Scheme Of Evaluation

Internal Assessment Test III – July 2021

Sub:	DIGITAL COMMUNICATION							Code:	18EC61
Date:	29/07/2021	Duration:	90 mins	Max Marks:	50	Sem:	VI	Branch:	ECE,TCE

Note: Answer Any 5 Questions

Question #		Description	Marks Distribution		Max Marks
1		Explain Gram-Schmidt orthogonalization procedure to obtain a set of orthonormal basis functions for the given set of signals.		10	10
		• Definition • Linear Independence • Basis Functions	2 2 6		
2	a	Consider a signal space with the following basis functions. $$\Phi_1(t) = \begin{cases} 1 \text{ for } 0 \le t \le 1 \\ 0 \text{ otherwise} \end{cases}$$ $$\Phi_2(t) = \begin{cases} 1 \text{ for } 0 \le t < 0.5 \\ -1 \text{ for } 0.5 \le t \le 1 \\ 0 \text{ otherwise} \end{cases}$$ Plot the signals with coordinates $(2, -1)$ and $(-2, 1)$.		06	10
		• Plotting $x_1(t)$ • Plotting $x_2(t)$	3 3		
2	b	Derive an expression for the energy of a signal in terms of its coordinates.		4	
		• Derivation	4		
3		Obtain a set of orthonormal basis functions for the following set of signals. $$x_1(t) = \begin{cases} 4 \text{ from } 0 \le t \le 4 \\ 0 \quad \text{otherwise} \end{cases}$$ $$x_2(t) = \begin{cases} 4 \text{ from } 0 \le t \le 2 \\ 0 \quad \text{otherwise} \end{cases}$$		10	10

		$$x_3(t) = \begin{cases} \mathbf{4 \text{ from } 2 \le t \le 4} \\ \mathbf{0 \quad\quad otherwise} \end{cases}$$ Express the signals as a linear combination of basis functions. Draw the signal space diagram (Constellation Diagram).			
		• Basis Function $\phi_1(t)$ • Basis Function $\phi_2(t)$ • Linear Combination • Constellation Diagram	2 3 3 2		
4		Find the output of the filter matched to the signal $x(t)$ when $x(t)$ is input to the filter where $$\mathbf{x(t)} = \begin{cases} \mathbf{1 \text{ for } 0 \le t \le 1} \\ \mathbf{0 \text{ otherwise}} \end{cases}$$		10	10
		• Impulse Response • Output for $0 \le t \le 1$ • Output for $1 \le t \le 2$	2 4 4		
5		Explain Binary Phase Shift Keying (BPSK) with neat block diagram of transmitter and receiver. Obtain the signal space diagram. What is the decision rule at the receiver?		10	10
		• Equations • Transmitter • Constellation Diagram • Receiver • Decision Rule	2 2 2 2 2		
6		Draw the block diagram of Coherent Binary Frequency Shift Keying (BFSK) receiver. State the decision rule at the receiver. Derive an expression for probability of error.		10	
		• Receiver • Decision Rule • Probability of Error	2 2 6		
7		Explain M-ary PSK with necessary equations. Draw the signal space diagram for M=8.		10	10
		• Definition • Equations • Basis Functions • Constellation Diagram	2 2 3 3		

SOLUTIONS

1. Gram-Schmidt orthogonalization procedure permits the representation of any set of M energy signals, $\{x_i(t)\}$, $i = 1, 2, \dots M$ as a linear combination of N orthonormal basis functions, where $N \leq M$.

That is to say,

$$x_i(t) = x_{i1}\phi_1(t) + x_{i2}\phi_2(t) + \dots + x_{iN}\phi_N(t)$$

$$= \sum_{j=1}^{N} x_{ij}\,\phi_j(t) \quad , \quad 0 \leq t \leq T$$

$$i = 1, 2, \dots M.$$

where the coefficients x_{ij} are defined by

②

$$x_{ij} = \int_0^T x_i(t)\,\phi_j(t)\,dt$$

$i = 1, 2, \ldots M$

$j = 1, 2, \ldots N$

$N \leq M.$

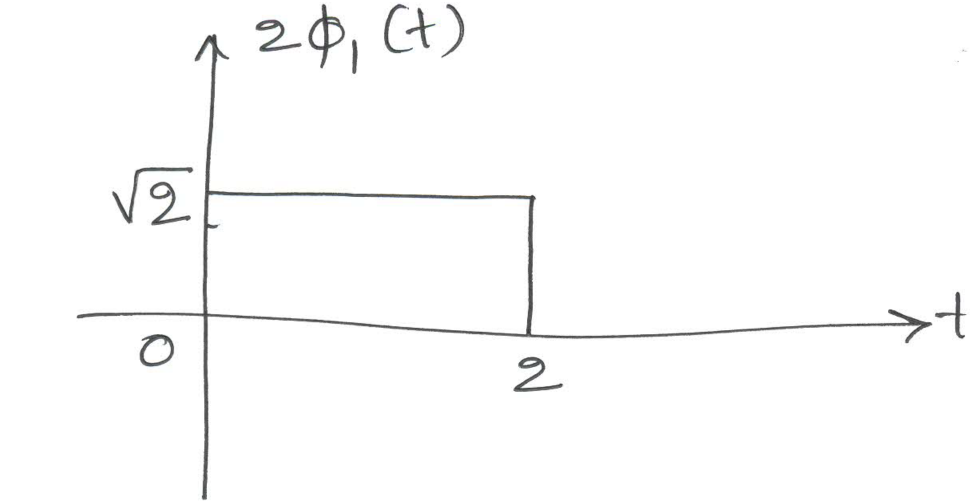
The real valued basis functions,

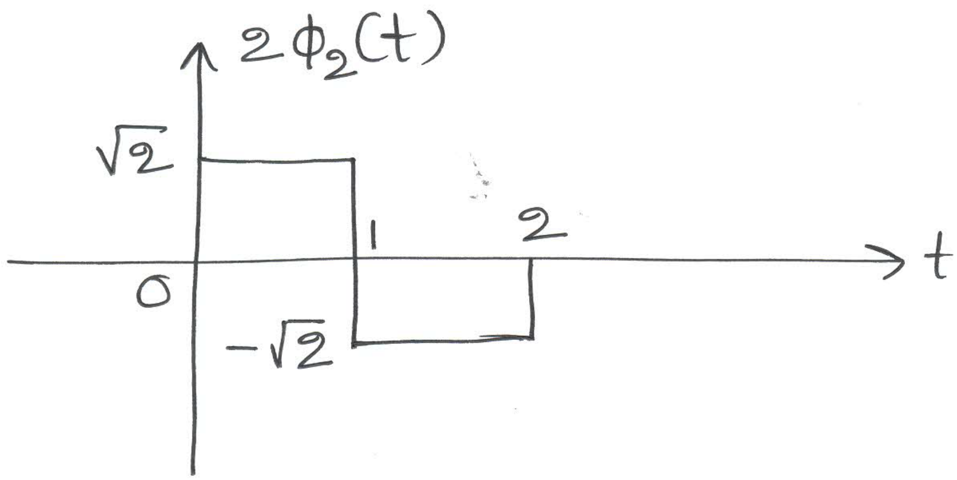
$\phi_1(t), \phi_2(t), \ldots \phi_N(t)$ are orthonormal,

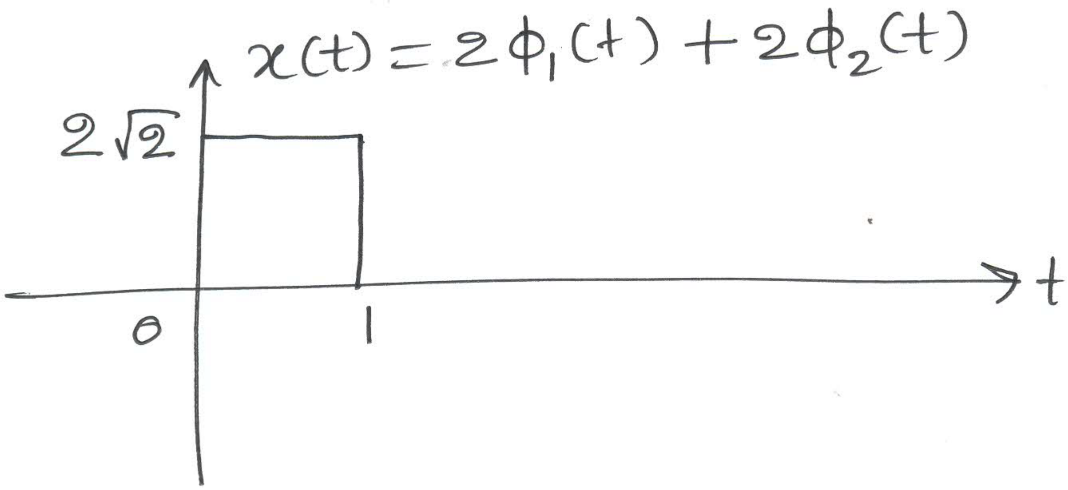
ie,

$$\int_0^T \phi_i(t)\,\phi_j(t)\,dt = \begin{cases} 1 & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

2a.


$2\phi_1(t)$
$\sqrt{2}$
0
2
t


$2\phi_2(t)$
$\sqrt{2}$
1
2
t
0
$-\sqrt{2}$


$x(t) = 2\phi_1(t) + 2\phi_2(t)$
$2\sqrt{2}$
t
0
1

2b.

$$E = \int_0^T x^2(t)\, dt$$

$$= \int_0^T \sum_{i=1}^{N} x_i \phi_i(t) \sum_{j=1}^{N} x_j \phi_j(t)\, dt$$

$$= \sum_{i=1}^{N} x_i \sum_{j=1}^{N} x_j \int_0^T \phi_i(t)\, \phi_j(t)\, dt \quad \ldots (1)$$

But $$\int_0^T \phi_i(t)\, \phi_j(t)\, dt = \begin{cases} 1 & \text{for } i=j \\ 0 & \text{for } i \neq j \end{cases} \quad \ldots (2)$$

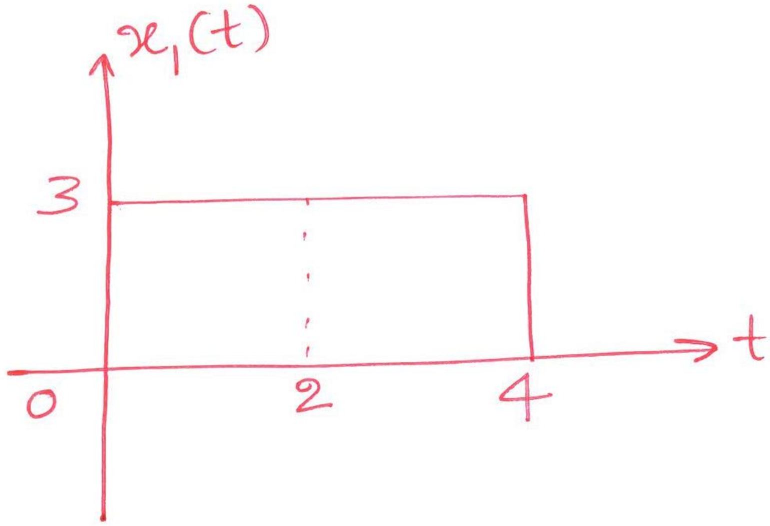
$\therefore$ (1) can be written as,

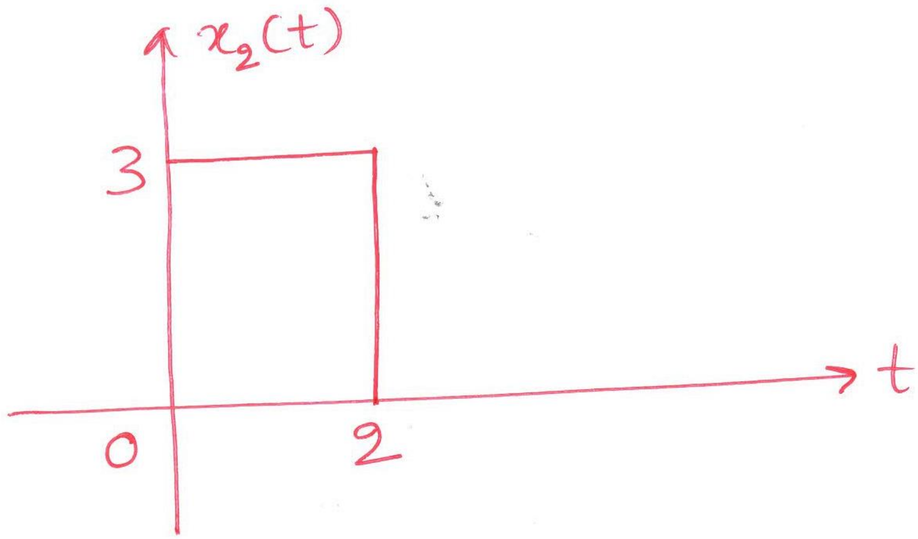
$$E = \sum_{i=1}^{N} x_i\, x_i$$

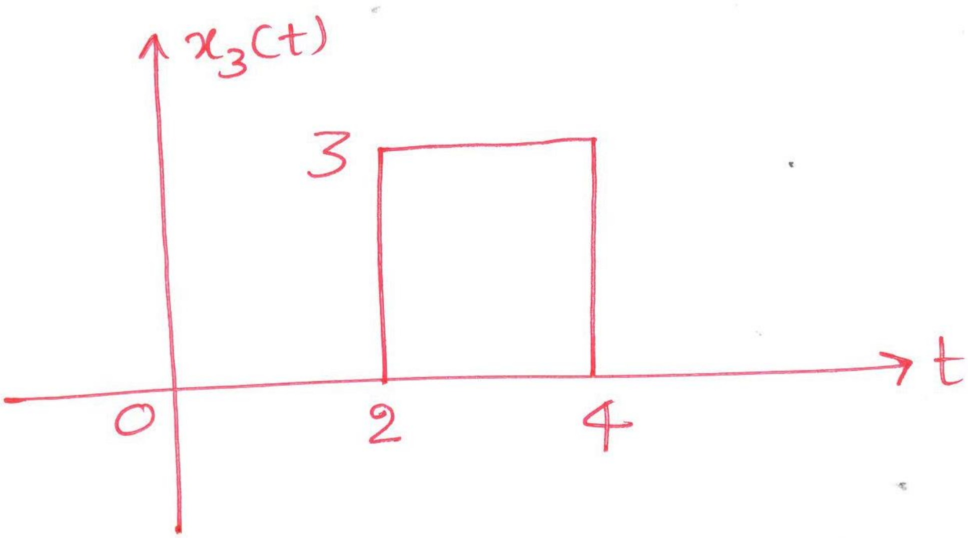
$$= \sum_{i=1}^{N} x_i^2 \quad \ldots (3)$$

This is the expression for energy of $x(t)$ in terms of its coordinates x_i, $i = 1, 2, \ldots N$.

3.







step i) Energy of $x_2(t)$,

$$E_2 = \int_0^2 3^2 \, dt$$

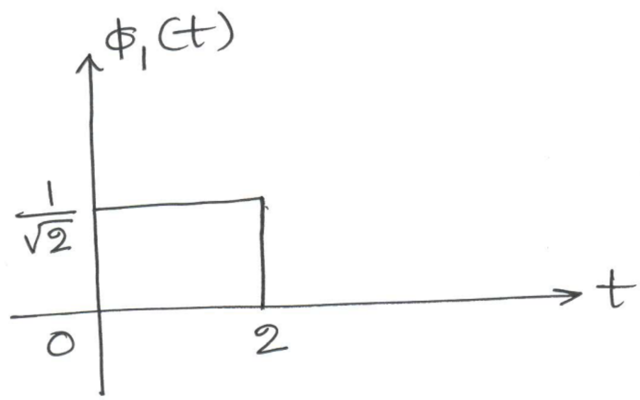
$$= 9t \Big|_0^2$$

$$= 9[2-0].$$

$$= 18$$

step ii) Basis function, $\phi_1(t) = \dfrac{x_2(t)}{\sqrt{18}}$

$$= \frac{x_2(t)}{3\sqrt{2}}$$



step iii) Energy of $x_3(t)$,

$$E_3 = \int_2^4 3^2 \, dt$$

$$= 9t \Big|_2^4$$

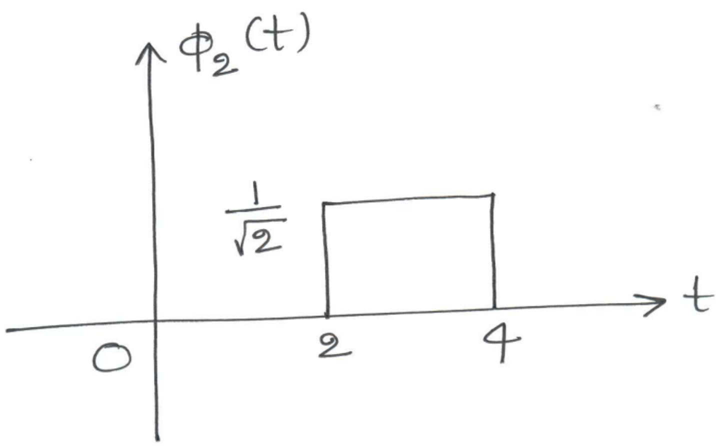
$$= 9[4-2]$$

$$= 18$$

step iv) Basis function, $\phi_2(t) = \dfrac{x_3(t)}{\sqrt{E_3}}$

$$= \frac{x_3(t)}{\sqrt{18}}$$

$$= \frac{x_3(t)}{3\sqrt{2}}$$

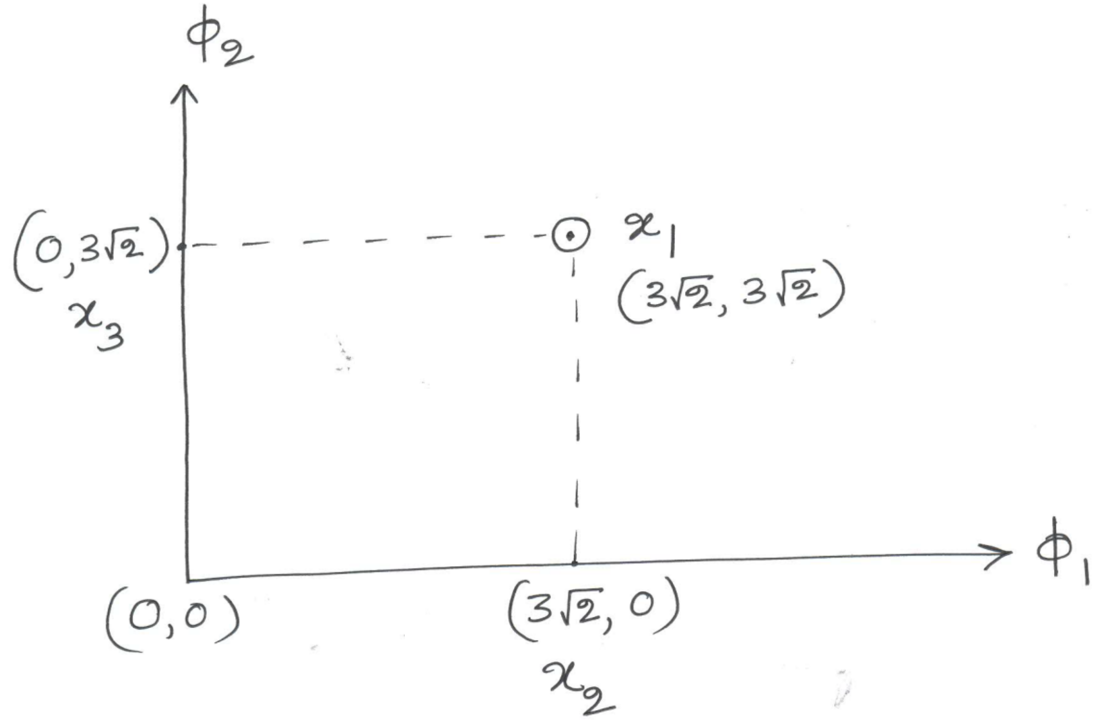


Expressing the signals as a linear combination of basis functions.

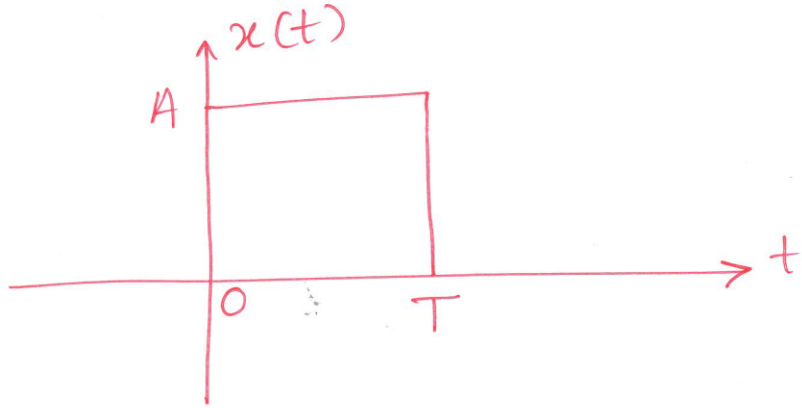
$$x_1(t) = 3\sqrt{2}\, \phi_1(t) + 3\sqrt{2}\, \phi_2(t)$$

$$x_2(t) = 3\sqrt{2}\, \phi_1(t) + 0\, \phi_2(t)$$

Constellation diagram
(signal-space diagram)

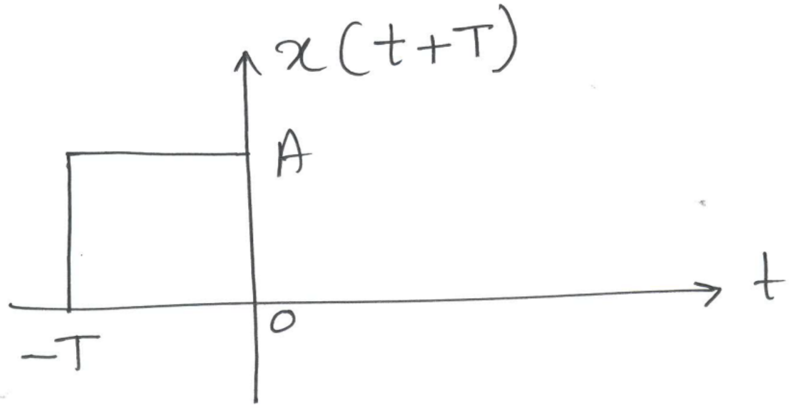


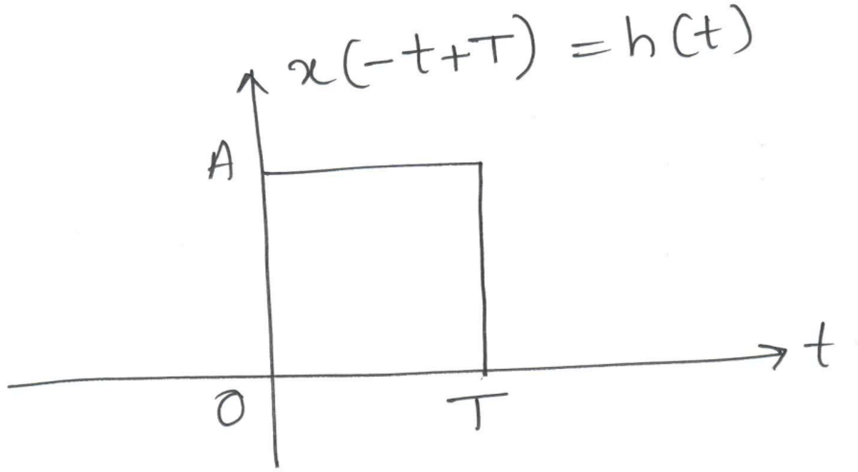
4.



Impulse response of the filter matched to $x(t)$ is given by,

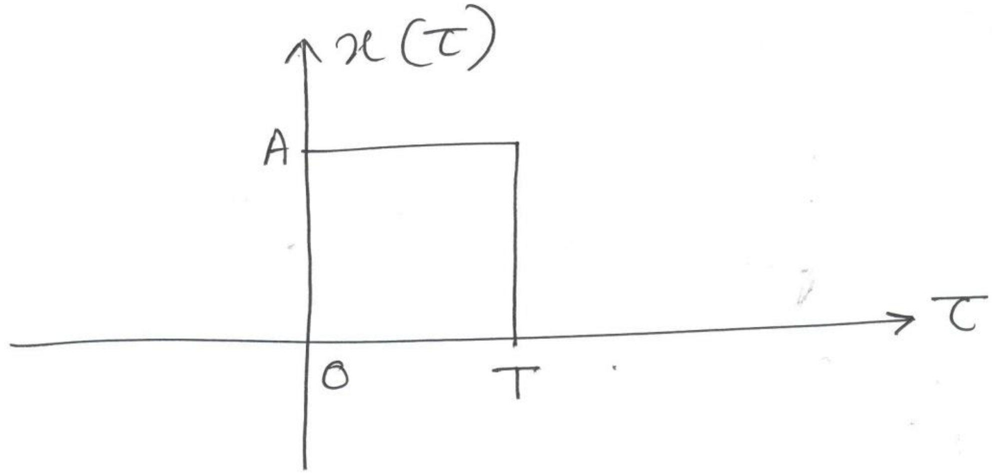
$$h(t) = x(T-t)$$

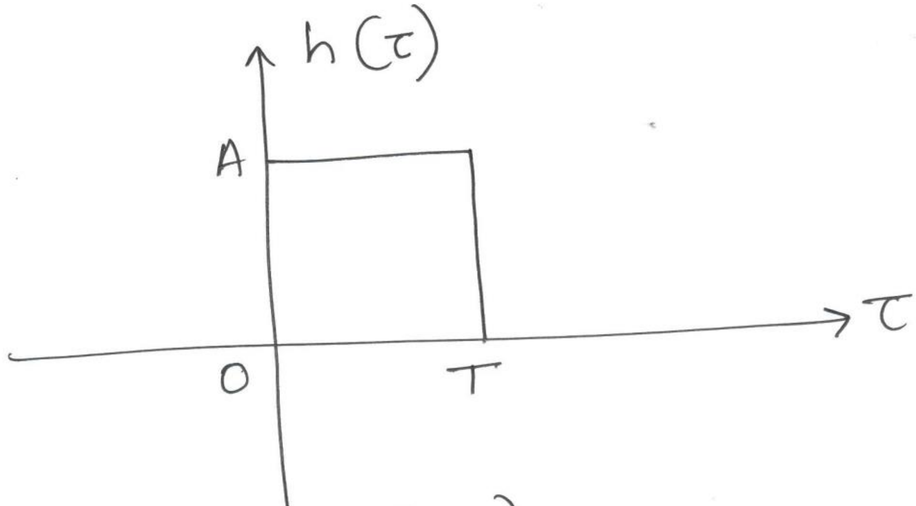


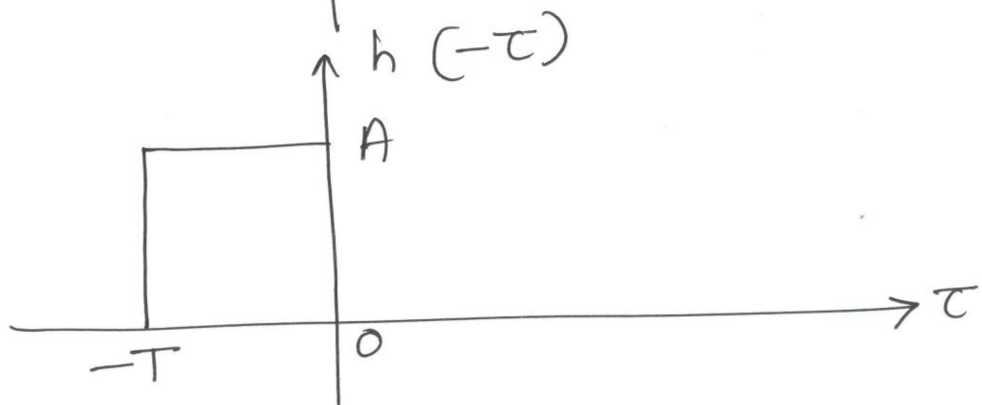


To find the output of matched filter, we have to find the convolution of input $x(t)$ and impulse response $h(t)$.

$$y(t) = x(t) * h(t)$$
$$= \int_0^T x(\tau)\, h(t-\tau)\, d\tau.$$

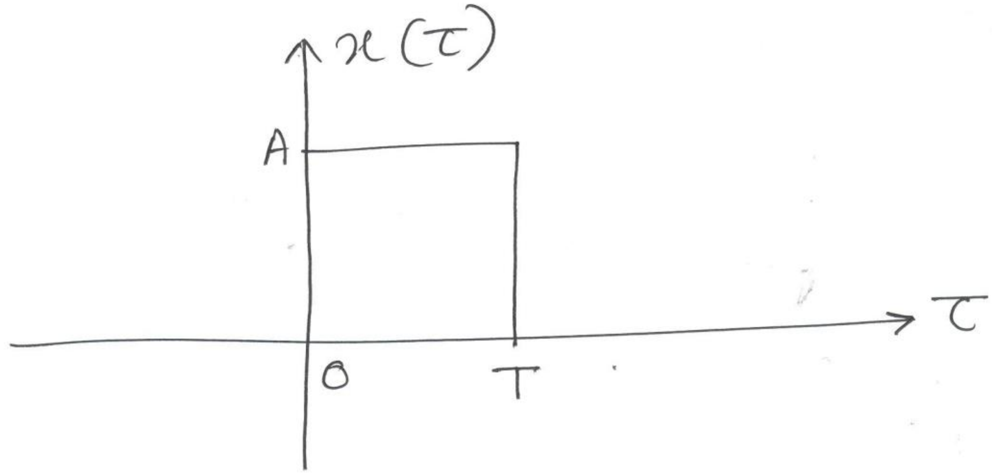


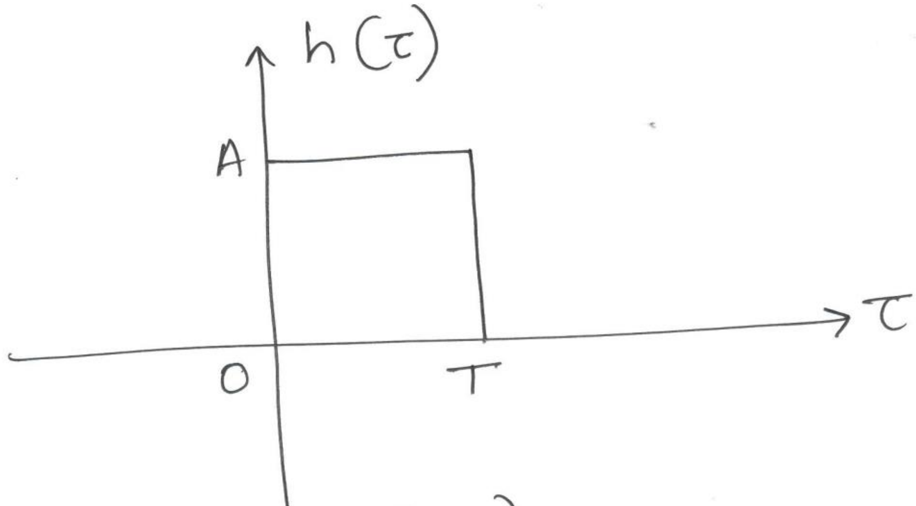


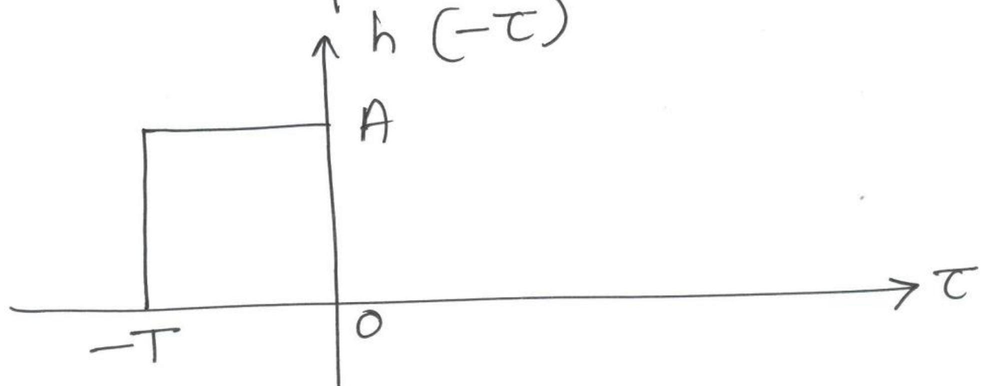


To find the output of matched filter, (54)
we have to find the convolution of input $x(t)$ and impulse response $h(t)$.

$$y(t) = x(t) * h(t)$$
$$= \int_0^T x(\tau)\, h(t-\tau)\, d\tau.$$



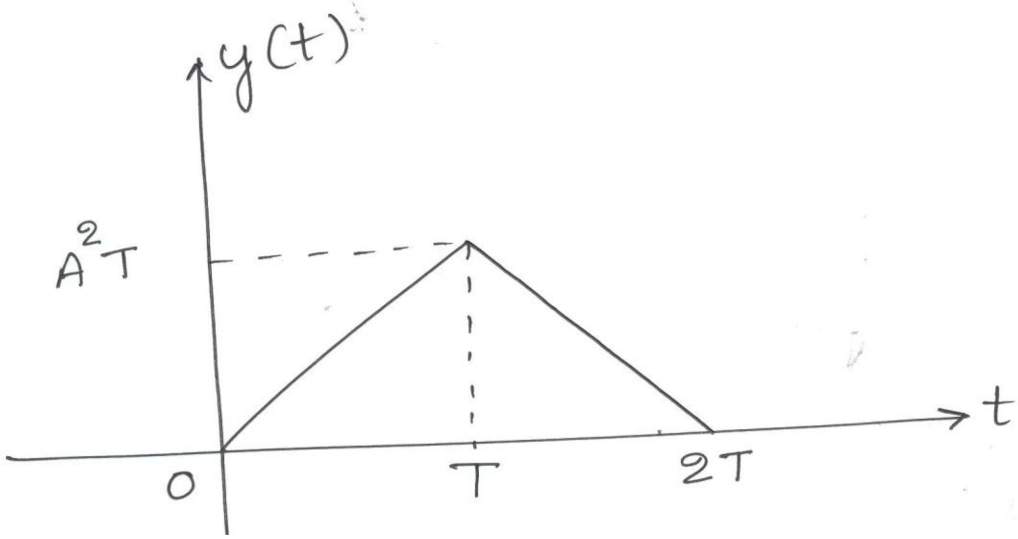




$\therefore\ y(t) = 0 \quad \text{for} \quad t > 2T.$

Hence,

$$y(t) = \begin{cases} 0 & \text{for } t < 0 \\ A^2 t & \text{for } 0 < t < T \\ 2A^2T - A^2t & \text{for } T < t < 2T \\ 0 & \text{for } t > 2T \end{cases}$$



5.

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t), \quad 0 \le t \le T_b$$

$$f_c = \frac{n}{T_b}$$

n - nonzero integer

T_b - bit duration

Bit 0 :

$$S_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t + \pi), \quad 0 \le t \le T_b$$

$$= -\sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_c t); \quad 0 \le t \le T_b$$

To find basis function.

$$\text{Energy of } s_1(t) = \int_0^{T_b} |s_1(t)|^2 \, dt$$

$$= \int_0^{T_b} \frac{2E_b}{T_b} \cos^2(2\pi f_c t) \, dt$$

$$= \frac{2E_b}{T_b} \int_0^{T_b} \frac{1 + \cos(4\pi f_c t)}{2} \, dt$$

$$= \frac{E_b}{T_b} \int_0^{T_b} 1 \, dt$$

$$= E_b$$

$\therefore$ Basis function, $\phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_b}}$

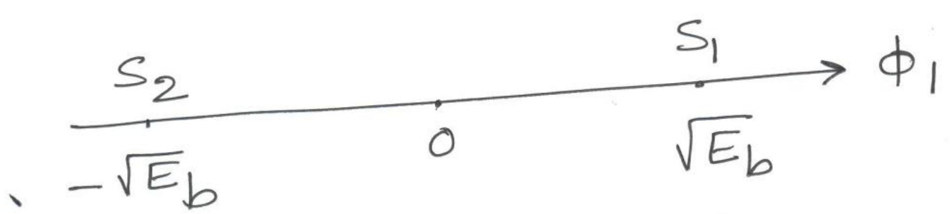
$$= \sqrt{\frac{2}{T_b}} \cos(2\pi f_c t)$$

$$0 \leq t \leq T_b$$

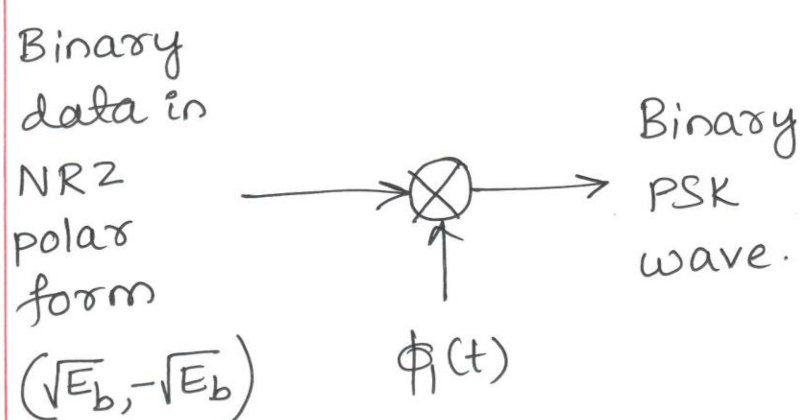
$\therefore \; s_1(t) = \sqrt{E_b}\, \phi_1(t), \quad 0 \leq t \leq T_b$

$s_2(t) = -\sqrt{E_b}\, \phi_1(t), \quad 0 \leq t \leq T_b$

Signal-space diagram

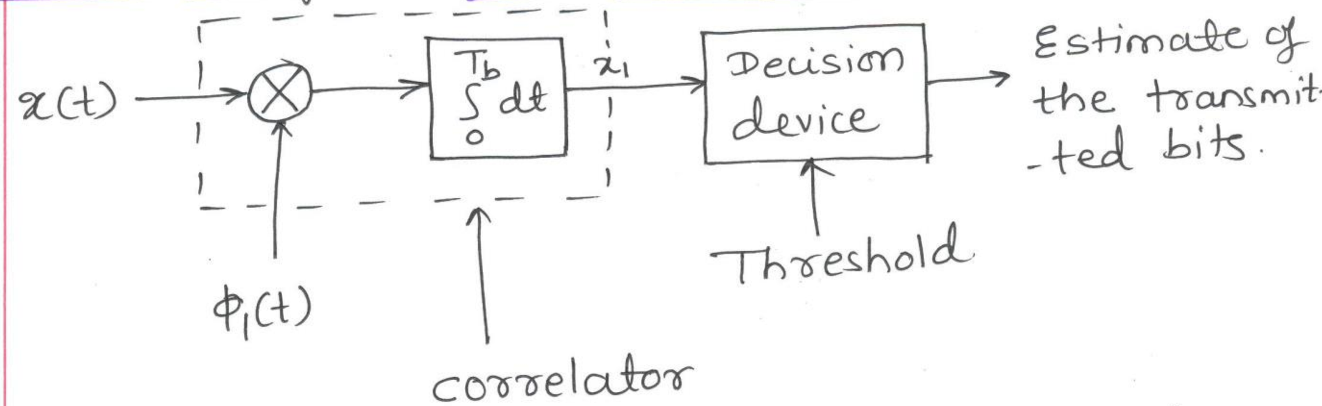


Block diagram of transmitter.



Block diagram of receiver

(13)

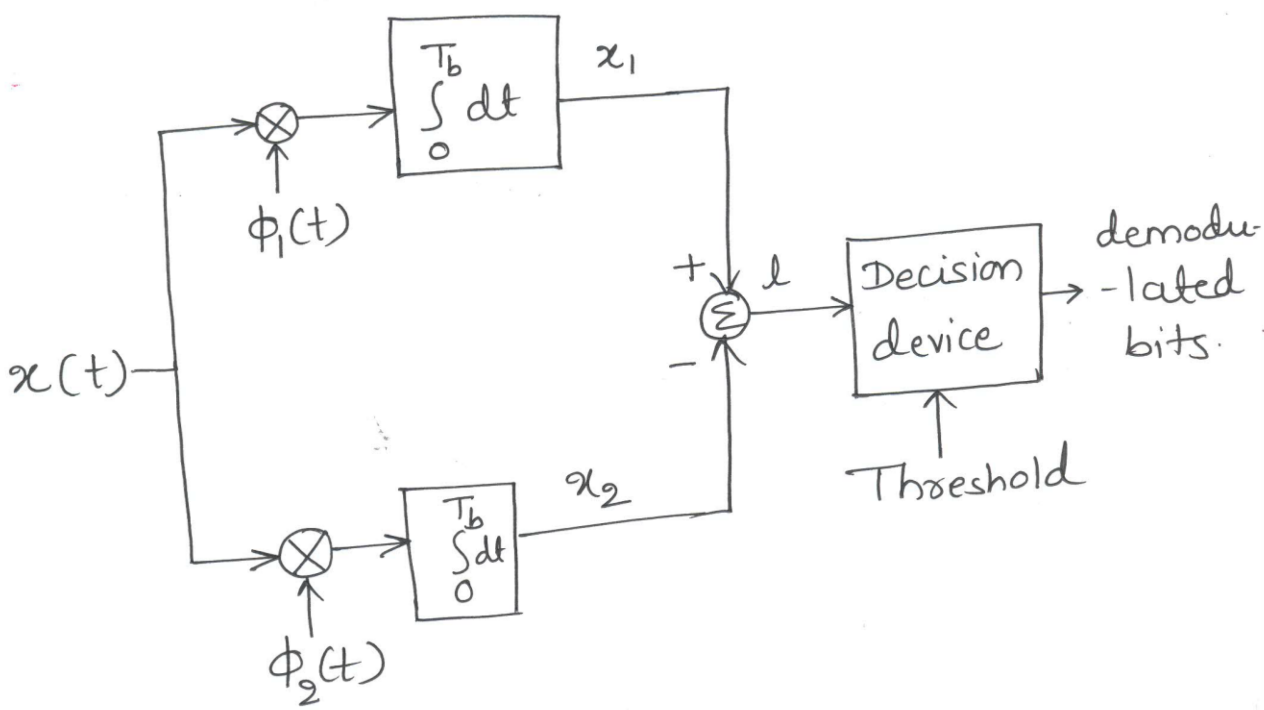


the received

6

Block diagram of receiver

(22)



$\therefore$ PDF of L when '0' was transmitted,

$$f_L(l/0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(l-\mu)^2}{2\sigma^2}}$$

$$= \frac{1}{\sqrt{2\pi N_0}} e^{-\frac{(l+\sqrt{E_b})^2}{2N_0}} \quad \ldots (10)$$

Wrong decision is made when $s_2(t)$ was transmitted and $l > 0$.

$\therefore$ Probability of error when bit '0' was transmitted,

$$P_e(0) = P\left(L>0 / 0\right)$$

$$= \int_0^\infty f_L(l/0)\, dl.$$

$$= \int_0^\infty \frac{1}{\sqrt{2\pi N_0}} e^{-\frac{(l+\sqrt{E_b})^2}{2N_0}} dl \quad \ldots (11)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-\frac{z^2}{2}} \, dz$$

$$= Q\left(\sqrt{\frac{E_b}{N_0}}\right) \quad \cdots (16)$$

Similarly, we may prove that probability of error when bit '1' was transmitted,

$$P_e(1) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) \quad \cdots (17)$$

$\therefore$ Average probability of error,

$$= \frac{1}{2} P_e(0) + \frac{1}{2} P_e(1)$$

(Assuming equiprobable 0s & 1s)

$$= \frac{1}{2} Q\left(\sqrt{\frac{E_b}{N_0}}\right) + \frac{1}{2} Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$= Q\left(\sqrt{\frac{E_b}{N_0}}\right) \quad \cdots (18)$$

7.

In M-ary PSK, phase of the carrier takes one of the M-possible values.

ie, $\theta_i = \frac{2\pi}{M}(i-1)$, $i = 1, 2, \cdots M$.

Accordingly. M-ary PSK modulated signal can be written as follows.

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + \frac{2\pi}{M}(i-1)\right] , \quad 0 \leq t \leq T$$

$i = 1, 2, \cdots M-1, M$.

E is the symbol energy

T is the symbol duration.

$f_c = \frac{n}{T}$ where n is a non-zero integer.

$$S_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\frac{2\pi}{M}(i-1)\right] \cos(2\pi f_c t)$$

$$- \sqrt{\frac{2E}{T}} \sin\left[\frac{2\pi}{M}(i-1)\right] \sin(2\pi f_c t)$$

$$0 \leq t \leq T.$$

Basis functions are,

$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \quad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \quad 0 \le t \le T$$

The coordinates of $s_i(t)$ are

$$\begin{bmatrix} \sqrt{E} \cos\left[\frac{2\pi}{M}(i-1)\right] \\ -\sqrt{E} \sin\left[\frac{2\pi}{M}(i-1)\right] \end{bmatrix}, \quad i = 1, 2, \dots M.$$

signal space diagram for $M = 8$