CBCS SCHEME

USN

16/17MCA15

First Semester MCA Degree Examination, Dec.2019/Jan.2020

Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. What are the contrapositive, the converse and the inverse of the conditional statement "The home team wins whenever it is raining". **(06 Marks)**

b. Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. **(05 Marks)**

c. Show that the following argument is valid :
$p \to q,\ \neg p \to r,\ r \to s\ \therefore\ \neg q \to s$. **(05 Marks)**

OR

2 a. Show that the premises "Everyone in the discrete mathematics class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science". Use rules of inference for quantitied statements. **(08 Marks)**

b. Show that $(p \wedge q) \to (p \vee q)$ is a tautology. **(04 Marks)**

c. Draw the truth table to show that $p \to q$ and $\neg p \vee q$ are logically equivalent. **(04 Marks)**

Module-2

3 a. What is the Cartesian product of $A \times B \times C$ where $A = \{0, 1\}$, $B = \{1, 2\}$, $C = \{0, 1, 2\}$? **(03 Marks)**

b. What are the truth sets of the predicates P(x), Q(x) and R(x), where the domain is the set of integers and P(x) is "$|x| = 1$", Q(x) is "$x^2 = 2$", R(x) is "$|x| = x$". **(06 Marks)**

c. Using set builder notation, prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$. **(07 Marks)**

OR

4 a. Use membership table to show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **(08 Marks)**

b. Let $f_1 : R \to R$, $f_2 : R \to R$ such that $f_1(x) = x^2$, $f_2(x) = x - x^2$. Find $f_1 + f_2$, $f_1 f_2$. **(04 Marks)**

c. What is the composite SoR of the relations R and S, where
R = {(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)}
S = {(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)}
Also find RoS. **(04 Marks)**

Module-3

5 a. In how many ways can we select three students from a group of five students to stand in a line for a picture? In how many ways can we arrange all five students in a line? **(05 Marks)**

b. Find the coefficient of $x^{12}y^{13}$ in the expansion of $(2x - 3y)^{25}$. **(05 Marks)**

c. What is the solution of recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$. **(06 Marks)**

OR

6 a. If N objects are placed in K boxes, then there is atleast one box containing atleast $\lceil N/K \rceil$ objects. **(06 Marks)**

b. A group of 30 people have been trained as astronauts to go on the first mission to Mars. How many ways are there to select a crew of 6 people to go on this mission? **(05 Marks)**

c. What is the solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with $a_0 = 1$ and $a_1 = 6$? **(05 Marks)**

Module-4

7 a. 5 cards are drawn at random from a deck of 52 cards. What is the probability of getting :
i) 3 Aces and 2 Jacks
ii) 3 Aces and a pair
iii) A full house (A full house means 3 of one kind and a pair). **(12 Marks)**

b. Box A contains 5 Red and 3 White marbles and Box B contains 4 Red and 6 White marbles. If a marble is drawn from each box, what is the probability that they both are of the same color? **(04 Marks)**

OR

8 a. A box contains 9 tickets numbered from 1 to 9 inclusive. If 3 tickets are drawn from the box one at a time (without replacement), find the probability that the three tickets are alternately either odd, even, odd or even, odd, even. **(08 Marks)**

b. A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further 103 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French and Russian. How many students have taken a course in all 3 languages? (Solve using principle of inclusion – Exclusion and Venn Diagram). **(04 Marks)**

c. If a fair die is rolled, what is the probability of getting i) a 5 or a 6 ii) an even number? **(04 Marks)**

Module-5

9 a. Define the following terms :
i) Simple graph
ii) Pseudograh
iii) Directed multi-graph
iv) Collaboration graph
v) Mixed graph. **(05 Marks)**

b. Develop the adjacency list for the following directed graph : (Ref. Fig. Q9(b)) **(05 Marks)**

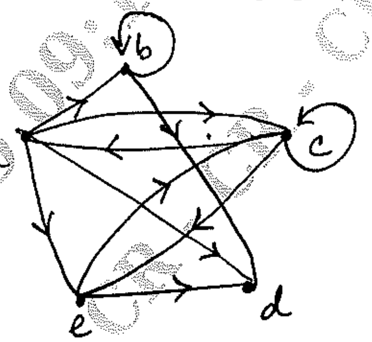

Fig.Q9(b)

c. Use Dijkstra's algorithm to find the length of a shortest path between the vertices a and z in the weighted graph shown below in Fig.Q9(c). **(06 Marks)**

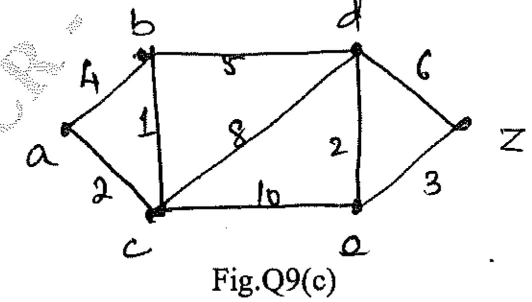

Fig.Q9(c)

OR

10 a. Use an adjacency matrix to represent the graph shown in Fig.Q10(a) below. **(04 Marks)**

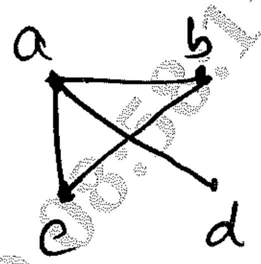

Fig.Q10(a)

b. Which of the simple graphs below have a Hamiltonian circuit or a Hamiltonian path? (Ref. Fig. Q10(b)). **(06 Marks)**

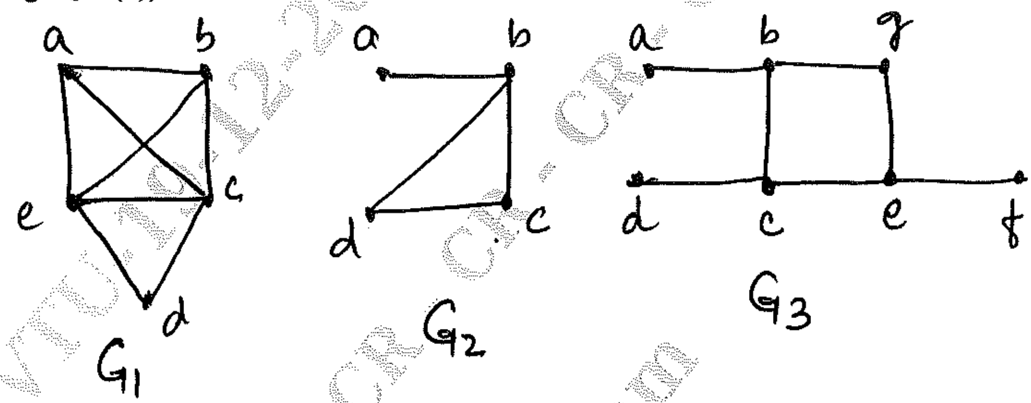

Fig.10(b)

c. Solve the travelling salesman problem for this graph by finding the total weight of all Hamiltonian circuits and determining a circuit with minimum total weight. (Ref. Fig.Q10(c)).

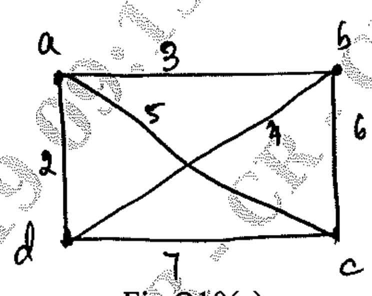

Fig.Q10(c)

(06 Marks)

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