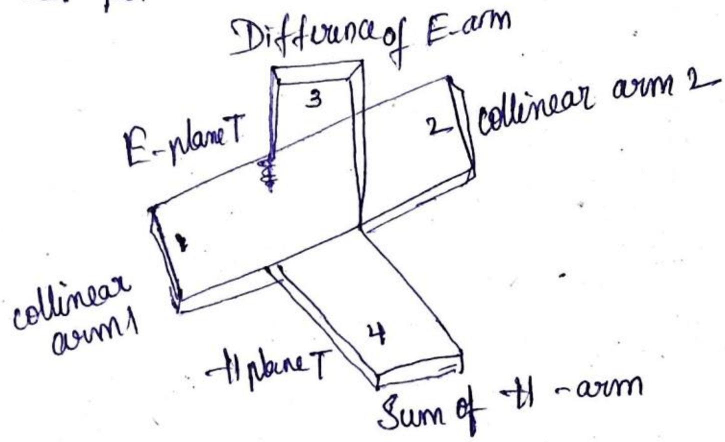
1) Magic tee is a combination of E plane tee and H-plane tee

2) In this waveguide is cut at both width and breadth and side arms in the direction of magnetic field and electric field are inserted respectively.

3) ports 1 and 2 form collinear ports. port 3 is called H-arm and port 4 is called E-arm



S matrix :-

$$[S]_{MT} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad -(1)$$

$S_{13} = S_{23}$ — (2) [∵ from H-plane tee action]

$S_{14} = -S_{24}$ — (3) [∵ from E-plane tee action]

Due to symmetry, power fed to the port 3 cannot come out of port 4 and vice versa

$$S_{34} = S_{43} = 0 \quad -(4)$$

Assume ports (3) and (4) are matched,

$$S_{33} = S_{44} = 0 \quad - (5)$$

Magic tee is reciprocal - Hence symmetric property also holds good. $S_{ij} = S_{ji} \quad - (6)$

Using (2) to (6) equations in eq-(1). The S-matrix simplifies to the following matrix:

$$[S]_{MT} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \quad - (7)$$

$$[S][S]^* = [U] \quad - (8)$$ from unitary property

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{13} & -S_{14} \\ S_{13} & S_{13} & 0 & 0 \\ S_{14} & -S_{14} & 0 & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* & S_{14}^* \\ S_{12}^* & S_{22}^* & S_{13}^* & -S_{14}^* \\ S_{13}^* & S_{13}^* & 0 & 0 \\ S_{14}^* & -S_{14}^* & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad - (9)$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad - (10)$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \quad - (11)$$

$$|S_{13}|^2 + |S_{13}|^2 = 1 \quad - (12)$$

$$|S_{14}|^2 + |S_{14}|^2 = 1 \quad - (13)$$

$$\rightarrow \quad S_{13} S_{11}^* + S_{13} S_{12}^* = 0 \quad - (14)$$

From (12), $2|S_{13}|^2 = 1$

$$S_{13} = 1/\sqrt{2} \quad - (15)$$

From (13), $2|S_{14}|^2 = 1$

$$S_{14} = 1/\sqrt{2} \quad - (16)$$

Using (15) & (16) in (10) & (11)

$$|S_{11}|^2 + |S_{12}|^2 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right) = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 0 \quad — (17)$$

$$|S_{12}|^2 + |S_{22}|^2 = 0 \quad — (18)$$

$$\rightarrow \quad S_{11} = -S_{22} \quad — (19)$$

Equation (17) is possible only when $S_{11} = S_{12} = 0$

$$[S]_{MT} = \begin{bmatrix} 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \end{bmatrix}$$

6) a) Since port 2 and 3 are matched, terminated

a_2 and $a_3 = 0$.

Total effective power input to port 1, $P_1 = |a_1|^2 - |b_1|^2$

$$P_1 = |a_1|^2 - |a_1|^2 |S_{11}|^2$$

$$= |a_1|^2 (1 - |S_{11}|^2)$$

$$= 20m \left(1 - \left(\frac{1}{2}\right)^2\right)$$

$$= 20 \times 10^{-3} \left(1 - \frac{1}{4}\right)$$

$$= \overset{5}{\cancel{20}} \times 10^{-3} \left(\frac{3}{\cancel{4}_1}\right)$$

$$= 15 mW$$

Power transmitted to port 3

$$P_3 = |a_1|^2 |S_{31}|^2$$

$$= 20 \times 10^{-3} \left(\frac{1}{\sqrt{2}}\right)^2$$

$$P_3 = 10 mW$$

Power delivered to port 2, $P_2 = |a_1|^2 |S_{21}|^2$

$$= 20 \times 10^{-3} \left(\frac{-1}{2}\right)^2$$

$$= \frac{20 \times 10^{-3}}{4}$$

$$= 5mW$$

b) i) Insertion loss = $10\log\left(\frac{P_i}{P_o}\right) = 10\log\left(\frac{|a_1|^2}{|b_2|^2}\right)$

$$= 10 \log \left(\frac{|a_1|}{|b_2|}\right)^2$$

$$= 20 \log \left(\frac{1}{(|b_2|/|a_1|)}\right) = 20\log\left(\frac{1}{S_{21}}\right)$$

ii) Transmission loss $= 10 \log \left(\frac{P_i - P_r}{P_o} \right)$

$$= 10 \log \left(\frac{P_i \left(1 - \frac{P_r}{P_i}\right)}{P_o} \right)$$

$$= 10 \log \left(\frac{1 - P_r/P_i}{P_o/P_i} \right)$$

$$= 10 \log \left(\frac{1 - |S_{11}|^2}{|S_{12}|^2} \right)$$

iii) Reflection loss $= 10 \log \left(\frac{P_i}{P_i - P_r} \right)$

$$= 10 \log \left(\frac{1}{1 - \frac{P_r}{P_i}} \right)$$

$$= 10 \log \left(\frac{1}{1 - |S_{11}|^2} \right)$$

iv) Return loss $= 10 \log \left(\frac{P_i}{P_r} \right)$

$$= 10 \log \left(\frac{1}{P_r/P_i} \right)$$

$$= 10 \log \left(\frac{1}{|S_{11}|^2} \right)$$

4) Given

$$[S] = \begin{bmatrix} 0.15 \angle 0^\circ & 0.85 \angle -45^\circ \\ 0.85 \angle 45^\circ & 0.2 \angle 0^\circ \end{bmatrix}$$

As [S] is not symmetrical. So the network is not reciprocal

To be lossless, the network must satisfy unitary property.

$$\sum_{K=1}^{N} S_{Ki} S_{Ki}^{*} = 1$$

If $i = 1$

$$|S_{11}|^2 + |S_{21}|^2 = (0.15)^2 + (0.85)^2$$

$$= 0.745 \neq 1.$$

So the network is not lossless.

Case 1:- When port 2 is terminated with a matched load, the reflection coefficient seen at port 1 is $\Gamma = S_{11} = 0.15$

$$\text{Return loss} = -20 \log |\Gamma|$$

$$= -20 \log (0.15)$$

$$= 16.5 \text{ dB}.$$

Case 2:- When port 2 is short circuited,

$$V_2 = 0 \Rightarrow V_2^+ + V_2^- = 0$$

$$V_2^+ = -V_2^-$$

from the scattering matrix,

As $\cancel{V_2^+ = S_{21} V_1^+ + S_{22} V_2^+}$ — (1) $\quad \cancel{V_1^+ = S_{11} V_1^+ + S_{12} V_2^+}$ — (3)

$\cancel{V_2^- = S_{21} V_1^+ - S_{22} V_2^-}$ — (2) $\quad \cancel{V_1^- = S_{11} V_1^+ - S_{12} V_2^-}$ — (4)

As $\Gamma = \dfrac{V_1^-}{V_1^+} \Rightarrow$ Divide (3) by V_1^+

from scattering matrix,

$$V_2^+ = S_{21} V_1^+ + S_{22} V_2^+ \quad \text{— (1)}$$

$$V_2^- = S_{21} V_1^+ - S_{22} V_2^- \quad \text{— (2)}$$

$$V_1^+ = S_{11} V_1^+ + S_{12} V_2^+ \quad \text{— (3)}$$

$$V_1^- = S_{11} V_1^+ - S_{12} V_2^+ \quad \text{— (4)}$$

for $\Gamma = \frac{V_1^-}{V_1^+}$

Divide ④ by V_1^+, we get :-

$$\frac{V_1^-}{V_1^+} = \frac{S_{11} V_1^+ + S_{12} V_2^+}{V_1^+}$$

$$\frac{V_1^-}{V_1^+} = S_{11} - S_{12} \frac{V_2^+}{V_1^+} \quad \text{— ⑤}$$

If we divide ② by V_1^+, we get

$$\frac{V_2^-}{V_1^+} = \frac{S_{21} V_1^+ - S_{22} V_2^-}{V_1^+}$$

$$\frac{V_2^-}{V_1^+} = S_{21} - S_{22} \frac{V_2^-}{V_1^+}$$

$$\frac{V_2^-}{V_1^+} (1+S_{22}) = S_{21}$$

$$\frac{V_2^-}{V_1^+} = \frac{S_{21}}{1+S_{22}}$$

$$\therefore \Gamma = \frac{V_1^-}{V_1^+} = S_{11} - S_{12} \left(\frac{S_{21}}{1+S_{22}}\right)$$

$$= 0.15 - \frac{(0.85 \angle 45^\circ)(0.85 \angle 45^\circ)}{1+0.2}$$

$$= 0.15 - (0.602)$$

$$= -0.452$$

Return loss $= -20 \log |\Gamma|$

$= -20 \log (+0.452)$

$=$ ~~69 dB~~ 6.9 dB

State and prove symmetric and unitary properities of S-matrix ?

The [S] matrix for a reciprocal network is symmetric.

Assuming the characteristic impedances (Z_{ON}) of all the ports are identical.

Also setting $Z_{on} = 1$

3) The total voltage and current at the n^{th} port can be written as,

$$V_n = V_n^+ + V_n^- \quad \text{——} \quad ①$$

$$I_n = V_n^+ - V_n^- \quad \text{——} \quad ②$$

Adding ① & ② we obtain,

$$2V_n^+ = V_n + I_n$$

$$V_n^+ = \frac{V_n + I_n}{2}$$

$$[V_n^+] = \frac{1}{2}([V] + [I])$$

$$= \frac{1}{2}([Z][I] + [I])$$

$$= \frac{1}{2}\{[Z] + [U]\}[I]$$

Subtracting eq ② from eq-①, we obtain

$$2V_n^- = V_n - I_n$$

$$V_n^- = \frac{1}{2}(V_n - I_n)$$

$$[V^-] = \frac{1}{2}([V] - [I])$$

$$= \frac{1}{2}\{[Z][I] - [I]\}$$

$$[V^-] = \frac{1}{2}\{[Z] - [U]\}[I] \quad - (4)$$

Dividing $\frac{(4)}{(3)}$

$$\frac{[V^-]}{[V^+]} = \{[Z] - [U]\}\{[Z] + [U]\}^{-1}$$

$$[V^-] = \{[Z] - [U]\}\{[Z] + [U]\}^{-1}[V^+] \quad - (5)$$

So that,

$$[S] = \{[Z] - [U]\}\{[Z] + [U]\}^{-1} \quad - (6)$$

Taking transpose of (6) gives

$$[S]^t = \{([Z] - [U])^t\}\{([Z] + [U])^t\}^{-1}$$

Now, $[U]$ is diagonal. So $[U]^t = [U]$ and if the network is reciprocal, $[Z]$ is symmetric. So that,

$$[Z]^t = [Z].$$

The above then reduces to

$$[S]^t = ([Z] - [U])([Z] + [U])^{-1} \quad - (7)$$

Comparing equations (6) and (7) we obtain $[S]^t = [S]$ for reciprocal networks.

the [S] matrix for a lossless network is unitary

→ If the network is lossless, then no real power can be delivered to this network.

→ Thus if the characteristic impedances of all the ports are identical and assumed to be unity. The average power delivered to the network is

$$P_{av} = \frac{1}{2} Re \{ [V^+]^t [I]^* \}$$

$$= \frac{1}{2} Re \{ ([V^+]^t + [V^-]^t)([V^+]^* - [V^-]^*) \}$$

$$= \frac{1}{2} Re \{ [V^+]^t [V^+]^* - [V^+]^t [V^-]^* + [V^-]^t [V^+]^* - [V^-]^t [V^-]^* \}$$

Since the terms $-[V^+]^t [V^-]^* + [V^-]^t [V^+]^*$ are of the form $A - A^*$ and so are pure imaginary

$$P_{av} = \frac{1}{2} [V^+]^t [V^+]^* - \frac{1}{2} [V^-]^t [V^-]^* = 0 \quad - (8)$$

In equation (8), the term $\frac{1}{2} [V^+]^t [V^+]^*$ represents the total incident power and $\frac{1}{2} [V^-]^t [V^-]^*$ represents the total reflected power.

So for a lossless network or junction, we have the intuitive result that the incident and reflected powers are equal.

$$[V^+]^t [V^+]^* = [V^-]^t [V^-]^* \quad - (9)$$

Using $[V^-] = [S][V^+]$. In (9) gives.

$$[V^+]^t [V^+]^* = [S]^t [V^+]^t [S]^* [V^+]^*$$

If $[V^+]$ is non-zero,

$$[S]^t [S]^* = [U] \text{ or } [S]^* = \{ [S]^t \}^{-1}$$

A matrix that sacrifies the condition of (11) is called a unitary matrix

The matrix equation of (10) can be written in summation form

as
$$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = \delta_{ij} \quad \text{for all } i, j \quad — (12)$$

where $\delta_{ij} = \begin{cases} 1 , & \text{if } i=j \\ 0 , & \text{if } i \neq j \end{cases}$ is the

Kronecker delta symbol.

Thus, if $i = j$ (12) reduces to

$$\sum_{k=1}^{N} S_{ki} S_{ki}^{*} = 1 \quad — (13)$$

where if $i \neq j$ (12) reduces to

$$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 0 \quad — (14)$$

(13) is the dot product of any column of [S] with the conjugate of that column gives unity, while (14) is that the dot product of any column with the conjugate of a different column gives zero.

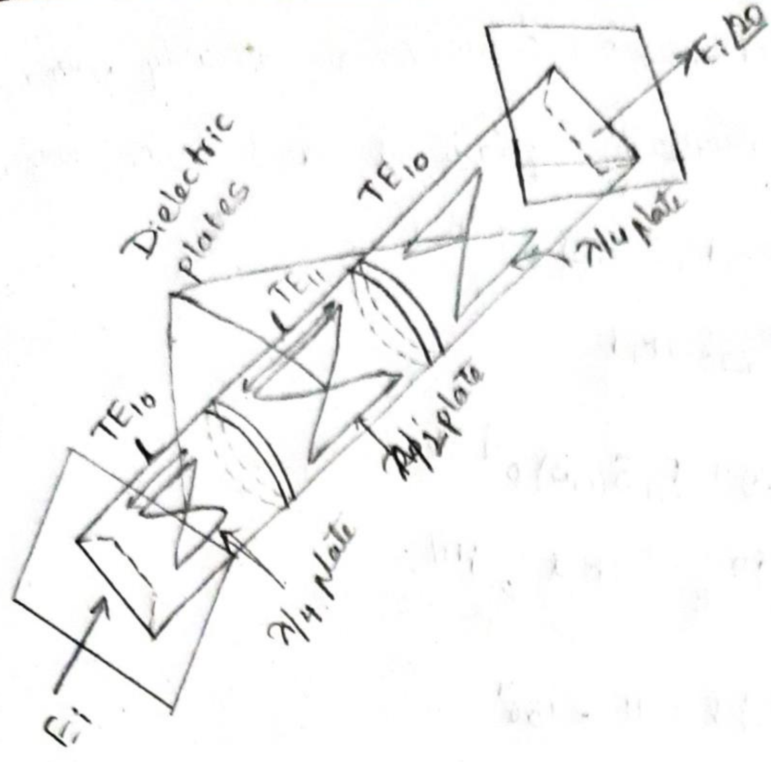
8) Precision type phase shifter :-

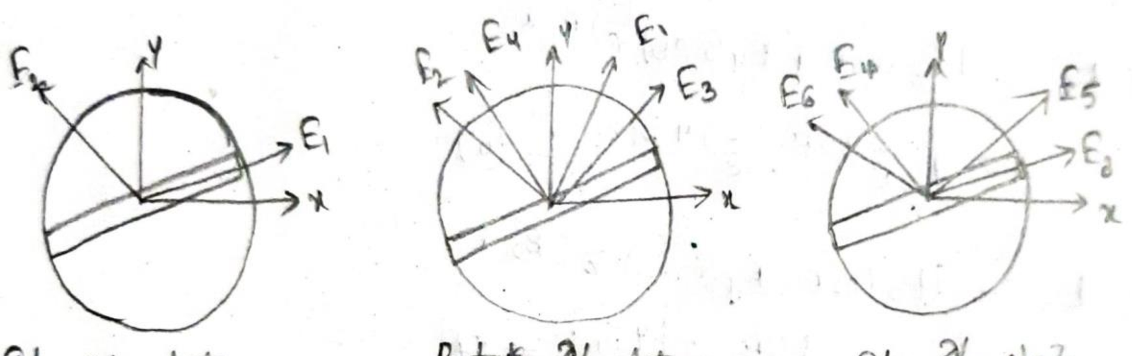
It uses a section of circular waveguide containing a lossless dielectric plate of length $2l = \frac{\lambda g}{2}$ called half wave section.

It can be rotated over 360° precisely between two sections of circular to rectangular waveguide transmissions A and B each has a dielectric plates of length $l = \frac{\lambda g}{4}$ called quarter-wave sections oriented at an angle of 45° with respect to the broad wall of rectangular wave guide ports at the input and output.

The incident TE_{10} wave in the rectangular guide becomes a TE_{11} wave in the circular guide the half wave section C produces a phase guide ~~the half wave~~ shift equal to twice the rotation angle θ with respect to the quarter wave section.

The dielectric plates are tapered through a length of quarter wavelength as both ends for reducing reflection due to dis-continuity.





The TE_{11} mode incident field E_i in the input quarter wave section can be decomposed into two transverse components on E_1 polarized parallel and other E_2 perpendicular to quarter wave plate

$$E_1 = E_i \cos 45^\circ e^{-j\beta_1 l} = E_0 e^{-j\beta_1 l}$$

$$E_2 = E_i \sin 45^\circ e^{-j\beta_2 l} = E_0 e^{-j\beta_2 l}$$

$$\rightarrow E_0 = \frac{E_i}{\sqrt{2}}$$

The difference in phase change $(\beta_1 - \beta_2) l = 90^\circ$

$$\therefore E_1 = E_0 e^{-j\beta_1 l}$$

$$E_2 = E_0 e^{-j\beta_1 l} \cdot e^{j\pi/2} = E_0 e^{-j(\beta_1 l - \pi/2)}$$

The quarter wave section converts a linearly polarized TE_{11} wave to circularly polarized and vice versa.

$$E_3 = (E_1 \cos\theta - E_2 \sin\theta)\, e^{-j2\beta_2 l}$$

$$= E_0 e^{-j\theta} e^{j3\beta_1 l}$$

$$E_4 = (E_2 \cos\theta + E_1 \sin\theta)\, e^{-j2\beta_2 l}$$

$$= E_0 e^{-j\theta} e^{-j3\beta_1 l} e^{-j\pi/2}$$

$$\rightarrow 2(\beta_1 - \beta_2) l = \pi = 180^\circ$$

$$-2\beta_2 l = \pi - 2\beta_1 l$$

$$\rightarrow E_5 = (E_3 \cos\theta + E_4 \sin\theta)\, e^{-j\beta_1 l}$$

$$= E_0 e^{-j2\theta} \cdot e^{-j4\beta_1 l} \quad - (6)$$

$$E_6 = (E_3 \cos\theta - E_4 \sin\theta)\, e^{-j\beta_2 l}$$

$$= E_0 e^{-j2\theta} e^{-j4\beta_1 l} \quad - (7)$$

$$\Rightarrow \frac{E_i}{\sqrt{2}} = E_0$$

$$\boxed{\sqrt{2} E_0 = E_i}$$

$$\rightarrow E_{out} = \sqrt{2} E_0 e^{-j2\theta} e^{-j4\beta_1 l}$$

$$\boxed{E_{out} = E_i e^{-j2\theta} e^{-j4\beta_1 l}} \quad - (8)$$

$\therefore$ Resultant E-field, $E_{out} = \sqrt{E_5^2 + E_6^2}$