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Fifth Semester B.E. Degree Examination, June/July 2017
Modern Control Theory

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1 a. Define state, state variable and state space. Form the state model for a single input single output continuous LTI system. (08 Marks)
- b. Construct the state diagram and state model for the following differential equation:

$$\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y(t) = 10u(t).$$
 (08 Marks)
- c. Mention any four advantages of state variable method over conventional control system. (04 Marks)
- 2 a. Obtain the state model for the armature controlled d.c. motor. (08 Marks)
- b. A feed back system is represented by the closed loop transfer function:

$$T(s) = \frac{s^2 + 3s + 3}{s^3 + 2s^2 + 3s + 1}.$$
 Draw a suitable signal flow graph and obtain the state model. (06 Marks)
- c. Obtain the state model in Jordan's canonical form of a system whose transfer function is

$$T(s) = \frac{1}{(s+2)^2(s+1)}.$$
 (06 Marks)
- 3 a. Obtain the transfer for the state model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} U \quad \text{and} \quad y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$
 (06 Marks)
- b. Consider a state model with matrix 'A' as

$$A = \begin{bmatrix} 0 & 2 & 0 \\ 4 & 0 & 1 \\ -48 & -34 & -9 \end{bmatrix}.$$

Obtain the modal matrix 'M' and also prove that the transformation $M^{-1}AM$ results in a diagonal matrix. (08 Marks)
- c. Mention any three properties of properties of Eigen values and prove the invariance of Eigen values. (06 Marks)
- 4 a. Mention any four properties of State Transition Matrix (STM). (04 Marks)
- b. Obtain the complete time response of the system given by

$$\dot{X}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix} X(t) \quad \text{and} \quad y(t) = [1, -1] X(t) \quad \text{where} \quad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$
 (10 Marks)
- c. Find $f(A) = A^{12}$ for $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$ using Cayley-Hamilton theorem. (06 Marks)

PART - B

- 5 a. A regulator system has the plant

$$\dot{x} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} 0 \quad \text{and} \quad y = [0 \ 0 \ 1]x.$$

Obtain the state feedback matrix by direct substitution method. Given the desired poles (closed loop) to be at $-2 + j\sqrt{12}$, $-2 - j\sqrt{12}$ and -5 . (08 Marks)

- b. Consider the regulator system given by $\dot{x} = Ax + Bu$ where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The system uses the state feedback control $u = -kx$. Determine the state feedback gain matrix K using Akerman and formula if the poles are located at $-2 + j4$; $-2 - j4$; -10 . (08 Marks)

- c. With a neat diagram, explain P-I controller. (04 Marks)

- 6 a. Explain briefly the characteristic features of non-linear system. (04 Marks)

- b. Give the satisfaction of non-linear systems with suitable examples. (04 Marks)

- c. Explain the following non-linearities:

- i) Saturation
- ii) Dead zone
- iii) On-Off non-linearity
- iv) Back-Lash in gears.

(12 Marks)

- 7 a. Explain the following with respect to non-linear systems:

- i) Limit cycles; ii) Jump resonance. (06 Marks)

- b. Find out the singular points for the following systems:

i) $\ddot{x} + 0.5\dot{x} + 2x = 0$

ii) $\ddot{y} + 3\dot{y} + 2y = 0$ and

iii) $\ddot{y} + 3\dot{y} - 10 = 0$. (06 Marks)

- c. Construct a phase trajectory by delta method for a non-linear system represented by the differential equation $\ddot{x} + 4|\dot{x}|\dot{x} + 4x = 0$. Choose initial conditions as $x_1(0) = 1.0$ and $\dot{x}(0) = 0$. (08 Marks)

- 8 a. Define the following: i) Positive definiteness; ii) Negative definiteness. and also give the sufficient condition for stability. (06 Marks)

- b. Use Krasovoski and theorem to show that the equilibrium state $x = 0$ of the system described by: $\dot{x}_1 = -3x_1 + x_2$; ii) $\dot{x}_2 = x_1 - x_2 - x_2^3$ is asymptotically stable in the large. (08 Marks)

- c. Determine whether or not the following quadratic form is positive definite:

$$V(x) = 10x_1^2 + 4x_2^2 + x_3^2 + 2x_1x_2 - 2x_2x_3 - 4x_1x_3.$$

(06 Marks)

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