USN

MATDIP401

Fourth Semester B.E. Degree Examination, June/July 2017
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the angle between any two diagonals of a cube. **(06 Marks)**

b. Find the angle between two lines whose direction cosines are given by $\ell + 3m + 5n = 0$ and $2mn - 6n\ell - 5\ell m = 0$. **(07 Marks)**

c. Find the coordinates of the foot of the perpendicular from A(1, 1, 1) to the line joining the points B(1, 4, 6) and C(5, 4, 4). **(07 Marks)**

2 a. Find the equation of the plane through (2, -1, 6) and (1, -2, 4) and perpendicular to the plane $x - 2y - 2z + 9 = 0$. **(06 Marks)**

b. Find the equation of a straight line through (7, 2, -3) and perpendicular to each of the lines. $\frac{x-1}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and $\frac{x+2}{4} = \frac{y-3}{5} = \frac{z-4}{6}$. **(07 Marks)**

c. Find the angle between the planes $x - y + z - 6 = 0$ and $2x + 3y + z + 5 = 0$. **(07 Marks)**

3 a. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three vectors then prove that
$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \left(\vec{a} \cdot \vec{c}\right)\vec{b} - \left(\vec{a} \cdot \vec{b}\right)\vec{c}$ **(06 Marks)**

b. If $\vec{A} = 4i + 3j + k$, $\vec{B} = 2i - j + 2k$ find a unit vector N perpendicular to the vectors $\vec{A}$ and $\vec{B}$ also show that $\vec{A}$ is not perpendicular to $\vec{B}$. **(07 Marks)**

c. Find the value of λ so that the points A(-1, 4, -3), B(3, 2, -5), C(-3, 8, -5) and D(-3, λ, 1) lie on the same plane. **(07 Marks)**

4 a. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$ where t is time. Find the components of its velocity and acceleration in the direction of the vector $i - 3j + 2k$ at t = 1. **(06 Marks)**

b. Find the angle between tangents to the curve $x = t^2 + 1$, $y = 4t - 3$, $z = 2t^2 - 6t$ at t = 1 and t = 2. **(07 Marks)**

c. Find the directional derivative of $x^2yz + 4xz^2$ at (1, -2, -1) in the direction of $2i - j - 2k$. **(07 Marks)**

5 a. Prove that $\text{div}(\text{curl}\,\vec{A}) = 0$. **(06 Marks)**

b. Find the divergence and curl of the vector.
$\vec{F} = (xyz + y^2z)i + (3x^2y + y^2z)j + (xz^2 - y^2z)k$ **(07 Marks)**

c. Find the constants a, b, c so that the vector,
$\vec{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational. **(07 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

6 Find :

a. $L[\sin 5t \sin 3t]$ **(05 Marks)**

b. $L[te^{8t}\cos 2t]$ **(05 Marks)**

c. $L\left[\frac{1-e^{at}}{t}\right]$ **(05 Marks)**

d. $L\left[\int_0^t e^{2t}\frac{\sin at}{t}dt\right]$ **(05 Marks)**

7 a. Find $L^{-1}\left[\frac{2s-1}{s^2+2s+17}\right]$. **(05 Marks)**

b. Find $L^{-1}\left[\frac{s+1}{(s-1)^2(s+2)}\right]$. **(05 Marks)**

c. Find $L^{-1}\left[\cot^{-1}\left(\frac{s}{a}\right)\right]$. **(05 Marks)**

d. Using convolution theorem evaluate $L^{-1}\left[\frac{s}{(s+2)(s^2+9)}\right]$. **(05 Marks)**

8 a. Using Laplace transforms, solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$ given $y(0)=y'(0)=0$. **(10 Marks)**

b. Using Laplace transforms, solve $\frac{dx}{dt}+y=\sin t$, $\frac{dy}{dt}+x=\cos t$, given x = 2, y = 0 when $t=0$. **(10 Marks)**

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