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Fourth Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – IV

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1 a. Find $y(0.1)$ by using Taylor's series method, given that $y' = \sqrt{x^2 + y}$, $y(0) = 0.8$. Consider upto third order derivative terms. (06 Marks)
- b. Given : $\frac{dy}{dx} = \frac{1}{1+x^2} - 2y^2$, $y(0) = 0$. Find $y(0.5)$, by taking $h = 0.25$, using Euler's modified method. (07 Marks)
- c. If $y' = \frac{1}{x+y}$, $y(0) = 2.0000$, $y(0.2) = 2.0933$, $y(0.4) = 2.1755$, $y(0.6) = 2.2493$, find $y(0.8)$ by using Adams-Bashforth method. (07 Marks)
- 2 a. Using the Picard's method, obtain the 2nd order approximate solution of the problem at $x = 0.2$, $\frac{dy}{dx} = x + yz$; $\frac{dz}{dx} = y + zx$, $y(0) = 1$ and $z(0) = -1$. (06 Marks)
- b. Using the R-K method, find the solution at $x = 0.1$ of an equation; $y'' - x^2y' - 2xy - 1 = 0$ with the conditions $y(0) = 1$, $y'(0) = 0$ and step size 0.1. (07 Marks)
- c. Given that $y'' + xy = 0$, $y(0) = 1$, $y(0.1) = 1.0998$, $y(0.2) = 1.1987$, $y(0.3) = 1.2955$, $y'(0) = 1$, $y'(0.1) = 0.9946$, $y'(0.2) = 0.9773$, $y'(0.3) = 0.946$, find $y(0.4)$, using Milne's method. (Apply corrector formula only once). (07 Marks)
- 3 a. Derive Cauchy-Riemann equations in the polar form. (06 Marks)
- b. If $f(z) = u + iv$ is an analytic function, then prove that the family of curves; $u(x, y) = C_1$, $v(x, y) = C_2$, C_1 and C_2 being constants, intersect orthogonally. Is the converse true? Justify your answer. (07 Marks)
- c. In a two dimensional fluid flow; if the velocity potential is $e^{-x} \cos y + xy$, find the stream function. (07 Marks)
- 4 a. Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Also find the invariant points. (06 Marks)
- b. Discuss the transformation, $w = z + \frac{K^2}{z}$, where $z \neq 0$, $K \neq 0$. (07 Marks)
- c. State and prove the Cauchy's theorem. (07 Marks)

PART – B

- 5 a. Obtain the series solution of Bessel's differential equation. (07 Marks)
- b. Derive the Rodrigue's formula. (07 Marks)
- c. Express the polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. (06 Marks)

- 6 a. 'A' can hit a target 3 times in 5 shots, 'B' 2 times in 5 shots and 'C' 3 times in 4 shots. They fire a volley. Find the probability that (i) 2 shots hit (ii) at least 2 shots hit. (06 Marks)
- b. If A and B are events with $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{4}$, $P(\bar{B}) = \frac{5}{8}$ find $P(A \cap B)$, $P(\bar{A} \cap \bar{B})$, $P(\bar{A} \cup \bar{B})$ and $P(\bar{A} \cap B)$. (07 Marks)
- c. State and prove Baye's theorem. (07 Marks)
- 7 a. (i) Is the function defined as follows a density function? $f(x) = e^{-x}$, $x \geq 0$, $f(x) = 0$, $x < 0$.
(ii) If so, determine the probability that the variate having this density will fall in the interval (1, 2).
(iii) Also find the cumulative probability function F(2). (06 Marks)
- b. Obtain the mean and standard deviation of the Poisson distribution. (07 Marks)
- c. The life of an electric bulb is normally distributed with mean life of 200 hours and S.D. of 60 hours. Out of 2500 bulbs, find the number of bulbs which are likely to last between 1900 and 2100 hours. Given that $P(0 < Z < 1.67) = 0.4525$. (07 Marks)
- 8 a. Explain the following terms briefly: (i) Null hypothesis (ii) Type I and Type II errors (iii) Confidence limits. (06 Marks)
- b. Two types of batteries are tested for their length of life and the following results are obtained:
Battery A : $n_1 = 10$, $\bar{x}_1 = 500$ hrs, $\sigma_1^2 = 100$
Battery B : $n_2 = 10$, $\bar{x}_2 = 560$ hrs, $\sigma_2^2 = 121$.
Find students 't' and test whether there is a significant difference in the two means. ($t_{0.05} = 2.10$ and $t_{0.01} = 2.88$). (07 Marks)
- c. Genetic theory states that children having one parent of blood type M and the other of blood type N will always be one of the three types M, MN, N and that the proportions of these types will on an average be 1 : 2 : 1. A report states that out of 300 children having one M parent and one N parent, 30% are found to be of type M, 45% of type MN and the remainder of type N. Test the theory by χ^2 (chi-square) test. (07 Marks)

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