USN | | | | | | | | | | | **10CS34**

Third Semester B.E. Degree Examination, June/July 2017
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Using the Venn diagram, prove that
$A\Delta(B\Delta C) = (A\Delta B)\Delta C$ **(06 Marks)**

b. In a survey of 60 people it was found that 25 read weekly magazines, 26 read forthrightly magazines, 26 read monthly magazines, 9 read both weekly and monthly magazines, 11 read both weekly and fortnightly magazines, 8 read fortnightly and monthly magazines and 3 read all three magazines, Find
  i) The number of people who read at least one of the three magazines and
  ii) The number of people who read exactly one magazine. **(07 Marks)**

c. An integer is selected at random from 3 through 17 inclusive. If A is the event that a number divisible by 3 is chosed and B is the event that the chosen number exceeds 10, determine the $P_r(A)$, $P_r(B)$, $P_r(A \cap B)$ and $P_r(A \cup B)$. **(07 Marks)**

2 a. Prove the following logical equivalence without using truth tables
$[(p \vee q) \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$ **(06 Marks)**

b. Define tautology. Examine whether the compound proposition is a tautology.
$[p \vee (q \wedge r)] \vee \neg[p \vee (q \wedge r)]$. **(07 Marks)**

c. State the converse, inverse and contra positive of the conditional "If two lines are parallel then they are equidistant" **(07 Marks)**

3 a. For the universe of all real numbers, define the following open statements,
$p(x) : x \geq 0$, $q(x) : x^2 \geq 0$, $r(x) : x^2 - 3 > 0$.
Determine the truth value of the following statements.
  i) $\exists x, p(x) \wedge q(x)$
  ii) $\forall x, p(x) \rightarrow q(x)$
  iii) $\forall x, q(x) \rightarrow r(x)$ **(06 Marks)**

b. Find whether the following argument is valid. If a triangle has two equal sides, then it is isosceles. If a triangle is isosceles, then it has two equal angles
the triangle ABC does not have two equal sides
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$\therefore$ ABC does not have two equal sides **(07 Marks)**

c. Give :
  i) a direct proof
  ii) an indirect proof and
  iii) Proof by contradiction for the following statement. "If m is an even integer, then m + 5 is an odd integer". **(07 Marks)**

4 a. Prove the following result by mathematical induction

$1^2 + 2^2 + 3^2 + \ldots\ldots. + n^2 = \frac{1}{6} n(n + 1)(2n + 1).$ **(06 Marks)**

b. Find an explicit definition of the sequence defined recursively by $a_1 = 7$, $a_n = 2a_{n-1} + 1$ for $n \geq 2$. **(07 Marks)**

c. Let F_n denote the n^{th} Fibonacci number prove that $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$. **(07 Marks)**

PART – B

5 a. Define Cartesian product of two sets, Let A = {a, b, c}, B = {1, 2} and C = {x, y, z}, Find $A \times (B \cup C)$ and $(A \times B) \cup C$. **(06 Marks)**

b. Let A = {1, 2, 3, 4} and B = {a, b, c} Find
   i) Number of relations from A to B
   ii) Number of one – to – one relations from A and B
   iii) Number of on to functions from A to B. **(07 Marks)**

c. Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 3x + 2$, $g(x) = \frac{1}{2}(x - 3)$. Find f^1, g^{-1} and $f^1 \, 0 \, g^{-1}$. **(07 Marks)**

6 a. Let A = {1, 2, 3, 4, 6} and R be a relation on A defined by aRb if and only if "a is a multiple of b". Write down the relation matrix M(R) and draw its diagraph. **(06 Marks)**

b. Define equivalence relation. Let S be the set of all non-zero integers and A = S×S on A, define the relation R by (a, b) R (c, d) if and only if ad = bc. Show that R is an equivalence relation. **(07 Marks)**

c. Let A = {1, 2, 3, 4, 6, 8, 12}. On A, define the partial orderly relation R by xRy if and only of "x divides y". Draw the Hasse diagram for R. **(07 Marks)**

7 a. If * is an operation on 2, defined by x*y = x + y + 1. Prove that (2, *) is an abelian group. **(06 Marks)**

b. Define subgroup of a group. Prove that the intersection of two subgroups of a group is a sub group of the group. **(07 Marks)**

c. For a group G, prove that the function $f : G \to G$ defined by $f(a) = a^{-1}$ is an isomorphism if and only if G is abelian. **(07 Marks)**

8 a. The encoding function $E : Z_2^2 \to Z_2^5$ is given by the generator matrix

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

   i) Determine all the code words.
   ii) Find the associated parity check matrix H. **(06 Marks)**

b. Prove that $(z, \oplus, \otimes)$ is a ring with binary operations. $x \oplus y = x + y + 1$, $x \otimes y = x + y + xy$, $\forall x, y \in z$. **(07 Marks)**

c. Show that Z_6 is an integral domain. **(07 Marks)**

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