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10EC55

Fifth Semester B.E. Degree Examination, June/July 2017

Information Theory & Coding

Time: 3 hrs. Max. Marks:100

Note: *Answer FIVE full questions, selecting at least TWO questions from each part.*

PART – A

1 a. A black and white TV picture consists of 525 lines of picture information. Assume that each line consists of 525 picture elements (pixels) and that each element can have 256 brightness levels. Picture are repeated at the rate of 30 frames/sec. calculate the average rate of information conveyed by a TV set to a viewer. **(04 Marks)**

b. Obtain an expression for maximum entropy of a system. **(06 Marks)**

c. Design a system to report the heading of a collection of 400 cars. The heading levels are : heading straight (S), turning left (L) and turning right (R). This information is to be transmitted every second. Construct a model based on the test data given below.

(i) On the average during a given reporting interval, 200 cars were heading straight, 100 were turning left and remaining were turning right.

(ii) Out of 200 cars that reported heading straight, 100 of them reported going straight during the next reporting period, 50 of them turning left and remaining turning right during the next period.

(iii) Out of 100 cars that reported as turning during a signaling period, 50 of them continued their turn and remaining headed straight during the next reporting period.

(iv) The dynamics of the cars did not allow them to change their heading from left to right or right to left during subsequent reporting periods.

- Find the entropy of each state.
- Find the entropy of the system.
- Find the rate of transmission. **(10 Marks)**

2 a. The source emits the messages consisting of two symbols each. These messages and their probabilities are given in Table 1. Design the source encoder using Shannon's encoding algorithm and also find encoder efficiency. **(10 Marks)**

Table 1

Message M_i	AA	AC	CC	CB	CA	BC	BB
Probability P_i	$\frac{9}{32}$	$\frac{3}{32}$	$\frac{1}{16}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{9}{32}$

b. Find the minimum number of symbols, 'r' in the coding alphabet for devising an instantaneous code such that W = {0, 5, 0, 5, 5}. Devise such a code (Note: W represents the set of the code words of length 1, 2, 3, ...) **(10 Marks)**

3 a. A non-symmetric binary channel is shown in Fig.Q3 (a),

i) Find H(X), H(Y), $H\left(\frac{X}{Y}\right)$ and $H\left(\frac{Y}{X}\right)$ given $P(X = 0) = \frac{1}{4}$, $P(X = 1) = \frac{3}{4}$, $\alpha = 0.75$, $\beta = 0.9$

ii) Find the capacity of the binary symmetric channel if $\alpha = \beta = 0.75$ **(10 Marks)**

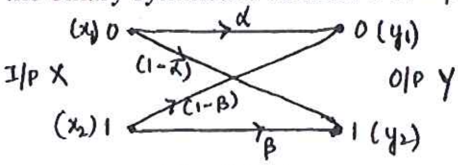

Fig. Q3 (a)

b. What is the entropy (η) of the image below, where numbers (0, 20, 50, 99) denote the gray level intensities? **(10 Marks)**

99	99	99	99	99	99	99	99
20	20	20	20	20	20	20	20
0	0	0	0	0	0	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	0	0	0	0	0	0

ii) Show step by step how to construct the Huffman tree to encode the above four intensity values in this image. Show the resulting code for each intensity value.
iii) What is the average number of bits needed for each pixel, using Huffman code?

4 a. State Shannon-Hartley law. Derive an expression for the upper limit on channel capacity as the bandwidth tends to infinity. **(10 Marks)**

b. An analog signal has a 4 kHz bandwidth. The signal is sampled at 2.5 times the Nyquist rate and each sample quantized into 256 equally likely levels. Assume that the successive samples are statistically independent.
   i) Find the information rate of this source.
   ii) Can the output of this source be transmitted without errors over a Gaussian channel of bandwidth 50 kHz and (S/N) ratio of 20 dB?
   iii) If the output of this source is to be transmitted without errors over an analog channel having (S/N) of 10 dB, compute the bandwidth requirement of the channel. **(10 Marks)**

PART – B

5 a. The parity check bits of a (7, 4) Hamming code are generated by,

$$C_5 = d_1 + d_3 + d_4$$
$$C_6 = d_1 + d_2 + d_3$$
$$C_7 = d_2 + d_3 + d_4$$

where d_1, d_2, d_3 and d_4 are the message bits.
   i) Find the generator matrix [G] and parity check matrix [H] for this code.
   ii) Prove that $GH^T = 0$.
   iii) The (n, K) linear block code so obtained has a "dual" code. This dual code is a (n, n-K) code having a generator matrix H and parity check matrix G. Determine the eight code – vectors of the "dual code" for the (7, 4) Hamming code described above.
   iv) Find the minimum distance of the dual code determined in part (iii). **(10 Marks)**

b. Design (n, K) Hamming code with a minimum distance of $d_{min} = 3$ and a message length of 4 bits. If the received code vector is R = [1 1 1 1 0 0 1]. Detect and correct the single error that has occurred due to noise. **(10 Marks)**

6 a. The expurgated (n, K-1) Hamming code is obtained from the original (n, K) Hamming code by discarding some of the code vectors. Let g(x) denote the generator polynomial of the original hamming code. The most common expurgated hamming code is the one generated by $g_1(x) = (1+x)g(x)$; where (1+x) is a factor of $1+x^n$. Consider the (7, 4) Hamming code generated by $g(x) = 1+x^2+x^3$,

(i) Construct the eight code vectors in the expurgated (7, 3) Hamming code, assuming a systematic format. Hence, show that the minimum distance of the code is 4.

(ii) Determine the generator matrix G_1 and the parity check matrix H_1 of the expurgated hamming code.

(iii) Devise the encoder for the expurgated hamming code and list the shift register contents in a tabular fashion for the message 011. Verify the code-vector so obtained using $[V] = [D][G_1]$.

(iv) Devise the syndrome calculator for the expurgated hamming code. Hence, determine the syndrome for the received vector 0111110. Also correct the error, if any, in that received vector. **(14 Marks)**

b. The generator polynomial for (15, 7) cyclic code is $g(x) = 1 + x^4+x^6+x^7+x^8$

(i) Find the code-vector in systematic form for the message $D(x) = x^2+x^3+x^4$.

(ii) Assume that the first and last bit of the code-vector V(x) for $D(x) = x^2+x^3+x^4$ suffer transmission errors. Find the syndrome of V(x). **(06 Marks)**

7 a. Consider the binary convolutional encoder shown in Fig. Q7 (a). Draw the state table, state transition table, state diagram and the corresponding code tree. Using the code tree, find the encoded sequence for the message (1 0 1 1 1). Verify the output sequence so obtained using transform domain approach. **(10 Marks)**

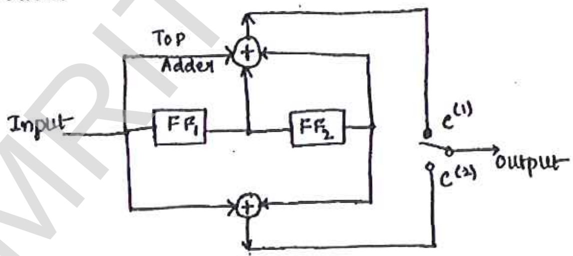

Fig. Q7 (a)

b. Consider the (3, 1, 2) convolutional code with $g^{(1)} = (1\ 1\ 0)$, $g^{(2)} = (1\ 0\ 1)$ and $g^{(3)} = (1\ 1\ 1)$.

(i) Draw the encoder block diagram.

(ii) Find the generator matrix.

(iii) Find the code-word corresponding to the information sequence (1 1 1 0 1) using time-domain and transform-domain approach. **(10 Marks)**

8 Write short notes on the following:

a. RS codes.

b. Golay codes.

c. BCH codes.

d. Burst error correcting codes. **(20 Marks)**

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