



Eighth Semester B.E. Degree Examination, July/August 2021
Design of Prestressed Concrete Elements

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions.
2. Use of IS-1343 is permitted.

1.
 - a. Distinguish between pretensioning and post-tensioning. (06 Marks)
 - b. List the advantages of PSC over RCC. (04 Marks)
 - c. Explain with sketch Hoyer's long line systems of pre-tensioning. (10 Marks)

2.
 - a. Explain the concept of load balancing in prestressed concrete design. (06 Marks)
 - b. A prestressed concrete beam made of T-section has flange (1000mm and 150mm) and web 200mm x 800mm. Beam supports super imposed load of 180kN/m over a simply supported span of 8m. If the prestressing force in the tendon is 6200kN at mid span and is located at a distance of 500mm from soffit, determine the resultant stresses at mid span for the following cases:
 - i) Prestress + self weight
 - ii) Prestress + self weight + super imposed load
 Take unit weight of concrete = 24kN/m³. (14 Marks)

3.
 - a. List the different types of losses in pretensioning and post tensioning separately. (06 Marks)
 - b. A prestressed concrete beam 300mm x 600mm is prestressed by tendons of area 800mm² at a constant eccentricity of 100mm with an initial stress of 1050N/mm². Span of the beam is 10.5m. With the following additional data, calculate losses for both pre and post-tensioning cases:

ES = 210kN/mm²
 EC = 35kN/mm²
 Anchorage slip = 1.5mm
 K = 0.0015m
 Ultimate creep strain = 40 x 10⁻⁶ per 1N/mm² stress for pretensioning and 20 x 10⁻⁶ per 1N/mm² for post tensioning
 Shrinkage strain = 300 x 10⁻⁶ for pretensioning
 = 200 x 10⁻⁶ for post-tensioning
 Relaxation steel = 2.5%

(14 Marks)

4.
 - a. What are the factors influencing deflections of PSC beams? Indicate how long term deflection is calculated? (07 Marks)
 - b. A concrete beam having a rectangular section 150mm x 300mm is prestressed by a parabolic cable at an eccentricity of 75mm at mid span towards bottom soffit and at an eccentricity of 25mm towards top at support sections. The effective prestressing force is 350kN. The beam supports a concentrated live load of 20kN at centre of span in addition to the self weight with a span 8m. Find the short term deflection at the centre of span under prestress, self weight, and live load. Find also the long term deflection if the loss ratio is 0.8 and the creep coefficient is 1.6. Take E_c = 38kN/mm². (13 Marks)

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- 5 a. Discuss the IS code method of determining the ultimate moment of resistance of rectangular and flanged sections PSC members. (08 Marks)
- b. A post-tensioned bonded prestressed concrete beam of T-section has a flange width of 1500mm and thickness of flange is 200mm. Thickness of the rib is 300mm. The area of high tensile steel is 5000mm², located at an effective depth of 1800mm. If the characteristic strength of concrete and steel are 40N/mm² and 1600N/mm² respectively. Calculate the flexural strength of T-section. (12 Marks)
- 6 Design a pretensioned roof purlin to suit the data given below:
Effective span = 6m, udl = 5kN/m, $f_{ck} = 50\text{N/mm}^2$, loss ratio = 0.8 permissible stresses at transfer are $\sigma_{ct} = 15\text{N/mm}^2$, $\sigma_{it} = -1.0\text{N/mm}^2$. At service load permissible stresses are, $\sigma_{cw} = 17\text{N/mm}^2$, $\sigma_{tw} = 0$, 7mm high tensile steel wires having an ultimate strength $f_{pu} = 1600\text{N/mm}^2$ are available for use. (20 Marks)
- 7 a. Explain the mechanism of shear failure in PSC beam. (04 Marks)
- b. A concrete beam of rectangular section 200mm wide and 650mm deep is prestressed by a parabolic cable located at an eccentricity of 120mm at mid span and zero at the supports. If the beam has a span of 12m and carries a udl of 4.5kN/m. Find the effective force necessary in the cable for zero shear stress at the support section. For this condition calculate the principal stresses. The density of concrete is 24kN/m³. (08 Marks)
- c. The support section of a PSC beam (150 × 300mm) is to resist a shear of 100kN. The prestress at centroidal axis is 5N/mm², $f_{ek} = 40\text{N/mm}^2$. The cover to tension reinforcement is 45mm. Check the section for shear and design suitable shear reinforcement, $f_t = 1.5\text{N/mm}^2$. (08 Marks)
- 8 A post tensioned beam of size 400mm width and 600mm depth is subjected to the following ultimate load conditions at service loads:
M = 350kN-m
T = 100kN-m
V = 100kN
If the area of prestressing tendons is 70mm² and effective prestressing force at service load condition is 800kN at an eccentricity of 200mm using provisions of IS:1343, design suitable transverse reinforcement. Take
 $f_{ck} = 40\text{N/mm}^2$
 $f_y = 415\text{N/mm}^2$
 $f_{pu} = 1600\text{N/mm}^2$ and cover = 50mm. (20 Marks)
- 9 a. What is meant by composite construction in PSC? What are the advantages of composite construction? (08 Marks)
- b. The precast prestressed unit has dimension 120mm × 200mm while cast-in-situ slab has 400mm × 40mm. The effective span is 6m and is prestressed with a force of 250kN with its centroid coinciding with bottom kern point. Determine the final stresses developed, if live load on slab is 10kN/m². Assume loss of prestress as 15 percent and modular ratio between pre cast and cast in concrete same. The beam was not propped while casting slab. (12 Marks)
- 10 A composite beam is made up of a precast rib of size 120mm × 200mm and a cast-in-situ slab of size 400mm × 40mm. It was prestressed with a force of 250kN with straight cables at an eccentricity of 35mm. Determine the deflection of the beam, if it is unsupported at the time of casting slab. Assume 15% loss.
Given : span = 6m, **CMRIT LIBRARY**
Live load = 4kN/m **BANGALORE - 560 037**
Modulus of elasticity for precast and cast-in-situ concrete = 30kN/mm². (20 Marks)

2 of 2

Solutions

1. a.

PRETENSIONING	POST TENSIONING
1. Concrete is prestressed with tendon before it is placed in position	1. Prestressing is done after concrete attains sufficient strength
2. Pretensioning is developed due to bonding between steel and concrete	2. Post tensioning is developed due to bearing
3. Preferred for small structural element and easy to transport	3. Preferred for large structural element and difficult to transport
4. Similar structural members are casted	4. Members are casted according to market requirements

5. Casted in moulds	5. Cables are used in place of wires and jacks for stretching
6. Greater certainty about the prestressing force	6. More economical to use a few cables or bars with large forces in than many small ones
7. Suitable for bulk production	7. Suited for medium to long-span in situ work where the tensioning cost is only a small proportion of the cost of the whole job

1. b

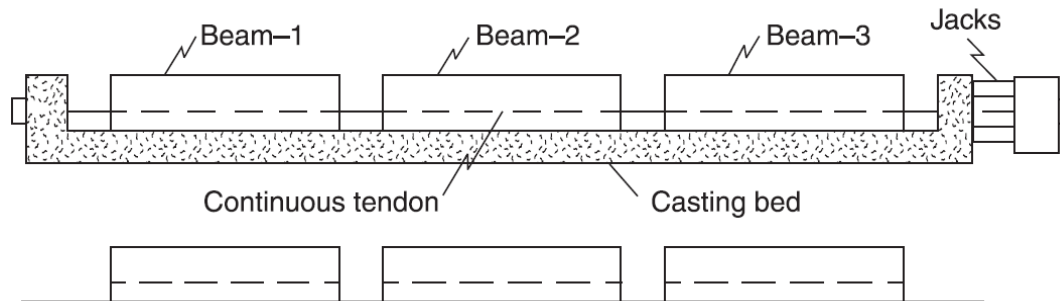
Differences between Prestressed Concrete and Reinforced Concrete:

1. In prestress concrete member steel play's active role. The stress in steel prevails whether external load is there or not. But in R.C.C., steel plays a passive role. The stress in steel in R.C.C members depends upon the external loads. i.e., no external load, no stress in steel.
2. In prestress concrete the stresses in steel is almost constant where as in R.C.C the stress in steel is variable with the lever arm.
3. Prestress concrete has more shear resistance, whereas shear resistance of R.C.C is less.
4. In prestress concrete members, deflections are less because the eccentric prestressing force will induce couple which will cause upward deflections, where as in R.C.C., deflections are more.
5. In prestress concrete fatigue resistance is more compare to R.C.C. because in R.C.C. stress in steel is external load dependent where as in P.S.C member it is load independent.
6. Prestress concrete is more durable as high grade of concrete is used which are denser in nature. R.C.C. is less durable.
7. In prestress concrete dimensions are less because external stresses are counterbalance by the internal stress induced by prestress. Therefore, reactions on column & footing are less as a whole the quantity of concrete is reduced by 30% and steel reduced by about 60 to 70%. R.C.C. is uneconomical for long span because in R.C.C. dimension of sections is large requiring more concrete & steel. Moreover, as self-weight increases more reactions acted on columns & footings, which requires higher sizes.

1.c

- ✓ Hoyer system or long line method is often adopted in pre-tensioning.
- ✓ Two bulk heads or abutments independently anchored to the ground are provided several meters apart, say, 100m. wires are stretched between the bulkheads.
- ✓ Moulds are placed enclosing the wires.
- ✓ Concrete is placed surrounding the wires.
- ✓ With this Hoyer system, several members can be produced along one line.
- ✓ This method is economical and is used in almost all pre-tensioning factories.

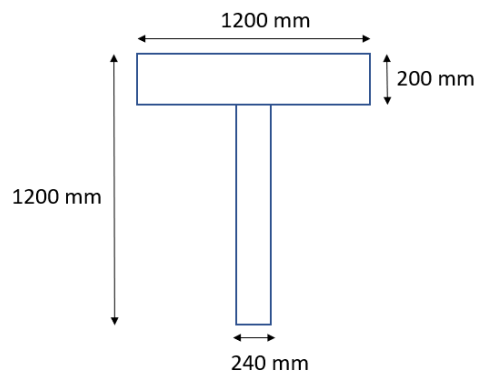
- ✓ For tensioning, a hydraulic jack is used.
- ✓ Wires are gripped at the bulkheads, using split-cone wedges.
- ✓ These wedges are made from tapered conical pins.
- ✓ Flat surface of the pin carries serrations to grip the wire.



2. a

- The concept is useful in selecting the tendon profile, which can provide the most desirable system of forces in concrete.
- The **cable profile in a prestressed member** corresponds to the **shape of the bending moment diagram** resulting from the external loads.
- Thus, if the beam supports two concentrated loads, the cable should follow a trapezoidal profile.
- If the beam supports uniformly distributed loads, the corresponding tendon should follow a parabolic profile.
- It is possible to select suitable cable profiles in a prestressed concrete member such that the transverse component of the cable force balances the given type of external loads.

2. b



Eff span = 16 m

$f_t = 0$, $f_b = 12 \text{ MPa}$

$A = 1200 \times 200 + 1000 \times 240 = 480000 \text{ mm}^2$

$$y_t = \frac{1200 \times 200 \times 100 + 1000 \times 240 \times (200 + 500)}{480000} = 400 \text{ mm}, y_b = 800 \text{ mm}$$

$$I_{xx} = \frac{1200 \times 200^3}{12} + (1200 \times 200) \times ((400 - 100)^2) + \frac{240 \times 1000^3}{12} + (240 \times 1000) \times ((700 - 400)^2)$$

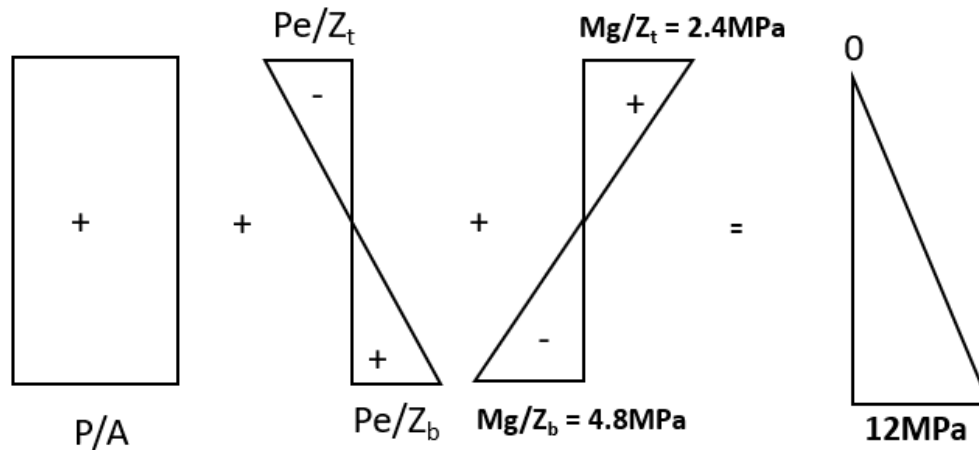
$$= 6.4 \times 10^{10} \text{ mm}^4$$

$$Z_t = \frac{I_{xx}}{y_t} = \frac{6.4 \times 10^{10}}{400} = 160 \times 10^6 \text{ mm}^3$$

$$Z_b = \frac{I_{xx}}{y_b} = \frac{6.4 \times 10^{10}}{800} = 80 \times 10^6 \text{ mm}^3$$

$$\text{Self-weight of the beam} = 25 \times 480000 / 1000^2 = 12 \text{ kN/m}$$

$$\text{Moment due to self-weight, } Mg = \frac{12 \times 16^2}{8} = 384 \text{ kNm}$$



At top stresses can be

$$\frac{P_i}{A} - \frac{P_i e}{Z_t} + \frac{Mg}{Z_t} = 0, \frac{P_i}{480000} - \frac{P_i e}{160 \times 10^6} + \frac{384 \times 10^6}{160 \times 10^6} = 0, 333.3 P - P e = -384 \times 10^6 \dots (1)$$

At bottom stresses can be

$$\frac{P_i}{A} + \frac{P_i e}{Z_t} - \frac{384 \times 10^6}{160 \times 10^6} = 12$$

$$166.67 P + P e = 1.344 \times 10^9 \dots (2)$$

Solving (1) and (2)

$$P = 1920 \text{ kN}, e = 533.33 \text{ mm}$$

3 a

The **various reductions of the prestressing force** are termed as the losses in prestress. The losses are broadly classified into two groups, **immediate and time-dependent**. The **immediate losses** occur during **prestressing of the tendons** and the transfer of prestress to the

concrete member. The **time-dependent losses** occur during the service life of the prestressed member.

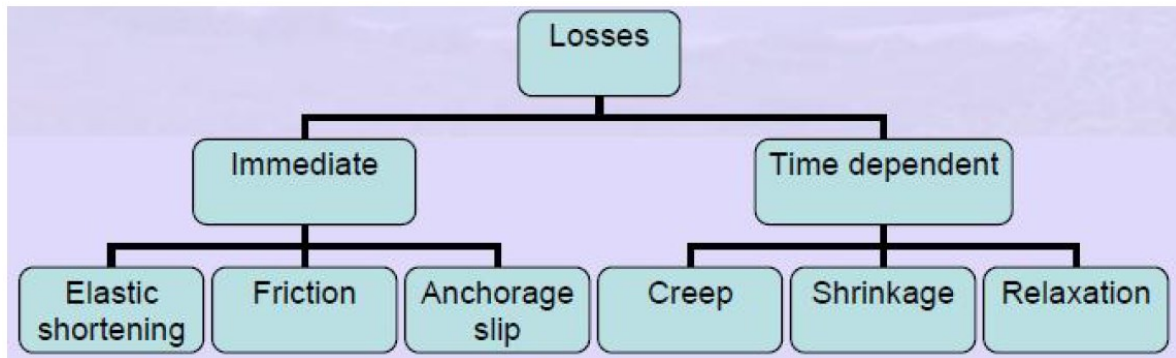


Table 5.1 Types of losses of prestress

S. No.	Pretensioning	S. No.	Post-tensioning
1.	Elastic deformation of concrete	1.	No loss due to elastic deformation if all the wires are simultaneously tensioned. If the wires are successively tensioned, there will be loss of prestress due to elastic deformation of concrete
2.	Relaxation of stress in steel	2.	Relaxation of stress in steel
3.	Shrinkage of concrete	3.	Shrinkage of concrete
4.	Creep of concrete	4.	Creep of concrete
		5.	Friction
		6.	Anchorage slip

IS1343 2012 Page 24, CL19.5.2

- **Elastic Shortening**

- i. **Pre-tensioned Members**

When the tendons are cut and prestress are transferred to concrete, member shortens and the prestressing steel also shortens along with it. This results in loss of pre-stress and is called loss due to elastic shortening of concrete.

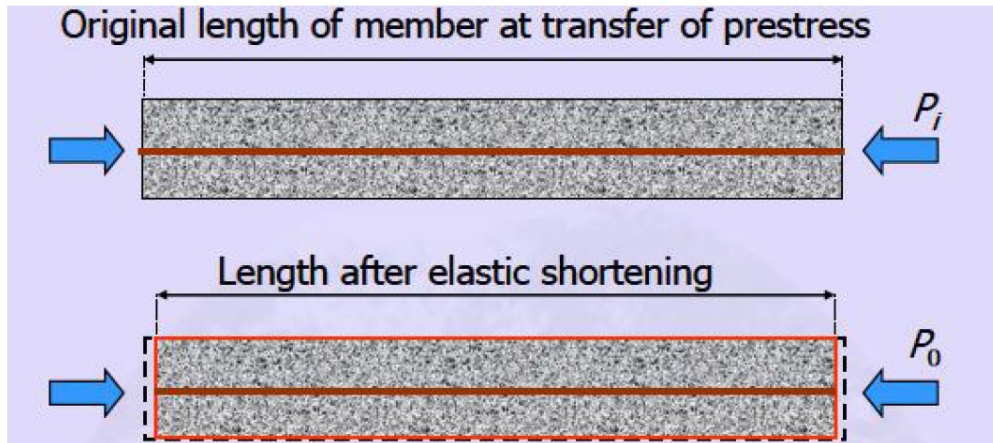
If f_c is the stress in concrete at the level of prestress and 'm' is the Modular ratio defined as modulus of elasticity of prestressing steel to modulus of elasticity of concrete. The strain at the level of Prestressing steel $e_c = f_c / E_c$

The stress in prestressing steel corresponding to strain will be equal to $m \times f_c$

$$\text{Loss in prestress} = m \times f_c = \frac{E_s}{E_c} \times f_c$$

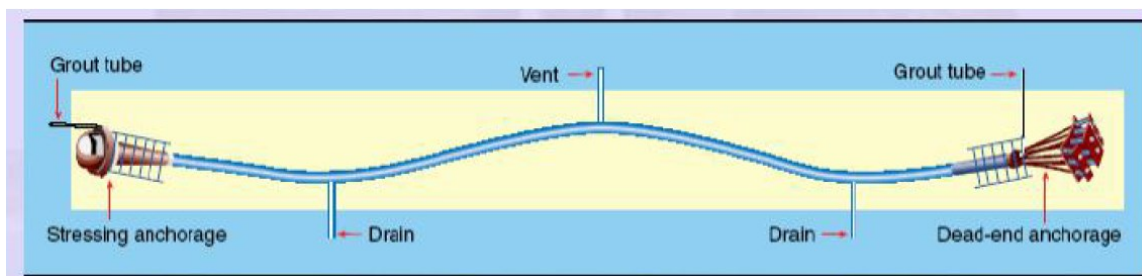
- ii. **Post-tensioned Members**

If there is only one tendon, there is no loss because the applied prestress is recorded after the elastic shortening of the member. For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons. Elastic shortening loss is quantified by the drop in prestress (Δf_p) in a tendon due to the change in strain in the tendon ($\Delta \epsilon_p$).



- **Friction**

The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member, leads to a drop in the prestress along the member from the stretching end. The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons. The friction is generated due to the curvature of the tendon and the vertical component of the prestressing force. The following figure shows a typical profile (laying pattern) of the tendon in a continuous beam.



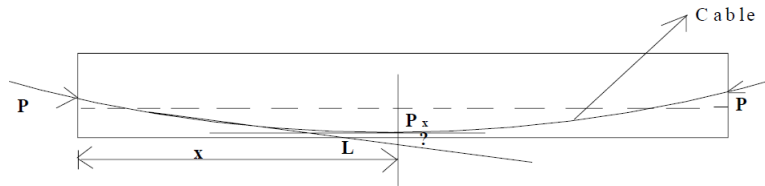
In addition to friction, the stretching must overcome the wobble of the tendon. The wobble refers to the change in position of the tendon along the duct. The losses due to friction and wobble are grouped together under friction.

The friction is proportional to the following variables.

- i. Coefficient of friction (μ) between concrete and steel.

- ii. Resultant of the vertical reaction from the concrete on the tendon (N) generated due to curvature.

The loss due to friction can be considerable for long tendons in continuous beams with changes in curvature. The drop in the prestress is higher around the intermediate supports where the curvature is high. The remedy to reduce the loss is to apply the stretching force from both ends of the member in stages.



$$P_x = P_o e^{-(\mu\alpha+kx)}$$

Where,

P_o → The Prestressing force at the jacking end.

μ → Coefficient of friction between cable and duct

α → The cumulative angle in radians through the tangent to the cable profile has turned between any two points under consideration.

k → Friction coefficient for wave effect.

• Anchorage Slip

In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space. The anchorage block also moves before it settles on the concrete. There is loss of prestress due to the consequent reduction in the length of the tendon.

The total anchorage slip depends on the type of anchorage system. Due to the setting of the anchorage block, as the tendon shortens, there is a reverse friction. Hence, the effect of anchorage slip is present up to a certain length.

The magnitude of loss of stress due to the slip in anchorage is computed as follows: -

If $\Delta \rightarrow$ Slip of anchorage, in mm
 $L \rightarrow$ Length of the cable, in mm
 $A \rightarrow$ Cross-sectional area of the cable in mm²
 $E_s \rightarrow$ Modulus of elasticity of steel in N/mm²
 $P \rightarrow$ Prestressing force in the cable, in N

$$\text{Then, } \Delta = \frac{PL}{AE_s}$$

Hence, Loss of stress due to anchorage slip = $\frac{P}{A} = \frac{E_s \Delta}{L}$;

- **Creep**

The sustained prestress in the concrete of a prestress member results in creep of concrete which is effectively reduces the stress in high tensile steel. The loss of stress in steel due to creep of concrete can be estimated if the magnitude of ultimate creep strain or creep-coefficient is known.

1. Ultimate Creep strain method

The loss of stress in steel due to creep of concrete = $\epsilon_{cc} f_c E_s$, where ϵ_{cc} is Ultimate creep strain for a sustained unit stress, f_c Compressive stress in concrete at the level of steel and E_s Modulus of elasticity of steel

2. Creep Coefficient Method

$$\text{Creep coefficient} = \frac{\text{Creep strain}}{\text{Elastic strain}} = \frac{\epsilon_c}{\epsilon_e}$$

$$\text{Therefore, loss of stress in steel} = \epsilon_c E_s = \phi \epsilon_e E_s = \phi \left(\frac{f_c}{E_c} \right) E_s = \phi f_c \alpha_e$$

Where, $\phi \rightarrow$ Creep Coefficient

$\epsilon_c \rightarrow$ Creep strain

$\epsilon_e \rightarrow$ Elastic strain

$\alpha_e \rightarrow$ Modular ratio

$f_c \rightarrow$ Stress in concrete

$E_c \rightarrow$ Modulus of elasticity of concrete

$E_s \rightarrow$ Modulus of elasticity of steel

The magnitude of creep coefficient varies depending upon the humidity, concrete quality, duration of applied loading and the age of concrete when loaded. The general value

recommended varies from 1.5 for watery situation to 4.0 for dry conditions with a relative humidity of 35%.

- **Shrinkage of Concrete**

1. The loss due to shrinkage of concrete results in shortening of tensioned wires & hence contributes to the loss of stress.
2. The shrinkage of concrete is influenced by the type of cement, aggregate & the method of curing used.
3. Use of high strength concrete with low water cement ratio results in reduction in shrinkage and consequent loss of prestress.
4. The primary cause of drying shrinkage is the progressive loss of water from concrete.
5. The rate of shrinkage is higher at the surface of the member.
6. The differential shrinkage between the interior surfaces of large member may result in strain gradients leading to surface cracking.

Hence, proper curing is essential to prevent cracks due to shrinkage in prestress members. In the case of pretensioned members, generally moist curing is restored in order to prevent shrinkage until the time of transfer. Consequently, the total residual shrinkage strain will be larger in pretensioned members after transfer of prestress in comparison with post-tensioned members, where a portion of shrinkage will have already taken place by the time of transfer of stress. This aspect has been considered in the recommendation made by the code (IS:1343) for the loss of prestress due to shrinkage of concrete and is obtained below:

If ϵ_{cs} is the total residual shrinkage strain = 300×10^{-6} for pretensioning and $\frac{200 \times 10^{-6}}{\log_{10}(t+2)}$ for post-tensioning

Where t is age of concrete at transfer in days

Then the loss of stress = $\epsilon_{cs} \times E_s$ where E_s – modulus of elasticity of steel

- **Relaxation of Steel**

Relaxation of steel is defined as the decrease in stress with time under constant strain. Due to the relaxation of steel, the prestress in the tendon is reduced with time. The relaxation depends on the type of steel, initial prestress and the temperature. The BIS recommends a

value varying from 0 to 90 N/mm² for stress in wires varying from 0.5 f_{pu} to 0.8 f_{pu} Where, f_{pu} is Characteristic strength of pre-stressing tendon.

The typical values for losses are as follows:-

Type of loss	Percentage loss of stress	
	Pre-tensioning	Post-tensioning
Elastic shortening & bending of concrete	4	1
Creep of concrete	6	5
Shrinkage of concrete	7	6
Creep in steel	8	8
Total	25	20

In these recommendation, it is assumed that temporary over stressing is done to reduce relaxation, and to compensate for friction and anchorage losses.

3.b

Relaxation of steel = 5% of initial stress

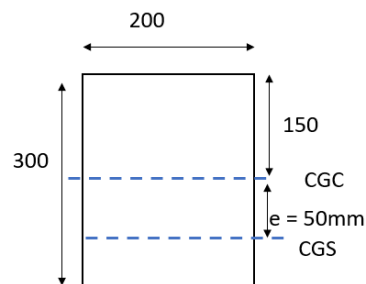
Shrinkage of concrete = 300×10^{-6} for pretensioning

= 200×10^{-6} for post tensioning

Creep coefficient C_c = 1.6

Slip at anchorage = 1 mm

Frictional coefficient for wave effects k = 0.0015/N



Solutions

$$A = 300 \times 200 = 60000 \text{ mm}^2$$

$$P = 1000 \times 320 / 1000 = 320 \text{ kN}, A = 240 \times 600 = 144000 \text{ mm}^2$$

$$I = 200 \times \frac{300^2}{12} = 450 \times 10^6 \text{ mm}^4$$

$$A_s = 320 \text{ mm}^2, l = 10 \text{ m}, m = \frac{E_s}{E_c} = 6.0$$

$$\text{*Stress in concrete at the level of steel} = f_c = \frac{P}{A} + \frac{P e}{I} y = \frac{320 \times 10^3}{320} + \frac{320 \times 10^3 \times 50 \times 50}{450 \times 10^6} = 7.11 \text{ N/mm}^2$$

	Pre tensioning	Post tensioning
Loss due to elastic shortening	$m f_c = 6 \times 7.11 = 42.66$	$\frac{1}{2} m f_c = 21.33 \text{ N/mm}^2$
Loss due to relaxation of steel	Table 6 IS 1343 Page 25 5% of initial stress = 5/100 $\times 1000 = 50 \text{ N/mm}^2$	5% of initial stress = 5/100 $\times 1000 = 50 \text{ N/mm}^2$
Loss due to creep	$Cc m f_c = 1.6 \times 6 \times 7.11 = 68.25 \text{ N/mm}^2$	68.25 N/mm ²
Loss due to shrinkage of concrete	$\delta s E_s = 300 \times 10^{-6} \times 210 \times 10^3 = 63 \text{ N/mm}^2$	$\delta s E_s = 200 \times 10^{-6} \times 210 \times 10^3 = 42 \text{ N/mm}^2$
Loss due to friction (wobble)		$k \times l \times \text{Initial prestress} = 0.0015 \times 10 \times 1000 = 15 \text{ N/mm}^2$
Loss due to anchorage slip		$\frac{\delta l}{l} \times E_c = \frac{1}{10000} \times 210 \times 10^3 = 21 \text{ N/mm}^2$
Total loss	223.916 N/mm ²	217.58 N/mm ²
% Loss	$223.916/1000 \times 100 = 22.3\%$	$217.586/1000 \times 100 = 21.75\%$

4 a

The deflection of a flexural member is calculated to satisfy a limit state of serviceability. It is the general practice, according to various national codes that structural concrete members should be designed to have adequate stiffness to limit deflections which may adversely affect the strength and serviceability of the structures at working load. Suitable control on deflectional is very essential for the following reasons.

- i. Excessive sagging of principal structural members is not only unsightly, but at times also renders the floor unsuitable for the intended use.

- ii. Large deflections under dynamic affects and under the influence of variable loads may cause discomfort to the users.
- iii. Excessive deflections are likely to cause damage to finishes, partitions and associated structures

Factors influencing deflections

1. Imposed load and self-weight
2. Magnitude of prestressing force
3. Cable profile
4. Second moment of area of cross section
5. Modulus of elasticity of concrete
6. Shrinkage, creep, and relaxation of steel stress
7. Span of the members
8. Fixity conditions

In the pre-cracking stage, the whole cross section is effective and the deflection in this stage are computed using the second moment of area of the gross concrete section. The computation of short term or instantaneous deflections which occur immediately after transfer of prestress and on applications of loads is conveniently done by using Mohr's theorems.

In the post cracking stage, a prestressed concrete beam behaves in a manner like that of a reinforced concrete beam and the computation of deflection in this stage is made by considering moment curvature relationships which involves the section, properties of cracked beam.

In both cases, the effect of creep and shrinkage of concrete is to increase the long term deflections under sustained loads which is estimated by using empirical methods that involve the use of effective or long-term modulus of elasticity or by multiplying short term deflection by suitable factors.

4 b

$$A = 120 \times 300 = 36000 \text{ mm}^2, \text{ Moment of Inertia, } I = 120 \times 300^3 / 12 = 270 \times 10^6 \text{ mm}^4$$

$$\text{Span } L = 6 \text{ m} = 6000 \text{ mm}$$

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$e = 50 \text{ mm}$$

$$\text{Modulus of elasticity of concrete, } E_c = 38 \times 10^3 \text{ N / mm}^2$$

Self weight of beam/dead load, $w_d = 24 \times 0.12 \times 0.3 = 0.864 \text{ kN/m} = 0.864 \text{ N/mm}$

Upward Deflection due to initial prestress $= \delta_{pi} = - \frac{P \times e \times L^2}{8 \times E_c \times I} = - 3.94 \text{ mm}$

Downward Deflection due to self weight/dead load $= \delta_d = + \frac{5}{384 \times E_c \times I} w_d L^4 = + 1.42 \text{ mm}$

i) Deflection due to prestress + self weight $= \delta_p + \delta_d = -3.94 + 1.42 = - 2.52 \text{ mm}$

Permissible upward deflection according to IS: 1343 = span/ 300 = 6000/300 = 20 mm.

Here deflection is $-2.52 \text{ mm} < 20 \text{ mm}$. Hence it is safe.

ii) Final deflection under prestress +self weight +imposed load or live load

$w_l = 4 \text{ kN/m}$ (given), $E_c = 38 \times 10^3 \text{ N/mm}^2$, $I = 270 \times 10^6 \text{ mm}^4$, $L = 6000 \text{ mm}$

Deflection due to live load(UDL), $\delta_l = + \frac{5}{384 \times E_c \times I} w_l L^4 = + 6.57 \text{ mm}$

Upward deflection of the beam due to prestress after loss of 20 % (only 80 % of Prestressing force is effective) $= 80 \% \times \delta_p = 0.8 \times - 3.94 = -3.152 \text{ mm}$

Final deflection under prestress +self weight + live load after the loss $= -3.152 + 1.42 + 6.57 = 4.838 \text{ mm}$

Long term deflection- (creep effects) – Use Formula by Lin

$$\alpha_f = \left[+\alpha_{il} - \alpha_{ip} \times \frac{P_t}{P_i} \right] \times (1 + \Phi)$$

initial deflection due to transverse loads(dead + live loads) $\alpha_{il} = \delta_d + \delta_l = +1.42 + 6.57 = 7.99 \text{ mm}$

initial deflection due to prestressing force $\alpha_{ip} = \delta_{pi} = -3.94 \text{ mm}$

$\frac{P_t}{P_i}$ or Loss ratio = 0.8 or 80 % , Creep coefficient , $\Phi = 1.8$, Then

Long term deflection , $\alpha_f =$

$$[+7.99 - 3.94 \times 0.8] \times (1 + 1.8) = 13.54 \text{ mm}$$

Check it with IS: 1343 code limit of span/ 250 = 6000/250 = 24 mm . It is safe against deflection since $13.54 \text{ mm} < 24 \text{ mm}$

The method of estimating the flexural strength of prestressed concrete section is based on the compatibility of strains and equilibrium of forces acting on the section at the stage of failure. The basic theory is applicable to all structural concrete sections, whether reinforced or prestressed, and generally the assumptions are given in IS 1343- 2012 /1999. The ultimate flexural strength of the section is expressed as

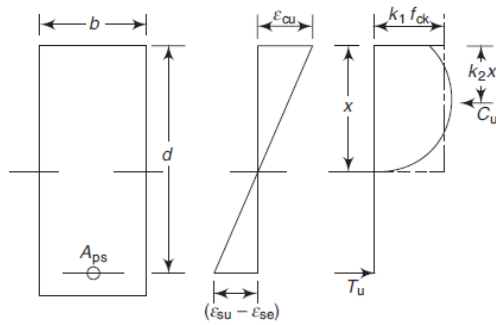


Fig. 7.2 Stress-Strain distribution at failure

Total compressive force $C_u = k_1 f_{ck} b x$

Total tensile force $T_u = A_{ps} f_{pb}$

The ultimate flexural strength of the section is expressed as

$$M_u = A_{ps} f_{pb} (d - k_2 x) = k_1 f_{ck} b x (d - k_2 x)$$

Page 59 Annex B - IS 1343-1980 or Page 51 Annex D IS 1343-2012 – Moment of resistance of PSC beams

D-1 The moment of resistance of rectangular sections or T-sections in which neutral axis lies within the flange may be obtained as follows:

$$M_u = f_{pb} A_{ps} (d - 0.42 x_u)$$

where

- M_u = moment of resistance of the section,
- f_{pb} = tensile stress in the tendon at failure,
- f_{pe} = effective prestress in tendon,
- A_{ps} = area of prestressing tendons in the tension zone,

d = effective depth to the centroid of the steel area, and

x_u = neutral axis depth.

For pretensioned members and for post-tensioned members with effective bond between the concrete and tendons, values of f_{pb} and x_u are given in Table 11. It shall be ensured that the effective prestress, f_{pe} after all losses is not less than $0.45 f_{pu}$, where f_{pu} is the characteristic tensile strength of tendon. Prestressing tendons in the compression zone should be ignored in the strength calculations when using this method.

Table 11 Conditions at the Ultimate Limit State for Rectangular Beams with Pre-tensioned Tendons or with Post-tensioned Tendons having Effective Bond

(Clause D-1)

Sl No.	$\frac{A_{ps} \cdot f_{pu}}{bd \cdot f_{ck}}$	Stress in Tendon as Proportion of the Design Strength		Ratio of the Depth of Neutral Axis to that of the Centroid of the Tendon in the Tension Zone	
		$\frac{f_{pb}}{0.87 f_{pu}}$		x_u/d	
		Pre-tensioning	Post-tensioning with Effective Bond	Pre-tensioning	Post-tensioning with Effective Bond
(1)	(2)	(3)	(4)	(5)	(6)
i)	0.025	1.0	1.0	0.054	0.054
ii)	0.05	1.0	1.0	0.109	0.109
iii)	0.10	1.0	1.0	0.217	0.217
iv)	0.15	1.0	1.0	0.326	0.316
v)	0.20	1.0	0.95	0.435	0.414 ¹⁾
vi)	0.25	1.0	0.90	0.542	0.488 ¹⁾
vii)	0.30	1.0	0.85	0.655	0.558 ¹⁾
viii)	0.40	0.9	0.75	0.783	0.653 ¹⁾

¹⁾ The neutral axis depth in these cases is too low to provide the necessary elongation for developing $0.87 f_{pu}$ stress level. Hence, it is essential that the strength provided exceeds the required strength by 15 percent for these cases.

The ultimate moment of resistance of flanged sections in which neutral axis falls outside the flange is computed by combining the moment of resistance of the web and flange portions and considering the stress blocks shown

If A_{pw} = area of prestressing steel for web

A_{pf} = area of prestressing steel for flange

D_f = thickness of flange

Then, $A_p = (A_{pw} + A_{pf})$

But, $A_{pf} = 0.45 f_{ck} (b - b_w) \left(\frac{D_f}{f_p} \right)$

After evaluating A_{pf} , the value of A_{pw} is obtained as,

$$A_{pw} = (A_p - A_{pf})$$

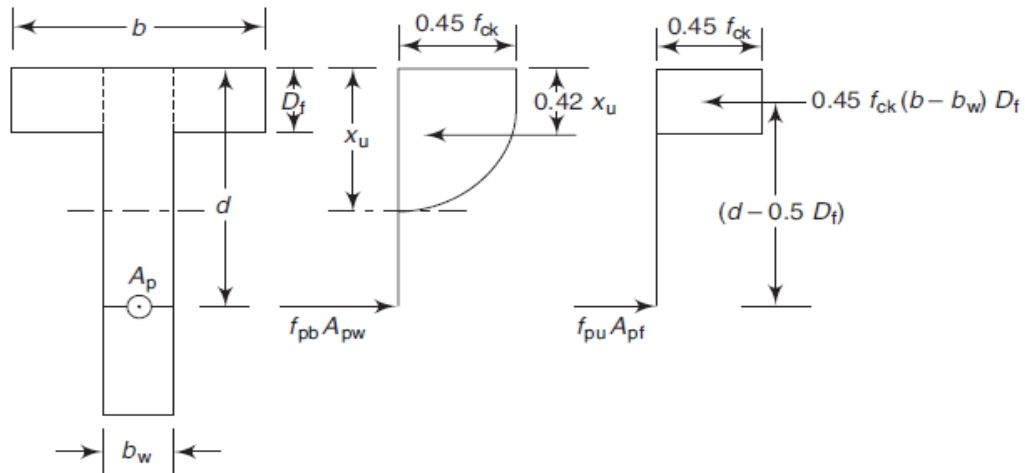


Fig. 7.8 Moment of resistance of flanged sections ($x_u > D_f$) (IS: 1343)

For the effective reinforcement ratio $\left(\frac{A_{ps} \cdot f_{pu}}{b \cdot d \cdot f_{ck}} \right)$, the corresponding values of $\left(\frac{f_{pb}}{0.87 f_{pu}} \right)$ and (x_u/d) are interpolated from Table 11. The ultimate moment of resistance of the flanged section is expressed as,

$$M_u = A_{fb} A_{pw} (d - 0.42 x_u) + 0.45 f_{ck} (b - b_w) D_f (d - 0.5 D_f)$$

The following examples illustrate the application of the IS Code provisions regarding the ultimate flexural strength of bonded prestressed concrete sections.

Steps to calculate ultimate flexural strength of the T section using the provisions of the Indian Standard Code for general cases

Step 1 . Assume depth of neutral axis x_u falling in the flange section, find effective reinforcement ratio

$$\frac{A_{ps} \times f_{pu}}{b \times d \times f_{ck}} =$$

From the Table 11 Page 51, IS 1343 -2012, through interpolation, find $\frac{f_{pb}}{0.87 \times f_{pu}}$, tensile stress in tendon at failure $f_{pb} =$ and depth of neutral axis $\frac{x_u}{d}$, $x_u =$ mm

Step 2. After finding x_u , Case 1 - check whether $x_u < D_f$ or Case 2 - check whether $x_u > D_f$

Step 3 . For Case 1 - Find *Ultimate flexural strength* , $M_U = f_{pb} \times A_{ps} \times (d - 0.42 \times x_u)$
 Available in IS code

For Case 2 - Find *Ultimate flexural strength* ,

$$M_U = f_{pb} \times A_{pw} \times (d - 0.42 \times x_u) + 0.45 \times f_{ck} \times (b - b_w) \times D_f \times (d - 0.5 \times D_f)$$

$$\text{where } A_{pf} = 0.45 \times f_{ck} \times (b - b_w) \times \frac{D_f}{f_{pu}}, \quad A_{pw} = (A_{ps} - A_{pf})$$

Design of bonded post tensioned concrete - Table 11, Page 60 Annex B - IS 1343-1980 or Table 11 Page 51, IS 1343- 2012

5 b

- STEP 1

Assume $x_u < D_f$, then put $b = b_f$ find $\frac{A_{ps} \times f_{pu}}{b_f \times d \times f_{ck}} = \frac{4700 \times 1600}{1200 \times 1600 \times 40} = 0.097$

Find $\frac{x_u}{d}$ through interpolation using Table 11, Page 51 corresponding to $\frac{A_{ps} \times f_{pu}}{b_f \times d \times f_{ck}} =$

$$0.1, \quad \frac{x_u}{d} \approx 0.217 \quad x_u \approx 0.217 \times 1600 \approx 347.2 \text{ mm}$$

But $x_u > D_f$, so assumption is wrong.

- STEP 2

Recalculate effective reinforcement ratio , by putting the values of b_w and A_{pw} in $\frac{A_{pw} \times f_{pu}}{b_w \times d \times f_{ck}}$

$$A_p = (A_{pw} + A_{pf})$$

and $A_{pf} = 0.45 f_{ck} (b - b_w) (D_f / f_{pu})$

$$= (0.45 \times 40)(1200 - 300) \left(\frac{150}{1600} \right)$$

$$= 1518 \text{ mm}^2$$

$$\therefore A_{pw} = (4700 - 1518) = 3182 \text{ mm}^2$$

Hence, the ratio $\left(\frac{A_{pw} \cdot f_{pu}}{b_w \cdot d \cdot f_{ck}} \right) = \left(\frac{3182 \times 1600}{300 \times 1600 \times 40} \right) = 0.265$

From Table 11, the corresponding values are interpolated as,

$$\left(\frac{f_{pb}}{0.87 f_{pu}} \right) = 1.0 \text{ and } \left(\frac{x_u}{d} \right) = 0.56$$

$$f_{pb} = (0.87 \times 1600) = 1392 \text{ N/mm}^2 \text{ and } x_u = (0.56 \times 1600) = 896 \text{ mm}$$

- STEP 3

Calculate Moment of resistance or flexural strength of the T-section

$$\left(\frac{f_{pb}}{0.87 f_{pu}} \right) = 1.0 \text{ and } \left(\frac{x_u}{d} \right) = 0.56$$

$$f_{pb} = (0.87 \times 1600) = 1392 \text{ N/mm}^2 \text{ and } x_u = (0.56 \times 1600) = 896 \text{ mm}$$

$$\begin{aligned} M_u &= f_{pb} A_{pw} (d - 0.42 x_u) + 0.45 f_{ck} (b - b_w) D_f (d - 0.5 D_f) \\ &= (1392 \times 3182)(1600 - 0.42 \times 896) \\ &\quad + (0.45 \times 40(1200 - 300)150(1600 - 0.5 \times 150)) \\ &= [(5420 \times 10^6) + (3705 \times 10^6)] \\ &= (9125 \times 10^6) \text{ Nmm} \\ &= 9125 \text{ kNm} \end{aligned}$$

6

Effective span = 20 m, Live load = 12 kN/m, $f_{ck} = 50 \text{ N/mm}^2$, $f_{ct} = 41 \text{ N/mm}^2$, Loss ratio $\eta = 0.85$, $f_p = 1500 \text{ N/mm}^2$, Area of one cable, $A_p = 12 \times \frac{\pi}{4} \times 7^2 = 461.58 \text{ mm}^2$, Area of the concrete section, $A = b \times d$

- Step 1

Assume breadth of the section as $b = 250 \text{ mm} = 0.25 \text{ m}$, let 'd' be the depth of the section in metres

$$\text{Self-weight of beam / gravity load} = 25 \times 0.25 \times d = 6.25 \times d$$

$$\text{Moment due to self-weight/ gravity, } M_g = \frac{6.25 \times d \times 20^2}{8} = 312.5 \times d \text{ kNm}$$

$$\text{Moment due to live load, } M_q = \frac{12.0 \times 20^2}{8} = 600 \text{ kNm}$$

$$\text{Loss ratio } \eta = 0.85$$

- Step 2

$$\text{Use expression for } Z_b = \frac{M_q + (1-\eta)M_g}{(\eta f_{ct} - f_{tw})} \dots (1)$$

$$\text{Also for rectangular section, } Z_b = \frac{b \times d^2}{6} = \frac{0.25 \times d^2}{6} \dots (2)$$

Equate (1) = (2), put $f_{ct} = 41000 \text{ kN/m}^2$, tensile stress at working load, $f_{tw} = 0$ since it is Type 1 members, all tensile stresses are zero

$$\frac{M_q + (1-\eta)M_g}{(\eta f_{ct})} = \frac{0.25 \times d^2}{6}$$

$$\frac{600 + (1-0.85) \times 312.5 \times d}{(0.85 \times 41000)} = \frac{0.25 \times d^2}{6}$$

Solve for 'd', $d = 0.659 \text{ m} \approx 0.700 \text{ m}$ (upper rounding) -

Dimension of the section is 250×700 mm

- Step 3

$$\text{Solve for } Z_t = Z_b = \frac{0.25 \times d^2}{6} = \frac{0.25 \times 0.7^2}{6} = 0.02 \text{ m}^3$$

- Step 4

Calculate Stress at top fibre f_t

$$f_t = f_{tt} - \frac{M_g}{Z_t} = 0 - \frac{312.5 \times 0.7}{0.02} = -10937 \text{ kN/m}^2 = -10.9 \text{ N/mm}^2$$

- Step 5

Calculate Stress at bottom fibre f_b

$$f_b = \frac{1}{\eta} \left(f_{tw} + \frac{M_g}{Z_b} + \frac{M_q}{Z_b} \right) = \frac{1}{0.85} \left(0 + \frac{312.5 \times 0.7}{0.02} + \frac{600}{0.02} \right) = 48161 \text{ kN/m}^2 = 48.16 \text{ N/mm}^2$$

- Step 6

$$\text{Prestressing Force, } P = \frac{A \times (Z_t f_t + Z_b f_b)}{(Z_t + Z_b)}, A = 0.25 \times 0.7 = \text{ , } Z_b = Z_t = 0.02, f_t = -10937 \text{ kN/m}^2, f_b = 48161 \text{ kN/m}^2$$

$$P = 3257.12 \text{ kN}$$

- Step 7 calculate the eccentricity of the tendon

$$e = \frac{(f_b - f_t) \times Z_t \times Z_b}{A \times (Z_t \times f_t + Z_b \times f_b)} = \frac{(48161 - (-10937)) \times 0.02 \times 0.02}{A \times (Z_t \times f_t + Z_b \times f_b)} = 0.18 \text{ m} = 180 \text{ mm}$$

- Step 8

No of cables required =

$$\text{Prestressing Force, } P = 3257.12 \text{ kN} = 3257000.12 \text{ N}$$

$$\text{Characteristic strength of tendon, } f_p = 1500 \text{ N/mm}^2$$

$$\text{Characteristic strength of tendon, } f_p = \frac{\text{Prestressing Force}}{\text{Total Area of cables}} = 1500 = \frac{3257000}{\text{Total Area of cables}},$$

$$\text{Total Area of cables} = \frac{3257000}{1500} = 2171.41$$

$$\text{Area of one cable} = 12 \times \pi/4 \times 7^2 = 461$$

$$\text{Number of cables} = \text{Total Area of cables} / \text{Area of one cable} = 2171.41 / 461$$

$$= 5 \text{ numbers of cables are needed}$$

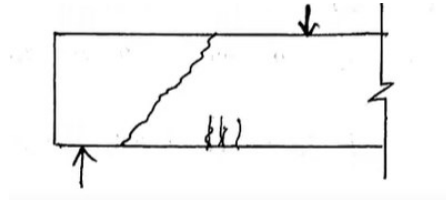
Design is over

7 a

The occurrence of mode of failure depends on span to depth ratio, loading, cross section and amount of anchorage reinforcement

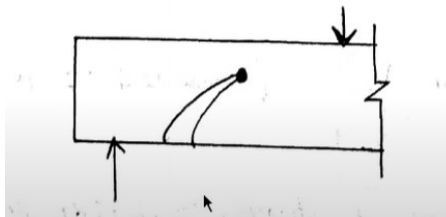
1. Diagonal tension failure

Inclined cracks propagate rapidly due to inadequate shear reinforcement



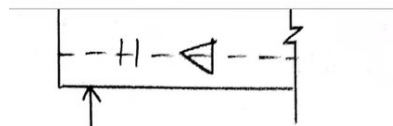
2. Shear compression failure

There is crushing of concrete near the compression flange above the tip of the inclined crack



3. Shear tension failure

Due to inadequate anchorage of longitudinal bars, the diagonal cracks propagate horizontally along the bar

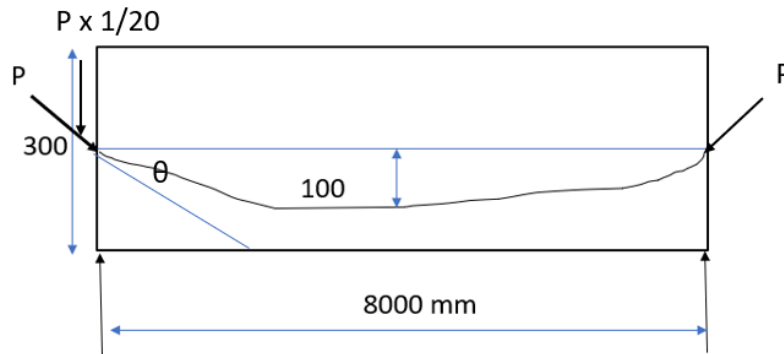


4. Web crushing failure

Concrete in the web crushes due to inadequate web thickness

5. Arched rib failure

For deep beams, web may buckle and subsequently crush. There can be failure due to bearing stress



Self-weight of the beam = $(0.15 \times 0.30 \times 24) = 1.08 \text{ kN/m}$

Total load = $(1.08 + 2.0) = 3.08 \text{ kN/m}$

Eccentricity of cable at the centre of span = 100 mm

Using the concept of load balancing, if P = effective prestressing force,

$$(P \times 100) = \left(\frac{3.08 \times 8000^2}{8} \right)$$

$$\therefore P = 246400 \text{ N} = 246.4 \text{ kN}$$

Calculation of slope of the cable

Calculation of slope at support $y = \frac{4 \times e}{L^2} (Lx - x^2)$

$$\frac{dy}{dx} = \frac{4 \times e}{L^2} (L - 2x)$$

At support $x = 0$, $\theta = \frac{dy}{dx} = \frac{4 \times e}{L} = \frac{4 \times 100}{8000} = \frac{1}{20}$, (for small angle and zero shear let us take directly

$$\sin \theta = \frac{1}{20}$$

Vertical component of prestressing force = $(246.4 \times 1/20) = 12.32 \text{ kN}$

Reaction at support due to dead and live loads = $\left(\frac{3.08 \times 8}{2} \right) = 12.32 \text{ kN}$

Hence, net shear force V at support = 0

Horizontal prestress at support = $\left(\frac{246400}{150 \times 300} \right) = 5.5 \text{ N/mm}^2$

Principal stress at support = 5.5 N/mm^2 (compression)

Solution. Given data:

$$\begin{aligned}b_w &= 150 \text{ mm} & f_{ck} &= 40 \text{ N/mm}^2 \\D &= 300 \text{ mm} & f_{cp} &= 5 \text{ N/mm}^2 \\d &= 250 \text{ mm} & f_y &= 415 \text{ N/mm}^2 \\V &= 130 \text{ kN} & f_t &= 0.24\sqrt{f_{ck}} = 0.24\sqrt{40} = 1.518 \text{ N/mm}^2\end{aligned}$$

According to the recommendations of the IS: 1343-2000 code, the ultimate shear strength of the section uncracked in flexure is given by

$$\begin{aligned}V_{cw} = V_c &= 0.67 b_w D \sqrt{f_t^2 + 0.8 f_{cp} f_t} \\&= [0.67 \times 150 \times 300 \sqrt{1.518^2 + (0.8 \times 5 \times 1.518)}] \\&= 87260 \text{ N} = 87.26 \text{ kN}\end{aligned}$$

Hence, balance shear = $[V - V_c] = [130 - 87.26] = 42.74 \text{ kN}$.

Using 8 mm diameter two-legged stirrups, the spacing of the stirrups is

$$s_v = \left[\frac{A_{sv} 0.87 f_y d}{(V - V_c)} \right] = \left[\frac{2 \times 50.26 \times 0.87 \times 415 \times 250}{42.74 \times 10^3} \right] = 212.28 \text{ mm}$$

Maximum permissible spacing = $(0.75 d) = (0.75 \times 250) = 187.5 \text{ mm}$

Adopt 8 mm diameter two-legged stirrups at 180 mm centres.

8

Solution.

$$\begin{aligned}M_O &= \left(\frac{0.8 f_{ep} I}{y_b} \right) = \left(\frac{0.8 \times 19.3 \times 665 \times 10^8}{755} \right) = 136 \times 10^7 \text{ N mm} \\&\left(\frac{100 A_p}{b_w d} \right) = \left(\frac{100 \times 2310}{150 \times 1100} \right) = 1.40\end{aligned}$$

TABLE 6 DESIGN SHEAR STRENGTH OF CONCRETE, ζ_c , N/mm²
(Clause 22.4.2)

$100 \frac{A_p}{bd}$	CONCRETE GRADE		
	M 30	M 35	M 40 and Above
(1)	(2)	(3)	(4)
0.25	0.37	0.37	0.38
0.50	0.50	0.50	0.51
0.75	0.59	0.59	0.60
1.00	0.66	0.67	0.68
1.25	0.71	0.73	0.74
1.50	0.76	0.78	0.79
1.75	0.80	0.82	0.84
2.00	0.84	0.86	0.88
2.25	0.88	0.90	0.92
2.50	0.91	0.93	0.95
2.75	0.94	0.96	0.98
3.00	0.96	0.99	1.01

NOTE — A_p is the area of prestressing tendon.

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From Table $\tau_c = 0.77 \text{ N/mm}^2$

The flexure-shear resistance of the section is

$$V_{cf} = (1 - 0.55 f_{pc}/f_p) \tau_c b_w d + (M_O/M) V$$

$$\text{or } V_{cf} = \left[\left(1 - \frac{0.55 \times 890}{1500} \right) 0.77 \times 150 \times 1100 \right] + \left[\left(\frac{136 \times 10^7}{2130 \times 10^6} \right) 237 \times 10^3 \right]$$

$$= (240 \times 10^3) \text{N} = 240 \text{ kN}$$

Since the actual shear (237 kN) is less than the ultimate shear resistance of the section, only the minimum shear reinforcements are required.

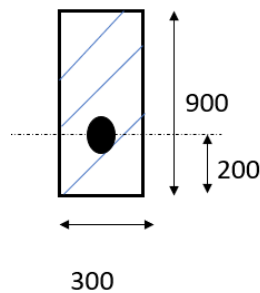
9 a

In a composite construction, precast prestressed members are used in conjunction with the concrete cast in situ, so that the members behave as monolithic unit under service loads. Generally, the high-strength prestressed units are used in the tension zone while the concrete, which is cast in situ of relatively lower compressive strength, is used in the compression zone of the composite members. The composite action between the two components is achieved by

roughening the surface of the prestressed unit on to which the concrete is cast in situ, thus giving a better frictional resistance, or by stirrups protruding from the prestressed unit into the added concrete, or by castellations on the surface of the prestressed unit adjoining the concrete which is cast in situ.

9 b

1. Section properties of the pretensioned beam



$$A = 300 \times 900 = \text{mm}^2$$

$$Z = \left[\frac{300 \times 900^2}{6} \right] = 40.5 \times 10^6 \text{ mm}^3$$

$$P = 2180 \text{ kN} = 2180000 \text{ N}$$

$$e = 900/2 - 200 = 250 \text{ mm}$$

Stresses due to prestressing force or direct stress $= + \frac{P}{A} = \frac{2 \times 2180000}{300 \times 900} = + 16.14 \text{ N/mm}^2$ at the bottom and top fibre

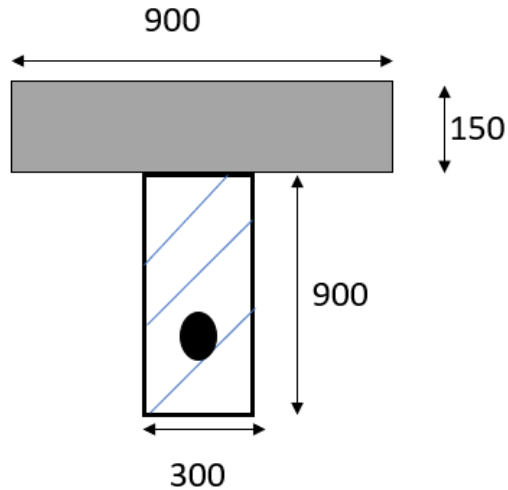
$$\text{Bending stress due to prestressing force at top fibre} = - \frac{P \times e}{Z} = - \frac{2180000 \times 250}{40.5 \times 10^6} = -13.4 \text{ N/mm}^2$$

$$\text{Bending stress due to prestressing force at bottom fibre} = + \frac{P \times e}{Z} = + \frac{2180000 \times 250}{40.5 \times 10^6} = +13.4 \text{ N/mm}^2$$

Moment due to the self-weight of the pretensioned beam, $M_D = 273 \text{ kNm}$ (**given**)

$$\text{Stresses at top and bottom fibre due to dead load of pretensioned beam} = \pm \frac{M_D}{Z} = \frac{273 \times 10^6}{40.5 \times 10^6} = \pm 2.25 \text{ N/mm}^2$$

2. Section properties of composite section



Distance of the centroid from the top fibre $y_t = \frac{900 \times 150 \times 75 + 900 \times 300 \times (150 + 450)}{900 \times 300 + 900 \times 150} = 425 \text{ mm}$

Moment of inertia , $I = \frac{900 \times 150^3}{12} + 900 \times 150 \times (425 - 75)^2 + \frac{900 \times 300^3}{12} + 900 \times 300 \times (600 - 425)^2 = 4.43 \times 10^{10} \text{ mm}^4$

Section moduli , $Z_t = \frac{4.43 \times 10^{10}}{425} = 101.88 \times 10^6 \text{ mm}^3$

$Z_b = \frac{4.43 \times 10^{10}}{(1050 - 425)} = 69.28 \times 10^6 \text{ mm}^3$

3. Live load stresses in the composite section

Maximum live load moment, $M_L = 750 \text{ kNm}$ (given)

At top $= \frac{M_L}{Z_t} = \frac{750 \times 10^6}{101.88 \times 10^6} = 7.36 \text{ N/mm}^2$ (compression)

At bottom $= \frac{M_L}{Z_b} = \frac{750 \times 10^6}{69.28 \times 10^6} = -10.82 \text{ N/mm}^2$ (tension)

Since the pretensioned beam is propped, the self-weight of the slab acts on the composite section

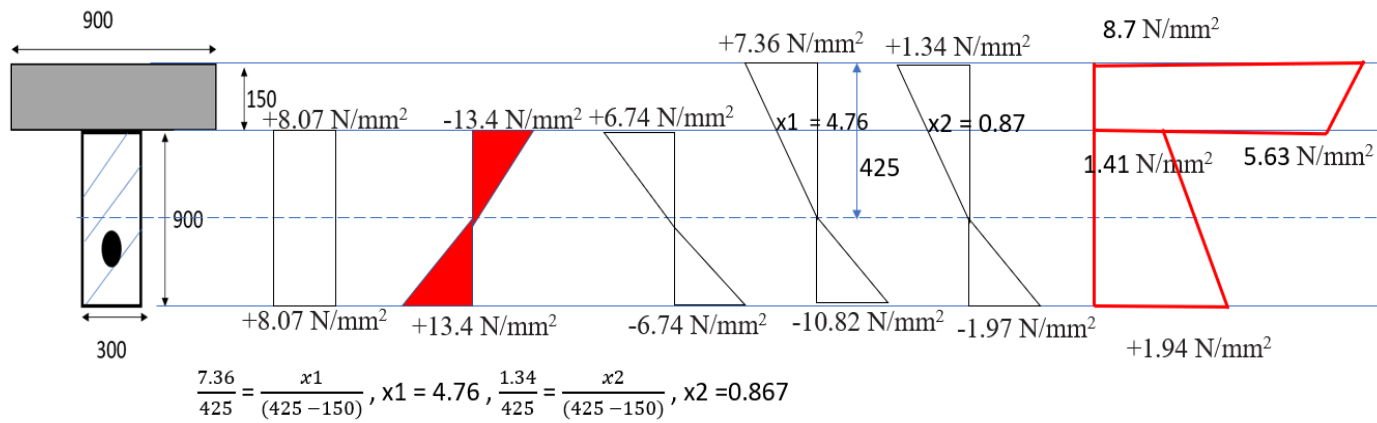
Moment due to dead load of the top slab, $M_d = 136.5 \text{ kNm}$ (given)

Stress due to this moment in the composite section

At top $= \frac{M_d}{Z_t} = \frac{136.5 \times 10^6}{101.88 \times 10^6} = 1.34 \text{ N/mm}^2$ (compression)

At bottom $= \frac{M_d}{Z_b} = \frac{136.5 \times 10^6}{69.28 \times 10^6} = - 1.97 \text{ N/mm}^2$ (tension)

The distribution of stresses for the various stages of loading for the propped and unpropped construction is shown in the figure below



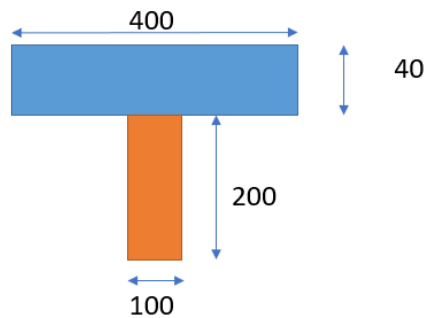
At the top level of PSC beam = $8.07 - 13.4 + 6.74 = 1.41 \text{ N/mm}^2$

At the bottom level of PSC beam = $8.07 + 13.4 - 6.74 - 10.82 - 1.97 = 1.94 \text{ N/mm}^2$

At top level of slab = $7.36 + 1.34 = 8.7 \text{ N/mm}^2$ at bottom level of slab = $4.76 + 0.87 = 5.63 \text{ N/mm}^2$

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Solution



$$P = 150 \text{ kN}$$

$$e = 33.33 \text{ mm}$$

$$\text{Self-weight of precast beam} = 0.48 \text{ N/mm}$$

$$\text{Self-weight of in situ cast slab} = 0.384 \text{ N/mm}$$

$$\text{Imposed load on composite section} = 3.2 \text{ N/mm}$$

$$\text{Moment of Inertia for pretensioned rib} = \frac{100 \times 200^3}{12} = 66.66 \times 10^{-6} \text{ mm}^4$$

$$y = \frac{400 \times 40 \times 20 + 200 \times 100 \times (40 + 100)}{400 \times 40 + 200 \times 100} = 86.67 \text{ mm}$$

$$I = \frac{400 \times 40^3}{12} + 400 \times 40 \times (86.67 - 20)^2 + \frac{100 \times 200^3}{12} + 100 \times 200 \times (140 - 86.67)^2 = 196.8 \times 10^6 \text{ mm}^4$$

$$\text{Moment of Inertia for composite members} = 196.8 \times 10^6 \text{ mm}^4$$

$$\text{Modulus of elasticity, } E = 35 \times 10^3 \text{ N/mm}^2$$

a) Unpropped construction:

$$\text{Deflection due to prestress} = \frac{PeL^2}{8EI} = \frac{150 \times 10^3 \times 33.33 \times 5000^2}{8 \times 35 \times 10^3 \times 66.66 \times 10^6} = -6.7 \text{ mm (upward)}$$

$$\text{Effective deflection after losses} = (0.85 \times 6.7) = -5.7 \text{ mm}$$

$$\text{Deflection due to self-weight of precast beam} = \frac{5}{384 \times E_C \times I} w_d L^4 = \frac{5 \times 0.48 \times 5000^4}{384 \times 35 \times 10^3 \times 66.66 \times 10^6} \\ = +1.7 \text{ mm}$$

$$\text{Deflection of precast beam due to self-weight of cast in situ slab acting on it} = \frac{1.7 \times 0.384}{0.48} = \\ +1.34 \text{ mm}$$

$$\text{Deflection of composite beam due to live load} = \frac{5 \times 3.2 \times 5000^4}{384 \times 35 \times 10^3 \times 196.8 \times 10^6} = +3.83 \text{ mm}$$

$$\text{Resultant deflection under service loads} = -5.7 + 1.7 + 1.34 + 3.83 = 1.17 \text{ mm}$$

b) Propped construction:

$$\text{Deflection due to prestress} = \frac{PeL^2}{8EI} = \frac{150 \times 10^3 \times 33.33 \times 5000^2}{8 \times 35 \times 10^3 \times 66.66 \times 10^6} = -6.7 \text{ mm (upward)}$$

$$\text{Effective deflection after losses} = (0.85 \times 6.7) = -5.7 \text{ mm}$$

$$\text{Deflection due to self-weight of precast beam} = \frac{5}{384 \times E_C \times I} w_d L^4 = \frac{5 \times 0.48 \times 5000^4}{384 \times 35 \times 10^3 \times 66.66 \times 10^6} \\ = +1.7 \text{ mm}$$

$$\text{Deflection of composite beam due to live load} = \frac{5 \times 3.2 \times 5000^4}{384 \times 35 \times 10^3 \times 196.8 \times 10^6} = +3.83 \text{ mm}$$

$$\text{Deflection of composite beam due to self-weight of cast in situ slab} = \frac{5 \times 0.384 \times 5000^4}{384 \times 35 \times 10^3 \times 196.8 \times 10^6} = \\ +0.47 \text{ mm}$$

$$\text{Resultant deflection under service loads} = -5.7 + 1.7 + 0.47 + 3.83 = 0.30 \text{ mm}$$

According to IS: 1343, the maximum permissible deflection under service loads is limited to a value of $(\text{span}/250) = (5000/250) = 20 \text{ mm}$. However, this value includes the long-term effects of creep and shrinkage. If the creep coefficient is assumed to be 3.0, final resultant deflections for unpropped and propped constructions are 3.51 and 0.9 mm, respectively, which are well within the permissible limits specified in the code.

