USN | | | | | | | | | | 10MAT31

# Third Semester B.E. Degree Examination, June/July 2017
# Engineering Mathematics - III

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

## PART – A

1 a. Obtain Fourier series for the function f(x) given by

$$f(x) = \begin{cases} 1+\dfrac{2x}{\pi}, & -\pi \le x \le 0 \\ 1-\dfrac{2x}{\pi}, & 0 \le x \le \pi \end{cases}.$$

Hence deduce that $\dfrac{1}{1^2}+\dfrac{1}{3^2}+\dfrac{1}{5^2}+.....=\dfrac{\pi^2}{8}$. **(06 Marks)**

b. Obtain Fourier half range Cosine series for the function f(x) = x sin x in (0, π). Hence show that $\dfrac{1}{1.3}-\dfrac{1}{3.5}+\dfrac{1}{5.7}-.....\infty=\dfrac{\pi-2}{4}$. **(07 Marks)**

c. Obtain the constant term and the co-efficient of the first sine and cosine terms in the Fourier series of f(x) as given in the following table. **(07 Marks)**

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| f(x) | 9 | 18 | 24 | 28 | 26 | 20 |

2 a. Find the Fourier transform of $e^{-a^2x^2}$, a < 0. Hence deduce that $e^{-x^2/2}$ is self reciprocal in respect of Fourier transform. **(06 Marks)**

b. Find the Fourier sine transform of $e^{-|x|}$. Hence show that

$$\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi e^{-m}}{2},\ m>0.$$ **(07 Marks)**

c. Find the Fourier Cosine transform of $f(x)=\dfrac{1}{1+x^2}$. **(07 Marks)**

3 a. Obtain various possible solutions of the one dimensional Heat equation $\dfrac{\partial u}{\partial t}=c^2\dfrac{\partial^2 u}{\partial x^2}$ by the method of separation of variables. **(06 Marks)**

b. Obtain the D'Alembert's solution of the wave equation $\dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2}$. Subject to the conditions u(x, 0) = f(x) and $\dfrac{\partial u}{\partial t}(x,0)=0$. **(07 Marks)**

c. Obtain various possible solutions of the two dimensional Laplace equation $u_{xx}+u_{yy}=0$ by the method of separation of variables. **(07 Marks)**

4 a. Fit a parabola $y = ax^2 + bx + c$ to the following data : **(06 Marks)**

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y | 1 | 3 | 7 | 13 | 21 | 31 |

b. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5,760 to invest and has space for at most 20 items. A fan and sewing machine cost Rs 360 and Rs 240 respectively. He can sell a fan at a profit of Rs 22 and sewing machine at a profit of Rs 18. Assuming that he can sell whatever he buys, how should he invest his money in order to maximize his profit? Translate the problem into LPP and solve it graphically. **(07 Marks)**

c. Use Simplex method to solve the following LPP

Minimize $Z = x_1 - 3x_2 + 3x_3$

Subject to $3x_1 - x_2 + 2x_3 \le 7$

$2x_1 + 4x_2 \ge -12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0.$ **(07 Marks)**

## PART – B

5 a. Using Newton – Raphson method, find the value of $\sqrt[3]{18}$ correct to 2 decimals, assuming 2.5 as the initial approximation. **(06 Marks)**

b. Apply Gauss – Seidal iteration method to solve the following equations :

$3x + 20y - z = -18$ ; $2x - 3y + 20z = 25$ ; $20x + y - 2z = 17$. **(07 Marks)**

c. Find the largest Eigen - value and the corresponding Eigen - vector for the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ with initial approximation $[1\ 1\ 0]^T$. **(07 Marks)**

6 a. Determine f(x) as a polynomial in x for the following data by using Newton's divided difference formula. **(06 Marks)**

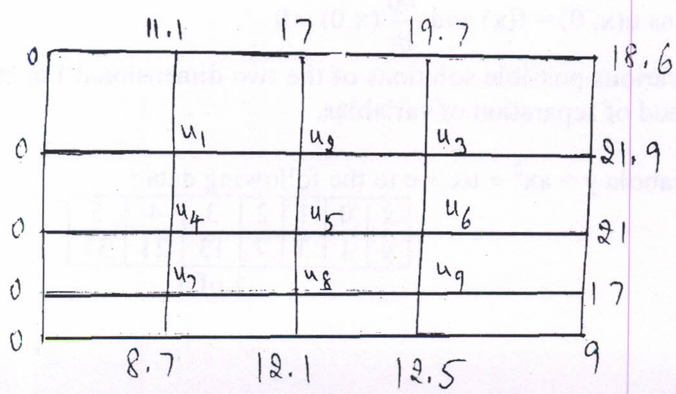
| x | -4 | -1 | 0 | 2 | 5 |
|---|---|---|---|---|---|
| f(x) | 1245 | 33 | 5 | 9 | 1335 |

b. From the data given in the following table, find the number of students who obtained i) less than 45 marks and ii) between 40 and 45 marks. **(07 Marks)**

| Marks | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|
| No. of students | 31 | 42 | 51 | 35 | 31 |

c. Evaluate $\int_4^{5.2} \log_e x \ dx$ by Weddle's rule. **(07 Marks)**

7 a. Solve the Laplace equation $u_{xx} + u_{yy} = 0$, given that the boundary values for the following square mash. **(06 Marks)**



b. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$, taking h = 1 upto t = 1.25. The boundary conditions are $u(0,t) = u(5, t) = 0$ , $u_t(x, 0) = 0$ and $u(x, 0) = x^2(5-x)$. **(07 Marks)**

c. Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in $0 < x < 5$ , $t \geq 0$, given that $u(x, 0) = 20$ , $u(0, t) = 0$, $u(5, t) = 100$. Compute u for the time – step with h = 1 by Crank – Nicholson method. **(07 Marks)**

**8** a. Find the Z – transform of the following :
i) $(n + 1)^2$ ii) $\sin(3n + 5)$ iii) $n_{c_p}$ $(0 \leq p \leq n)$. **(06 Marks)**

b. If $u(z) = \frac{2z^2 + 3z + 12}{(z-1)^4}$. Find $u_0$, $u_1$, $u_2$, $u_3$. **(07 Marks)**

c. Solve $y_{n+2} + 4y_{n+1} + 3y_n = 3^n$ with $y_0 = 0$ , $y_1 = 1$, using Z - transforms. **(07 Marks)**



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