CBCS Scheme

USN | | | | | | | | | | |

15CS36

Third Semester B.E. Degree Examination, June/July 2017

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Define the following with an example for each
   i) Proposition
   ii) Tautology
   iii) Contradiction
   iv) Dual of statement. **(06 Marks)**

b. Establish the validity of the following argument using rules of inference. If the band could not play rock music or the refreshments were not served on time, then the new year party could have been cancelled and Alica would have been angry. If the party were cancelled, then refunds would have to be made. No refunds were made, therefore the band could play rock music. **(05 Marks)**

c. Determine the truth value of the following statements if the universe comprises all nonzero integers :
   i) $\exists x \exists y [xy = 2]$
   ii) $\exists x \forall y [xy = 2]$
   iii) $\forall x \exists y [xy = 2]$
   iv) $\exists x \exists y [(3x + y = 8) \wedge (2x - y) = 7]$
   v) $\exists x \exists y [(4x + 2y = 3) \wedge (x - y = 1)]$ **(05 Marks)**

OR

2 a. Find the possible truth values for p, q and r if
   i) $p \to (q \vee r)$ – FALSE
   ii) $p \wedge (q \to r)$ – TRUE. **(05 Marks)**

b. Show that $(p \wedge (p \to q)) \to q$ is independent of its components. **(06 Marks)**

c. Give a direct proof for each of the following :
   i) For all integers k and ℓ, if k and ℓ are both even, then $k + \ell$ is even
   ii) For all integers k and ℓ, if k and ℓ are both even, then $k * \ell$ is even. **(05 Marks)**

Module-2

3 a. Prove by mathematical induction, for every positive integer 8 divides $5^n + 2 \cdot 3^{n-1} + 1$. **(06 Marks)**

b. Assuming PASCAL language is case insensitive, an identifier consists of a single letter followed by upto seven symbols which may be letters or digits (26 letters, 10 digits). There are 36 reserved words. How many distinct identifiers are possible in this version of PASCAL? **(05 Marks)**

c. Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(05 Marks)**

OR

4 a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(05 Marks)**

b. Lucas numbers are defined recursively as $L_0 = 2$, $L_1 = 1$ and $L_n = L_{n-1} + L_{n-2}$ for $n \geq 2$. If Fi^s are fibonacci numbers and Li^s are the Lucas numbers, prove that $L_n = F_{n-1} + F_{n+1}$ for all positive integers n. **(05 Marks)**

c. Find the number of distinct terms in the expansion of $(w + x + y + z)^{12}$. **(06 Marks)**

Module-3

5 a. Let A = {a, b, c, d} and B = {1, 2, 3, 4, 5, 6}.

i) How many functions are there from A to B? How many of these are one-to-one? How many are onto?

ii) How many functions are there from B to A? How many of these are one-to-one? How many are onto? **(06 Marks)**

b. Prove that if $f : A \to B$, $g : B \to C$ are invertible functions, then $g \circ f : A \to C$ is invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. **(06 Marks)**

c. For the Hasse diagram, given in Fig. Q5(c), write i) maximal ii) minimal iii) greatest and iv) least element (s). **(04 Marks)**

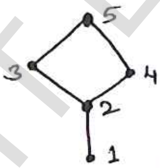

Fig. Q5(c)

OR

6 a. Let $f, g : z^+ \to z^+$, where for all $x \in z^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x - 1\}$.

i) What is the range of f?

ii) Is f a onto function?

iii) Is f one-to-one?

iv) What is the range of g?

v) Is g an onto function? **(05 Marks)**

b. If $f : A \to B$ and $B_1, B_2 \leq B$, then prove the following :

i) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$

ii) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$

iii) $f^{-1}(\overline{B}_1) = \overline{f^{-1}(B_1)}$ **(06 Marks)**

c. Let A = {1, 2, 3, 4}, R = {(1, 3), (1, 1), (3, 1), (1, 2), (3, 3), (4, 4)} be the relation on A. Determine whether the relation R is reflexive, irreflexive, symmetric, antisymmetric or transitive. **(05 Marks)**

Module-4

7 a. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(05 Marks)**

b. Describe the expansion formula for rook polynomials. Find the rook polynomial for 3×3 board using the expansion formula. **(05 Marks)**

c. Solve the recurrence relation $b_n = bD_{n-1} - b^2 D_{n-2}$, $n \geq 3$ given $D_1 = b > 0$ and $D_2 = 0$. **(06 Marks)**

OR

8 a. In how many ways can we arrange the letters in the CORRESPONDENTS so that ;
   i) There is no pair of consecutive identical letters?
   ii) There are exactly two pairs of consecutive identical letters
   iii) There are atleast 3 pairs of consecutive identical letters **(06 Marks)**

b. Find the recurrence relation and the initial conditions for the sequence 0, 2, 6, 12, 20, 30, 42 ...... Hence find the general term of the sequence. **(05 Marks)**

c. Find the general solution of the equation $S(k) + 3S(k-1) - 4S(k-2) = 4^k$. **(05 Marks)**

Module-5

9 a. Define the following with an example
   i) Simple graph
   ii) Regular graph
   iii) Subgraph
   iv) Maximal subgraph
   v) Induced subgraph. **(05 Marks)**

b. Show that there exists no simple graphs corresponding to the following degree sequences
   i) 0, 2, 2, 3, 4
   ii) 1, 1, 2, 3
   iii) 2, 3, 3, 4, 5, 6
   iv) 2, 2, 4, 6. **(04 Marks)**

c. Let T = (V, E) be a complete m-ary tree with |V| = n. If T has ℓ leaves and i internal vertices, then prove the following :
   i) $n = m \cdot i + 1$
   ii) $\ell = (m-1)i + 1$
   iii) $i = \dfrac{(\ell - 1)}{(m-1)} = \dfrac{(n-1)}{m}$ **(07 Marks)**

OR

10 a. In the graph shown in Fig. Q10(a). Determine
   i) a walk from b to d that is not a trail
   ii) b – d trail that is not a path
   iii) a path from b to d
   iv) a closed walk from b to b that is not a circuit
   v) a circuit from b to b that is not a cycle
   vi) a cycle from b to b **(06 Marks)**

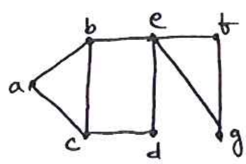

Fig. Q10(a)

b. Determine the order |V| of the graph G = (V, E) in the following cases
   i) G is cubic graph with 9 edges
   ii) G is regular with 15 edges
   iii) G has 10 edges with 2 vertices of degree 4 and all other of degree 3. **(06 Marks)**

c. Obtain the optimal prefix code for the string ROAD IS GOOD. **(04 Marks)**

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