}\sin 4x + 2^x$. **(05 Marks)**

c. Solve $y'' - 2y' + y = xe^x\sin x$. **(05 Marks)**

d. Solve $(D+3)x + (D+1)y = e^t$ and $(D+1)x + (D-1)y = t$. **(06 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) Particular solution of $(D^2-1)y=1$ using variation of parameters is $y_p =$ _______ .

A) -1 B) -2 C) -3 D) -4

ii) The differential equation, $x^3y''' + x^2y'' = \log x$ reduces to the form when $x = e^t$ as,

A) $(D+1)^3 y = t$ B) $D(D-1)^2 y = t$ C) $D^3 y = 0$ D) $D^2 y = 0$

iii) The complementary function of, $(1+x)^2 y'' + (1+x)y' + y = 2\sin\log(x+1)$ with $(1+x) = e^t$ is $y_c =$ ________ .

A) $C_1\cos t - C_2 \sin t$ B) $C_1 \cos 2t + C_2 \sin 2t$

C) $C_1 \cos t + C_2 \sin t$ D) $C_1 \cos 2t - C_2 \sin 2t$

iv) In $P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$, if $P_0(x) = 0$, then it has

A) Singular B) Regular singularity C) Exact D) Homogeneous

b. Solve $(D^2 - 3D + 2)y = \dfrac{1}{1+e^{-x}}$ using variation of parameters. **(05 Marks)**

c. Solve $x^2y'' - x\dfrac{dy}{dx} + y = \log x$. **(05 Marks)**

d. Solve $xy'' + y' + xy = 0$ using Frobenius series solution method. **(06 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) The partial differential equation of the relation $z = ax + a^2y^2 + b$ is $q =$ __________ .

A) p^2y B) $2p^2y$ C) p^2y^2 D) $2py^2$

ii) The solution of $\dfrac{\partial^2 z}{\partial x^2} = z$ is $z =$ ______ .

A) $C_1(x)e^y + C_2(x)e^{-y}$ B) $C_1(x)e^y - C_2(x)e^{-y}$

C) $C_1(y)e^x + C_2(y)e^{-x}$ D) $C_1(y)e^x - C_2(y)e^{-x}$

iii) The solution of $yq - xp = z$ by Lagrange's method is ________ $= 0$

A) $f\left(\dfrac{x}{y}, \dfrac{y}{z}\right)$ B) $f\left(\dfrac{y}{x}, \dfrac{y}{z}\right)$ C) $f\left(xyz, \dfrac{y}{z}\right)$ D) $f\left(xy, \dfrac{y}{z}\right)$.

iv) The solution of $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y}$ by separation of variables with K as the common solution is $u =$ _________.

A) $Ce^{K(x+y)}$ B) Ce^{Kxy} C) $Ce^{K(x+y)}$ D) Ce^{y}

b. Form the partial differential equation from the relation $f(x+y+z, x^2+y^2+z^2) = 0$. **(05 Marks)**

c. Solve $y^2p - xyq = x(z-2y)$ using Lagrange's linear form. **(05 Marks)**

d. Solve $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = 0$ using separation of variables. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of $\int_0^1\int_0^6 xy\,dx\,dy$ is ______.

A) 6 B) 7 C) 8 D) 9

ii) Area of the ellipse by double integration is = ________.

A) $\pi(a+b)$ B) $\pi(a-b)$ C) πab D) $\pi(b-a)$

iii) The value of $\beta\left(\frac{1}{2},\frac{1}{2}\right) =$ ________.

A) $\sqrt{\pi}$ B) π C) $\pi+1$ D) $\frac{\pi}{2}$

iv) The value of $\Gamma\left(\frac{1}{4}\right)\times\Gamma\left(\frac{3}{4}\right) =$ ______

A) $\pi\sqrt{2}$ B) $2\sqrt{\pi}$ C) $\sqrt{2\pi}$ D) 2π

b. Change the order of integration in, $I = \int_0^1 \int_{x^2}^{2-x} xydydx$ and hence evaluate. **(05 Marks)**

c. Evaluate $\int_1^e \int_1^{\log_e y} \int_1^{e^x} \log zdzdxdy$. **(05 Marks)**

d. Define Beta and Gamma functions, derive the relation as $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) If $\int_C F.d\vec{r} = 0$ then F is called,

A) Singular B) Irrotational C) Solenoidal D) Domain

ii) In Green's theorem $\iint_S \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial x}\right)dxdy =$ ______ .

A) $F_1 + F_2$ B) $\int_C (F_1dx + F_2dx)$ C) $\int_S F_1dx + F_2dy$ D) $\int_c (F_1dx + F_1dy)$

iii) In Stoke's theorem $\int_C F.dR =$ ______ .

A) $\int_C curlF.Nds$ B) $\int_C divF.Nds$ C) $\int_C gradF.Nds$ D) $\int_S curlF.Nds$

iv) If $\vec{F} = x^3i + y^3j + z^3k$ then divF = ________

A) $x^2 + y^2 + z^2$ B) $2(x^2 + y^2 + z^2)$ C) $3(x^2 + y^2 + z^2)$ D) $3(x^2i + y^2j + z^2k)$

b. If $F = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C F.d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the curve given by $x = t$, $y = t^2$, $z = t^3$. **(05 Marks)**

c. Use Green's theorem to evaluate $\int_C (y - \sin x)dx + \cos xdy$, where C is the triangle in xy-plane bounded by the lines $y = 0$, $x = \frac{\pi}{2}$ and $y = \frac{2x}{\pi}$. **(05 Marks)**

d. Use Gauss divergence theorem to evaluate $\int_S F.Nds$ where $F = 4xyi + yzj - xzk$ and S is the surface of the cube bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$. **(06 Marks)**

7 a. Choose the correct answers for the following : **(04 Marks)**

i) $L\{e^{-t}\cosh t\} =$ ________

A) $\frac{s+1}{(s-1)^2+1}$ B) $\frac{s-1}{(s+1)^2+1}$ C) $\frac{s+1}{(s-1)^2-1}$ D) None of these

ii) $L\{t^{-1}f(t)\} =$ ________

A) $\int_s^\infty F(s)ds$ B) $\int_s^\infty f(t)dt$ C) $\int_t^\infty F(s)ds$ D) $\int_0^\infty f(t)dt$

iii) When T denotes period of the function f(t) then, $\frac{1}{1-e^{-ST}}\int_0^T e^{-st}f(t)dt =$ ________ .

A) f(t) + C B) $L\{f(t)e^{-st}\}$ C) $L\{f(t)\}$ D) $L\{e^t\}$

iv) In unit step function if $u(t-a)=0$ then,

A) $t < a$ B) $t \geq a$ C) $t = a$ D) $t \leq a$

b. Find the Laplace transform of the function $f(t) = te^{-t}\sin^2 3t$. (05 Marks)

c. Find the Laplace transform of the function,

$$f(t) = \begin{cases} t, & \text{for } 0 < t \leq a \\ 2a - t, & \text{for } a < t < 2a \end{cases}, \text{ 2a is the period.}$$ (05 Marks)

d. Express f(t) in terms of unit step function and find the Laplace transform when,

$$f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & 2 < t < 4. \\ 8, & t > 4 \end{cases}$$ (06 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) The value of $L^{-1}\left\{\frac{s^2-3s+4}{s^3}\right\} =$ ________ .

A) $1-3t^2+2t$ B) $\frac{1+3t}{2t^2}$ C) $1-3t+2t^2$ D) $\frac{1+2t^2}{3t}$

ii) The value of $L^{-1}\left\{\frac{s}{(s-2)^2}\right\} =$ ________

A) $e^{2t}(1-2t)$ B) $e^{2t}(1+2t)$ C) $e^{2t}(2+2t)$ D) $2+2t$

iii) By convolution theorem $L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\} =$ ________ .

A) $\int_0^\infty e^{-t}e^{t-2}dt$ B) $\int_0^t e^{-t}e^{-(t^2-2)}dt$ C) $\int_0^t e^{-u}e^{-(t-u)(2)}du$ D) $\int_0^\infty e^{-2t}e^{t^2+1}dt$

iv) Laplace transform of $\frac{dy}{dt}+y=0$ with y(0) = 1 is = ________

A) e^{-t} B) e^t C) te^t D) $\frac{e^t}{t}$

b. Find the inverse Laplace transform of, $F(s) = \frac{2s^2-6s+5}{s^3-6s^2+11s-6}$. (05 Marks)

c. Find $L^{-1}\left\{\frac{s}{(s-1)(s^2+4)}\right\}$ by convolution theorem. (05 Marks)

d. Solve by Laplace transform method, $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+2y = 4t+12e^{-t}$ with $y(0) = 6$, $y'(0) = -1$ (06 Marks)

* * * * *