CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test - III

Sub:	**CONTROL SYSTEMS**							Code:	17EE61 / 18EE61
Date:	02/08/2021	Duration:	90 mins	Max Marks:	50	Sem:	6th	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	Sketch the root locus of the transfer function whose open loop transfer function is $G(s)=\frac{K}{s(s+2)(s+4)}$	10	CO4	L4
2	For the closed loop transfer function when the input is unit step $\frac{C(s)}{R(s)}=\frac{16}{s^2+4s+16}$. Determine undamped natural frequency, damping ratio, Maximum overshoot, Peak time and settling time.	10	CO3	L3
3	A unity feedback system is characterized by open-loop transfer function $G(s)=\frac{10}{s^2(1+0.4s)(1+0.3s)}$. Determine steady state errors for unit step, unit ramp and unit parabola.	10	CO3	L3
4	Determine the range of K for stability of unity feedback system whose open loop transfer function is $G(s)=\frac{K}{s(s+1)(s+2)}$ using Routh Hurwitz criteria.	10	CO3	L4
5	Determine if the pole nearest to the imaginary axis is at least -0.75 away from imaginary axis, for the system given by characteristic equation $s^4+5s^3+10s^2+10s+4=0$	10	CO3	L4
6	A unity feedback control system has an open loop transfer function $$G(s)=\frac{K}{s\left(s^2+4s+13\right)}$$ Sketch the root locus.	10	CO4	L4
7	Determine the frequency domain specifications for the unity feedback system $G(s)=\frac{225}{s(s+6)}$	10	CO4	L3

Control Systems IAT 3 Solution

1.

Step:-1

Starting points are $s=0, s=-2, s=-4$

Ending points are $s=\infty, \infty, \infty$.

Step:-2.

Number of branches = 3.

Step:-3 Symmetrical.

Step:-4.

Angle of Asymptotes $\phi_a = \frac{(2q+1)180}{n-m}$.

$n=3, m=0, n-m-1 = 3-0-1 = 2$.

If $q=0, \phi_a = 60^\circ$.

$q=1, \phi_a = 180^\circ$.

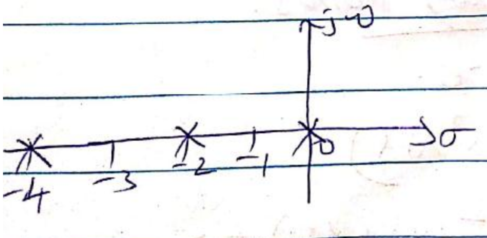
$q=2, \phi_a = -60^\circ$.

Intersection of Asymptote

$\sigma_a = \frac{(0-2-4)-0}{3-0}$

$= -\frac{6}{3} = -2$.

Step:-5. root locus of real axis.



$-\infty$ to $-4 \Rightarrow$ odd $\Rightarrow$ Yes.

-4 to $-2 \Rightarrow$ Even $\Rightarrow$ No.

-2 to $0 \Rightarrow$ odd $\Rightarrow$ Yes.

0 to $\infty \Rightarrow$ Even $\Rightarrow$ No

Step:-6. No complex pole of complex zero
∴ No Angle of Departure/Arrival.

Step:-7

$$\frac{C(s)}{R(s)} = \frac{K}{s(s+2)(s+4)+K}$$

$$s(s+2)(s+4)+K=0.$$

$$s^3+6s^2+8s+K=0.$$

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	
s^0	K	

$s^0 \Rightarrow K>0.$

$s^1 \Rightarrow K<48$

For stable,

$0<K<48.$

$$K=48, \quad 6s^2+K=0.$$

$$6s^2+48=0.$$

$$s^2+8=0.$$

$$s^2=-8.$$

$$s=\pm 2.8j$$

Step:- 8. Breakaway points.

$$s^3+6s^2+8s+K=0.$$

$$K=-(s^3+6s^2+8s)$$

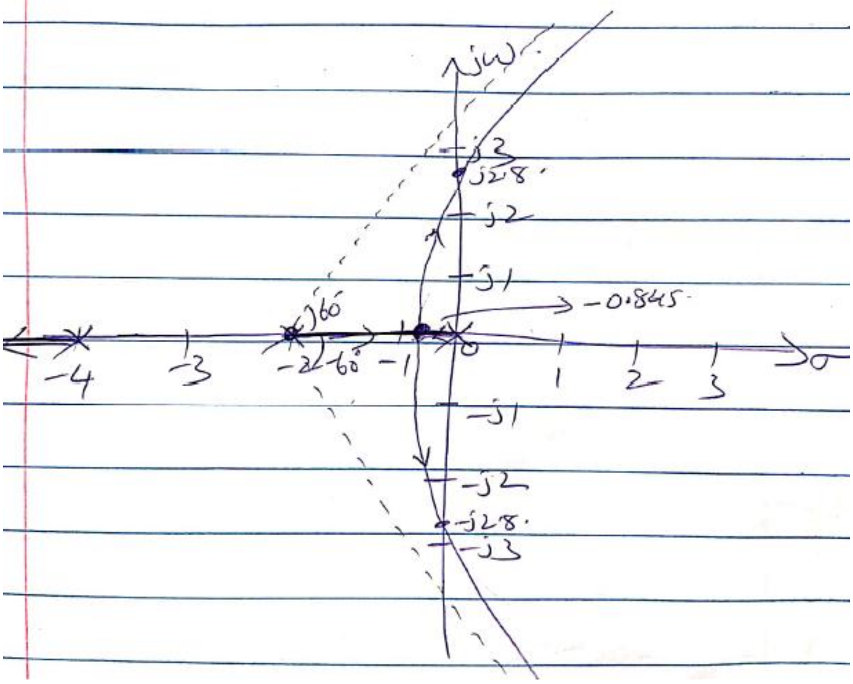
$$\frac{dK}{ds} = -(3s^2+12s+8)=0.$$

$$s=-0.845, -3.154$$

$s=-0.845$ is valid breakaway point

$s=-3.154$ is invalid breakaway point

sketch root locus.



2.

$$\frac{C(s)}{R(s)} = \frac{16}{s^2+4s+16}$$

$s^2+2\delta\omega_n s+\omega_n^2 \Leftrightarrow s^2+4s+16$

$\omega_n^2 = 16$ | $2\delta\omega_n = 4$

$\omega_n = 4\,rad/sec$ | $\delta = 0.5$

∴ undamped natural freq, $\omega_n = 4\,rad/sec$

Damping ratio, $\delta = 0.5$

/ Maximum overshoot,

$\%M_p = e^{-\delta\pi/\sqrt{1-\delta^2}} \times 100$

$= 16.3\%$

3.

$$G(s) = \frac{10}{s^2(1+0.4s)(1+0.3s)}$$

Unit step

$$K_p = \lim_{s \to 0} G(s) = \infty, \quad e_{ss} = \frac{1}{1+K_p} = 0.$$

Unit Ramp

$$K_v = \lim_{s \to 0} sG(s) = \infty, \quad e_{ss} = \frac{1}{K_v} = 0.$$

Unit Parabola

$$K_a = \lim_{s \to 0} s^2 G(s) = \lim_{s \to 0} \cancel{s^2} \frac{10}{\cancel{s^2}(1+0.4s)+(1+0.3s)}$$

$$= 10.$$

$$e_{ss} = \frac{1}{K_a} = \frac{1}{10} = 0.1$$

4.

characteristic equation is,

$$1+G(s)H(s) = 1 + \frac{K}{s(s+1)(s+2)} = 0$$

$$s(s+1)(s+2)+K = 0.$$

$$s^3+3s^2+2s+K = 0.$$

Routh table,

s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	
s^0	K	

For stable system,

s^0 row $\Rightarrow K > 0.$

s^1 row $\Rightarrow \frac{6-K}{3} > 0 \Rightarrow K < 6$

$\therefore$ Range of K for stable system is

$$0 < K < 6.$$

5.

Put $s=(z-0.75)$

$(z-0.75)^4+5(z-0.75)^3+10(z-0.75)^2+10(z-0.75)+4=0$

$z^4-2z^3+11.24z^2-5z-7.043=0$

z^4	1	11.24	-7.043
z^3	-2	-5	
z^2	8.74	-7.043	
z^1	-6.6		
z^0	-7.043		

Unstable system.

$\therefore$ 3 roots are less negative than -0.75.

6.

Step 1 Starting and Ending Points.

Starting points are Poles.

$S=0,\ S^2+4S+13=0$

$S=-2\pm j3$.

Ending points are zeros $\Rightarrow$ No zeros.

$\Rightarrow S=\infty,\infty,\infty$

Step 2 Number of branch = Number of Poles.

= 3.

Step 3 Symmetrical with respect to real-axis.

Step 4

Angle of Asymptote $\phi_a=\frac{(2q+1)180}{n-m}$

where $q=0,1,\dots,n-m-1$.

$n=3,\ m=0,\ n-m-1=3-0-1=2$.

If $q=0$, $\phi_a=\frac{180}{3-0}=60°$.

If $q=1$, $\phi_a=\frac{3\times180}{3}=180°$.

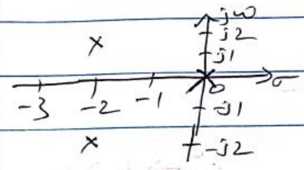
If $q=2$, $\phi_a=\frac{5\times180}{3}=300=-60°$.

Intersection Point of Asymptote,

$$\sigma = \frac{\Sigma \text{ finite poles} - \Sigma \text{ finite zeros}}{n-m}$$

$$= \frac{(0-2-2)-0}{3-0} = \frac{-4}{3} = -1.33$$

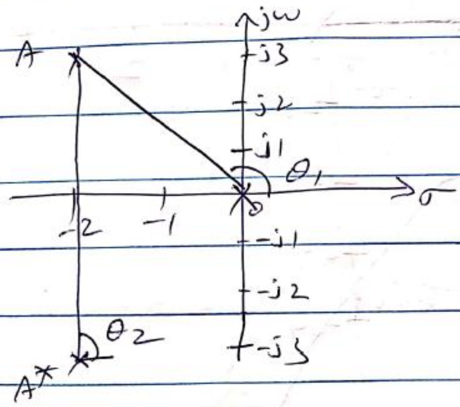
Step 5 Root locus on real axis



$-\infty$ to $0 \Rightarrow$ Odd roots $\Rightarrow$ Yes

0 to $\infty \Rightarrow$ Even $\Rightarrow$ No

Step:-6 Angle of Departure.



$\theta_1 = 180 - \tan^{-1}\frac{3}{2}$

$= 117.4^\circ$

$\theta_2 = 90^\circ$

Angle of Departure from A $\Big\}$ $= 180 - (\theta_1 + \theta_2) + (0)$

$= 180 - 117.4 - 90 = -27.4^\circ$

Angle of Departure from A* $\Big\}$ $= 27.4^\circ$

Step:-7 Intersection of root locus with Imaginary axis.

$$\frac{C(s)}{R(s)} = \frac{K}{s(s^2+4s+13)+K}$$

$s(s^2+4s+13)+K=0.$

$s^3+4s^2+13s+K=0.$

Routh table.

s^3	1	13
s^2	4	K
s^1	$\frac{52-K}{4}$	
s^0	K	

$s^0 \Rightarrow K>0.$

$s^1 \Rightarrow K<52$

For stable $0<K<52.$

Critical value of $K=52.$

$4s^2+K=0.$

$4s^2+52=0 \Rightarrow s^2+13=0.$

$s^2=-13$

$s=\pm j3.6$

Step:-8 Breakaway point.

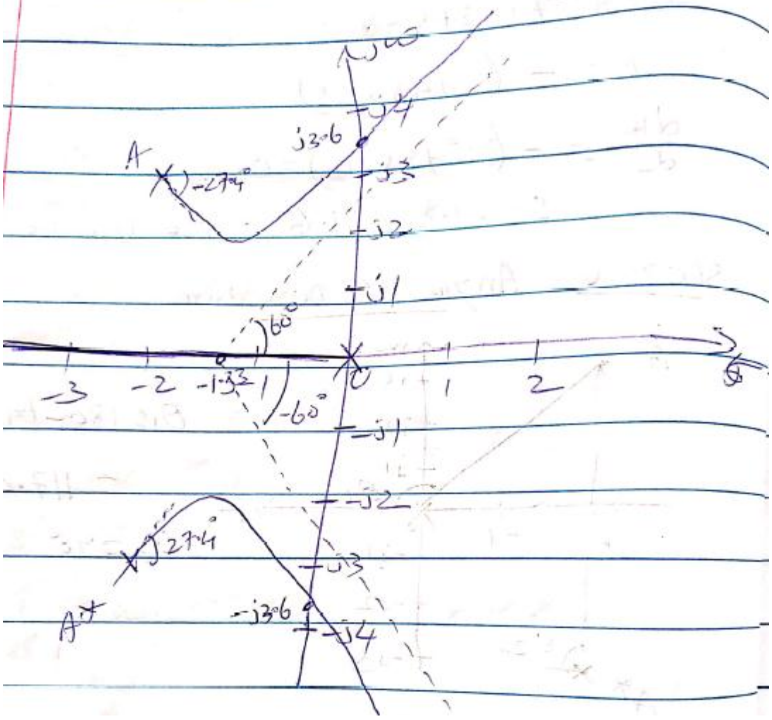
$s^3+4s^2+13s+K=0.$

$K=-(s^3+4s^2+13s)$

$\frac{dK}{ds}=-(3s^2+8s+13)=0.$

$s=-1.33\pm j1.6$ ⇒ No Breakaway points

Sketch root locus:



7.

$$\therefore \frac{C(s)}{R(s)} = \frac{225}{s^2+6s+225} \longrightarrow (1)$$

* By Comparing Equation (1) with $\dfrac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$, we have

$$\omega_n^2 = 225$$

$$\omega_n = \sqrt{225}$$

$$\boxed{\omega_n = 15 \text{ rad/sec}}$$

$$2\xi\omega_n = 6$$

$$\xi = \frac{6}{2\omega_n} = \frac{6}{2\times 15}$$

$$\boxed{\xi = 0.2}$$

Resonant Peak.

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

$$= \frac{1}{2*0.2*\sqrt{1-(0.2)^2}}$$

$$\boxed{M_r = 2.60}$$

Resonant frequency

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

$$= 15*\sqrt{1-2*(0.2)^2}$$

$$\boxed{\omega_r = 14.39 \text{ rad/sec}}$$

Band width

$$\omega_b = \omega_n\sqrt{(1-2\xi^2)+\sqrt{2-4\xi^2+4\xi^4}}$$

$$= 15\sqrt{(1-2(0.2)^2)+\sqrt{2-4(0.2)^2+4(0.2)^4}}$$

$$= 15\sqrt{(1-2(0.2)^2)+1.3588}$$

$$\boxed{\omega_b = 22.656 \text{ rad/sec.}}$$