CBCS SCHEME

USN | | | | | | | | | | | 18EC45

Fourth Semester B.E. Degree Examination, July/August 2021
Signals and Systems

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

1 a. Determine each of the following:

i) $f(t) = e^{-3(t-1)}\delta(t)$

ii) $f(t) = \int_{-1}^{1} \delta(t^2 - 4)dt$

iii) $f(t) = \int_{-\infty}^{\infty} \delta(t^2 - 4)dt$ **(06 Marks)**

b. Sketch the waveforms for the following signals:

i) $x_1(t) = u(t) - u(t-3)$

ii) $x_2(t) = u(t+2) - 2u(t) + u(t-2)$

iii) $x_3(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

iv) $x_4(t) = r(t+1) - r(t) + r(t-2)$

v) $x_5(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$ **(06 Marks)**

c. Find the even and odd parts of i) $x(n) = u(n)$ ii) $x(n) = \alpha^n u(n)$. **(08 Marks)**

2 a. A trapezoidal pulse, x(t) is defined by

$$x(t) = \begin{cases} 5-t, & 4 \le t \le 5 \\ 1, & -4 \le t \le 4 \\ t+5, & -5 \le t \le -4 \\ 0, & \text{otherwise} \end{cases}$$

This pulse is applied to a differentiation having the input-output relation:

$y(t) = \dfrac{dx(t)}{dt}$. Find the energy of the signal y(t). **(06 Marks)**

b. Evaluate the following integrals:

i) $\int_{-1}^{1} (3t^2 + 1)dt$

ii) $\int_{-\infty}^{\infty} (t^2 + \cos\pi t)\delta(t-1)dt$

iii) $\int_{-\infty}^{\infty} e^{-t}\delta(2t-2)dt$ **(06 Marks)**

c. i) Sketch x(n) and then plot its even and odd parts x(n) = 1 + u(n).
ii) For the signal x(t) shown in Fig.Q.2(c), sketch $x_e(t)$ and $x_0(t)$. **(08 Marks)**

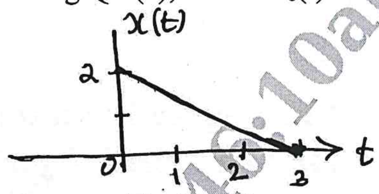

Fig.Q.2(c)

3 a. Prove the following:
i) $x_1(n)*h_1(n) = h_1(n)*x_1(n)$
ii) $[x_1(n)*h_1(n)]*h_2(n) = x_1(n)*[h_1(n)*h_2(n)]$
iii) $x_1(n)*[h_1(n)+h_2(n)] = x_1(n)*h_1(n) + x_1(n)*h_2(n)$ **(08 Marks)**

b. Prove the following:
i) $x(n)*\delta(n-n_0) = x(n-n_0)$
ii) $\delta(n-\alpha)*\delta(n-\beta) = \delta(n-\alpha-\beta)$ **(04 Marks)**

c. Convolute the two continuous-time signals:
i) $x_1(t) = e^{-2t}u(t)$
ii) $x_2(t) = u(t+2)$ **(08 Marks)**

4 a. An LTI system is characterized by an impulse response, $h(n) = \left(\frac{3}{4}\right)^n u(n)$. Find the step response of the system. **(08 Marks)**

b. Convolute the sequences given below:
i) $x_1(n) = \alpha^n u(n)$
ii) $x_2(n) = \beta^n u(n)$ **(08 Marks)**

c. Show that:
$[x_1(t)*h_1(t)]*h_2(t) = x_1(t)*[h_1(t)*h_2(t)]$ **(04 Marks)**

5 a. Show that an LTI system is BIBO stable if and only if its impulse response h(n) is absolutely summable. That is, $\sum_{k=-\infty}^{\infty}|h(k)| < \infty$. **(06 Marks)**

b. State and prove the following properties of Fourier series:
i) Time-shift ii) Time differentiation. **(06 Marks)**

c. Find the Fourier coefficient X(k) for a periodic signal $x(t) = |\sin \pi t|$. **(08 Marks)**

6 a. Investigate causality and stability of the following systems:
(i) $h(t) = e^{-2|t|}$ (ii) $h(n) = 2^n u(n-1)$ **(06 Marks)**

b. Prove the convolution property of Fourier series. **(04 Marks)**

c. Find the Fourier coefficient X(k) for the periodic signal shown in Fig.Q.6(c). Also, plot magnitude and phase spectra. **(10 Marks)**

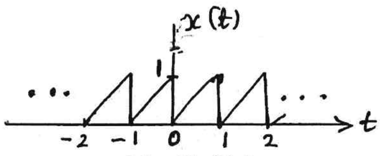

Fig.Q.6(c)

7 a. Prove the following properties of Fourier transform:
i) Time-domain convolution ii) Frequency – shift. (06 Marks)

b. Find the Fourier transform of

$$x(t)=\begin{cases}1, & |t|<a\\ 0, & |t|>a\end{cases}$$

Also, sketch magnitude and phase spectra. (08 Marks)

c. Find Fourier transform of signum function, $x(t)=\text{sgn}(t)$. (06 Marks)

8 a. Prove the following properties of Discrete-Time Fourier Transform (DTFT):
i) Time-shift ii) Time-domain convolution. (06 Marks)

b. Find the DTFT of $x(n)=a^{|n|}, |a|<1$ Also, sketch magnitude and phase spectra. (08 Marks)

c. Find inverse DTFT of $X(e^{j\Omega})=\dfrac{3-\frac{1}{4}e^{-j\Omega}}{-\frac{1}{16}e^{-j2\Omega}+1}$ (06 Marks)

9 a. Prove the following properties of Z-transform:
i) Multiplication by 'n' (Ramp) ii) Time-domain convolution. (06 Marks)

b. Find the inverse Z-transform of $X(z)=\dfrac{z}{3z^2-4z+1}$ for the ROCs:

i) $|z|>1$ ii) $|z|<\frac{1}{3}$ iii) $\frac{1}{3}<|z|<1$ (08 Marks)

c. Find Z-transform of the sequences given below, using properties

i) $x(n)=\left(\frac{1}{2}\right)^n u(n)*\left(\frac{1}{3}\right)^n u(n)$

ii) $x(n)=n\,u(n)$ (06 Marks)

10 a. Find the Z-transform of $x(n)=\sin(\Omega_0 n)u(n)$. (04 Marks)

b. Find the inverse Z-transform of $X(z)=\dfrac{z+1}{(z-1)^2\left(z-\frac{1}{2}\right)}$; ROC: $|z|>1$. (08 Marks)

c. A causal LTI system is described by $y(n)-\frac{3}{4}y(n-1)+\frac{1}{8}y(n-2)=x(n)$, where x(n) and y(n) are the input and output of the system, respectively.
i) Find the system function, H(z).
ii) Find the impulse response, h(n).
iii) Find the BIBO stability of the system.
iv) Find the frequency response of the system. (08 Marks)

* * * * *