USN | | | | | | | | | | **10CV661**

Sixth Semester B.E. Degree Examination, June/July 2017
Theory of Elasticity

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. When a body is subjected to stresses σ_x , σ_y and σ_z in x, y and z directions respectively, Obtain an expression for σ_x as $\sigma_x = \lambda\in + 2G\in_x$. **(10 Marks)**

Where , $\lambda = \dfrac{\mu E}{(1-2\mu)(1+\mu)}$ and $\in = \in_x + \in_y + \in_z$.

Hence derive $(\lambda + G)\dfrac{\partial \in}{\partial x} + G\nabla^2 u + x = 0$.

b. The possible state of stress in a solid is given by

$\sigma_x = c_1 x^2 yz$
$\sigma_y = c_2\, xyz^3$
$\sigma_z = 2(x^3 + y^3 - 2yz)$
$\tau_{xy} = -3xy^2 z$
$\tau_{yz} = c_3[6y^2z^2 - 5xz^4 + 8(x^2 + y^2)]$
$\tau_{zx} = -3xyz^2$. Find the values of c_1, c_2 and c_3. **(10 Marks)**

2 a. Derive the two sets of compatibility equations in terms of strains for three dimensional cases. **(10 Marks)**

b. Find the constants of c_1, c_2 and c_3 at point (2, -1) for the stress distribution given as :

$\sigma_x = -2xy^2 + c_1 x^3$
$\sigma_y = -1.5c_2 xy$
$\tau_{xy} = -c_2 y^3 - c_3 x^2 y$. **(10 Marks)**

3 a. If E is replaced by $\dfrac{E_1}{1-\mu_1^2}$ and μ by $\dfrac{\mu_1}{1-\mu_1}$ in plane stress constitutive relations, prove that

$\nabla^2(\sigma_x + \sigma_y) = -\dfrac{1}{(1-\mu_1)}\left(\dfrac{\partial X}{\partial x} + \dfrac{\partial Y}{\partial y}\right)$. **(10 Marks)**

b. Determine the principal strains and their directions for an equiangular strain rosette.
Given : $\in_0^0 = 550 \times 10^{-6}$ $\in_{60^0} = -100 \times 10^{-6}$ $\in_{120^0} = 150 \times 10^{-6}$. **(10 Marks)**
Also determine the principal stresses given $\mu = 0.3$ and E = 200GPa.

4 a. For a simply supported beam of length 2L , depth 2h and unit width loaded by concentrated load W at midspan, the stress function satisfying the loading condition is **(10 Marks)**
$\phi = \dfrac{b}{6}xy^3 + cxy$. Determine the constants "b" and "c". Also find the stresses in the beam.

b. Check whether the following is a stress function. If it is, investigate what problem it can solve when applied to region y = 0 , y = d and x = 0 and x ≥ 0. **(10 Marks)**

$\phi = -\dfrac{F}{d^3}xy^2(3d - 2y)$.

PART – B

5 a. Derive equation of equilibrium in polar co-ordinates. **(10 Marks)**

b. Show that $\phi = \frac{-py}{\pi} \tan^{-1} \frac{y}{x}$ is a stress function. Also prove that it represents a case of simple radial stress distribution. **(10 Marks)**

6 a. Prove that for a solid rotating disk, the maximum stresses are given by

$(\sigma_r)_{max} = (\sigma_\theta)_{max} = \left(\frac{3+\mu}{8}\right)\rho\, w^2 b^2.$ **(10 Marks)**

b. Also prove that for a hollow disk of inner radius "a" and outer radius "b",

$(\sigma_r)_{max} = \left(\frac{3+\mu}{8}\right)\rho\, w^2\, (b-a)^2$. Show that $(\sigma_\theta)_{max} > (\sigma_r)_{max}$. **(10 Marks)**

7 Discuss the effect of a circular hole on the stress distribution in a rectangular plate subjected to a tensile stress s in x – direction only and hence evaluate stress concentration factor. **(20 Marks)**

8 a. Prove that for non – circular sections subjected to torsion T = GJθ.
Where, GJ = Torsional rigidity. **(10 Marks)**

b. A 2 – celled thin walled tube, each cell having dimensions of a × a with uniform wall thickness δ. Show that there will be no stress in the central web when the tube in twisted. **(10 Marks)**

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