CBCS SCHEME

USN | | | | | | | | | | | 18EC44

Fourth Semester B.E. Degree Examination, July/August 2021
Engineering Statistics and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: *1. Answer any FIVE full questions.*
2. Use of normalized Gaussian Random variable table is permitted.

1 a. Define a random variable and discuss the following terms associated with random variables:
i) Sample space
ii) Probability Mass function
iii) Probability density function
iv) Cumulative distribution function. **(05 Marks)**

b. Given the data in the following table:

K	Y_K	$P(Y_K)$
1	2.1	0.20
2	3.2	0.21
3	4.8	0.19
4	5.4	0.14
5	6.9	0.26

i) Plot the Pdf and Cdf of the discrete random variable Y.
ii) Compute the mean and variance of Y. **(08 Marks)**

c. If 'X' is a random variable uniformly distributed in the interval [a b], obtain the expression for mean, variance and mean square value. **(07 Marks)**

2 a. The Pdf for a random variable Y is $f_y(Y) = 1.5(1 - y^2)$ $0 < y < 1$ what are mean, mean square, variance of Y. **(08 Marks)**

b. It is given that $E[X] = 2.0$ and that $E[X^2] = 6$.
i) Find the standard deviation of X
ii) If $Y = 6X^2 + 2X - 13$ find μ_Y **(06 Marks)**

c. The normalized Gaussian random variable is given as

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad -\alpha < x < \alpha$$

Obtain the characteristic function for this random variable. **(06 Marks)**

3 a. A bivariate Pdf is given as
$f_{XY}(x, y) = 0.2\delta(x)\,\delta(y) + 0.3\delta(x-1)\,\delta(y) + 0.3\delta(x)\,\delta(y-1) + C\delta(x-1)\,\delta(y-1)$
i) What is the value of the constant C?
ii) What are the Pdfs for X and Y?
iii) What is $F_{XY}(x, y)$ when $(0 < x < 1)$ and $(0 < y < 1)$?
iv) What are $F_{XY}(x, \alpha)$ and $F_{XY}(\alpha, y)$
v) Are X and Y independent? **(08 Marks)**

b. The mean and variance of random variable X are -1 and 2. The mean and variance of random variable Y are 3 and 4. The correlation coefficient $\rho_{XY} = 0.5$. What are the covariance COV[XY] and the correlation E[XY]. **(05 Marks)**

c. Write a short note on Chi-square random variable and students random variable. **(07 Marks)**

4 a. X is a random variable, $\mu_X = 4$ and $\sigma_X = 5$, Y is a random variable, $\mu_Y = 6$ and $\sigma_Y = 7$. The correlation coefficient is 0.2. If U = 3X + 2Y. What are var[u], cov[uX] and cov[uY]? **(08 Marks)**

b. Let 'X' and 'Y' be exponentially distributed random variable with $f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$

Obtain the characteristic function and Pdf of W = X + Y. **(06 Marks)**

c. The Random variables X_i have same mean of $m_x = 4$ and variance of $\sigma_X^2 = 1.5$. For $w = \sum_{i=1}^{150} X_i$, determine m_w and σ_w^2. Also for $w = \frac{1}{150} \sum_{i=1}^{150} X_i$, determine m_y and $\sigma^2 y$. Comment on the result. **(06 Marks)**

5 a. Briefly explain the following terms:
i) Random process
ii) Stationary process
iii) Ergodic process. **(04 Marks)**

b. Discuss Auto correlation and Auto covariance function of a random process. Also mention properties of Auto correlation function. **(06 Marks)**

c. A random process is described by $X(t) = A \cos(w_c t + \theta)$ where A, w_c are constants and 'θ' is a random variable uniformly distributed between $\pm\pi$. Is X(t) wide sense stationary? If so, then what are the mean and the auto correlation function for the random process? **(10 Marks)**

6 a. Let X(t) and Y(t) be two independent jointly wide sense stationary random process defined as $X(t) = A \cos(w_1 t + \theta_1)$, $Y(t) = B\cos(w_2 t + \theta_2)$. Here θ_1 and θ_2 are dependent random variables distributed uniformly between $-\pi$ to π obtain the auto correlation function of the multiplication process. w(t) = Y(t) X(t). **(06 Marks)**

b. Establish a relationship between power spectrum and Auto-correlation function. **(06 Marks)**

c. The power spectral density (Psd) of a wide sense stationary random process is given as

$$S_X(w) = \begin{cases} 2\cos\left(\frac{\pi w}{2w_m}\right) & \text{for } -w_{m-} < w \leq w_m \\ 0 & \text{otherwise} \end{cases}$$

The Psd of a carrier random process is given as $s_c(w) = 10\pi[\delta(w - w_0) + \delta(w + w_0)]$ where $w_0 >> w_m$. Obtain the Psd of the modulated signal. **(08 Marks)**

7 a. Write the vector U = (1, 3, 9) as a linear combination of the vectors u_1 = (2, 1, 3), u_2 = (1, -1, 1), u_3 = (3, 1, 5). **(05 Marks)**

b. Determine whether or not each of the following forms a basis, x_1 = (2, 2, 1), x_2 = (1, 3, 7), x_3 = (1, 2, 2) in R^3. **(05 Marks)**

c. Determine whether the given transformation is linear or not
$T : R^2 \to R^3$
T(x, y) = [x-y, x+y, 2x] **(10 Marks)**

8 a. What is a vector space? Define the four fundamental vector spaces. **(05 Marks)**

b. Apply Gramschmidt process to the vectors v_1 = (2, 2, 1), v_2 = (1, 3, 1), v_3 = (1, 2, 2) to obtain an orthonormal basis for $v_3(R)$ with standard inner product. **(10 Marks)**

c. Let w be the subspace of R^5 spaced by x_1 = (1, 2, -1, 3, 4), x_2 = (2, 4, -2, 6, 8), x_3 = (1, 3, 2, 2, 6), x_4 = (1, 4, 5, 1, 8), x_5 = (2, 7, 3, 3, 9). Find the dimension of w. **(05 Marks)**

9 a. If a 4 × 4 matrix has det $A = \frac{1}{2}$, find: i) det (2A) ii) det (-A) iii) det (A^2) iv) det (A^{-1}). **(04 Marks)**

b. By applying row operations, produce an upper triangular matrix and hence compute determinant of the matrix.

$$B = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

(06 Marks)

c. Find a matrix P which dragonalizes a matrix $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Hence verify that $D = P^{-1}AP$ and compute A^4. **(10 Marks)**

10 a. If $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ find eigen values and eigen vectors for the matrix A. Can the matrix be diagonalized? **(10 Marks)**

b. Compute A^TA and AA^T. Find eigen values and unit eigen vectors for $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$, multiply the three matrices $u \Sigma V^T$ to recover A. **(10 Marks)**

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