15CV82 DESIGN OF PRESTRESSED CONCRETE ELEMENTS

1 a. Define Prestressed Concrete. Explain briefly Pretensioned and Post tensioned members. **(03 Marks)**

b. A PSC unsummetrical I section beam span 8m support a load 20kN/m , Top flange 300 × 60mm ; Bottom flange 100 × 60mm ; Web 80 × 280mm ; P = 100kN located at 50mm from bottom. Find stress at mid span. Given $A = 46.4 \times 10^3 mm^2$, NA 156mm from top $I_{xx} = 760.45 \times 10^6 mm^4$. **(05 Marks)**

c. A PSC inverted T section web 300 × 900mm , Flange 300 × 600mm , Simply supported over a span of 15m. It is tensioned by 3 cable each containing 12 wires of 7mm diameter placed at 150mm from Soffit. Calculate Max UDL the beam can carry if Max tension and compression is limited to 1MPa and 15Mpa. Loss of pre stress 15%. **(08 Marks)**

2 a. Explain Load Balancing Concept. **(03 Marks)**

b. A PSC section 400 × 600mm is prestressed by 1920kN by a parabolic cable having max eccentricity 200mm at mid span 100mm at support. Find stress at mid span only by load balancing concept. **(07 Marks)**

c. A PSC beam with single overhanging is simply supported at A, Continuous over B span AB 8m and over hanging BC 2m , C/S of beam 300 × 900mm , Live load at 3.52kN/m. Suggest a suitable cable profile. Take prestressing force 500kN. **(06 Marks)**

3 a. Define Loss of Pre-stress. Briefly explain different loss with suitable formula. **(05 Marks)**

b. A post tensioned PSC beam 250 × 400mm is prestressed by 12 wires of 7mm diameter stressed to $1200 N/mm^2$. The cable is parabolic with eccentricity 120mm at centre and zero at support span 10m. Calculate loss of pre-stress and % loss of pre-stress. Take $\mu = 0.55$, $K = 0.0015/m$, $\varepsilon_{cs} = 1.354 \times 10^{-4}$, $\phi = 1.6$, $E_s = 2 \times 10^5 N/mm^2$, $E_c = 31.6 \times 10^3 N/mm^2$. , Relaxation 5% , Slip 2mm. **(06 Marks)**

c. A post tensioned PSC member 400 × 400mm span 12m is pre-stressed by 4 – cable each having area $200mm^2$ initial pre-stress $1000 N/mm^2$. Find the loss of pre-stress when cable is tensioned one by one. Take $\varepsilon_{cs} = 0.003$, $\phi = 2.5$, $m = 6$, $\Delta = 3mm$, $E_s = 2.1 \times 10^5 N.mm^2$. Eccentricity of cable is zero. **(05 Marks)**

4 a. A simply supported 6m beam post tensioned by two cable having 100mm eccentricity below NA at centre. The first cable is parabolic with an eccentricity 100mm above NA at support. The second cable is straight. C/s of each cable is $100mm^2$, Initial pre-stress is $1200 N/mm^2$, $A = 2 \times 10^4 mm^2$, Radius of gyration 120mm. The beam support a load of 20kN each at middle third point $E_c = 38 kN/mm^2$. Calculate Short term and Long term deflection. Take $\phi = 2$. Loss of pre-stress 20%. **(10 Marks)**

b. A PSC beam 200 × 400mm span 10m is pre-stressed by a parabolic cable at 80mm from bottom at mid span and 125mm from top at support force in the cable 400kN , $E_c = 35 kN/mm^2$. Calculate i) Deflection at mid span to support its self weight.
ii) Point load to be applied at centre for zero deflection. **(06 Marks)**

5 a. A pretensioned T – section flange 1200mm × 150mm , Web 300mm × 1500mm , Steel area 4700mm^2 , located at a depth 1600mm M40 conc. Find Ultimate moment tensile strength of steel 1600N/mm^2. **(10 Marks)**

b. A post tension unbounded rectangular beam 400mm × 800mm effective depth cross sectional area of cable 2840mm^2 , Effective pre-stress 900N/mm^2 , Span 16m. Find Ultimate moment. Take M40 conc. **(06 Marks)**

6 Design a PSC beam E-span 15m live load 20kN/m , Loss of pre-stress 20% , Permissible comp stress in conc at transfer and at working load 15N/mm^2 and 12N/mm^2. No tensioned is allowed. Take b = 400mm. **(16 Marks)**

7 a. Explain Shear failure is PSC member. **(04 Marks)**

b. A post tensioned beam 200 × 400mm span 10m , Load 8kN/m , P = 500kN. The cable is parabolic with 140mm eccentricity at mid span and zero at support. Calculate
i) Principal stress at support ii) Find principal stress in absence of pre-stress. **(12 Marks)**

8 a. The cross section of a bridge girder T beam, top flange 600mm × 230mm , Web 150mm , NA is at 545mm from top of area 328500mm^2 , MI = 665 × 10^{8}mm^4 , Second moment of area , $a\bar{y}$ = 665 × 10^8 mm^3 , Span 25m , Cable area 2300mm^2 , Parabolic cable with e = 650mm at mid span and 285 at support effective pre stress 900N/mm^2 , Tensile stress is concrete 1.6N/mm^2. Find Max UDL the beam can support if load factor is 2.0. Assume no loss of pre-stress. **(08 Marks)**

b. A PSC beam 250mm × 1500mm carries an effective pre-stress 1362kN , Shear force 771kN Slope of cable at support $\theta = \frac{1}{6}$, Extreme fiber stress 7N/mm^2 at top and zero at bottom principal tensile stress 0.7N/mm^2. Design Shear reinforcement. **(08 Marks)**

9 a. Explain Anchorage Zone stresses and stress distribution in end block with suitable figure. **(04 Marks)**

b. What are the methods available for calculating Anchorage Zero stress? Explain Indian Code provision. **(04 Marks)**

c. The end block of a post tensioned beam 300 × 300mm subjected to a anchorage force of 32.8kN by a Freyssinet anchorage area 11720mm^2. Design Anchorage reinforcement. **(08 Marks)**

10 a. Explain Composite Construction in PSC. Mention the advantages of precast PSC member. **(04 Marks)**

b. A precast pre-tensioned beam 100mm × 200mm E-span 5m is pre-stressed by a force of 150kN. Loss of pre-stress 15%. The beam is incorporated in a composite T beam by casting a top flange of breadth 400mm thickness 40mm. Live load 8kN/m^2. Assuming unproved condition. Find the stress developed. **(12 Marks)**

SOLUTIONS

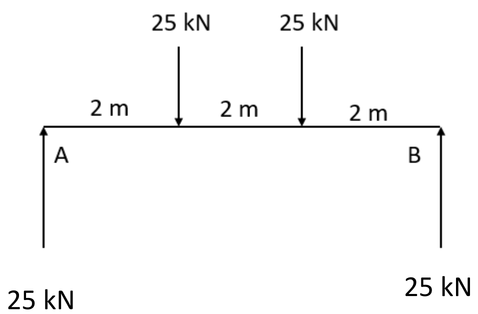
1 a Prestressed concrete is basically concrete in which internal stresses of a suitable magnitude and distribution are introduced so that the stresses resulting from external loads are counteracted to a desired degree. The initial load or 'prestress' is applied to enable the structure to counteract the stresses arising during its service period.

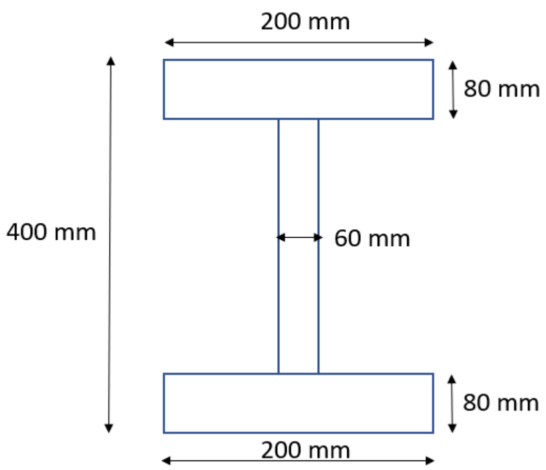
PRETENSIONING	POST TENSIONING
1. Concrete is prestressed with tendon before it is placed in position	Prestressing is done after concrete attains sufficient strength
Pretensioning is developed due to bonding between steel and concrete	2. Post tensioning is developed due to bearing

Preferred for small structural element and easy to transport	3. Preferred for large structural element and difficult to transport
Similar structural members are casted	4. Members are casted according to market requirements
Casted in moulds	5. Cables are used in place of wires and jacks for stretching
Greater certainty about the prestressing force	6. More economical to use a few cables or bars with large forces in than many small ones
Suitable for bulk production	7. Suited for medium to long-span in situ work where the tensioning cost is only a small proportion of the cost of the whole job

1 b

Solutions (Diagram should be drawn using the data from the given question)





Centroid from top as well as bottom, **top, $y = y_t = y_b$ = 200 mm**

BM @ Mid span due to external load/service load, $M_L = 25 \times 3 - 25 \times 1 = 50$ kNm

Calculate moment of interia , I (only for symmetric section)

$$I = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{200 \times 400^3}{12} - \frac{140 \times 240^3}{12} = 905.387 \times 10^6 \text{ mm}^4$$

$$Z_t = Z_b = \frac{I}{y} = 4.52 \times 10^6 \text{ mm}^3 \text{ (symmetric section)}$$

A = 2 × 200×80 +60× 240 = 46400mm^2 =0.0464 m^2

Self-weight of beam = 0.0464 × 25 = 1.16 kN/m

BM due to self-weight @ midspan, $M_D = \frac{1.16 \times 6^2}{8}$ = 5.22 kNm

Total BM @ Midspan, $M_s = M_D + M_L$ = 50 + 5.22 = 55.22 kNm

Let P_i be initial prestressing force and 'e' be the corresponding eccentricity, the final pre-stressing force at service condition(WHEN WE APPLY EXTERNAL LOAD) it will be 0.8 P_i (because there is a loss of prestress by 80 % = 80 % x P_i = 0.8 P_i)

At transfer (only prestress + self-weight)

At transfer(while applying prestressing force) the maximum permissible tensile stress of 1N/mm^2 will be at top of beam, since the tension due to prestressing force will be greater than the tension due to self-weight.(M_s = 5.22kNm)

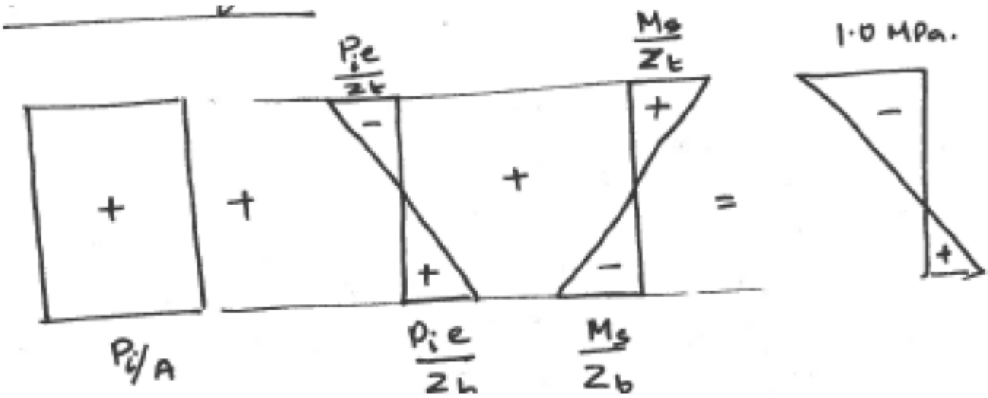
(Tension - negative sign)

At top, stresses can be expressed as

$$\frac{P_i}{A} - \frac{P_i e}{Z_t} + \frac{M_S}{Z_t} = -1 \text{ .... (1)}$$

$$\frac{P_i}{46400} - \frac{P_i e}{4.52 \times 10^6} + \frac{5.22 \times 10^6}{4.52 \times 10^6} = -1$$

$$\frac{P_i}{46400} - \frac{P_i e}{4.52 \times 10^6} = -2.153 \text{........(1)}$$



At service (prestress + self-weight + live load)

The BM due to live load shall be added to this self wt, M_S = 55.22 kNm. In addition, the effective prestressing force will be 0.8 P_i

The max tensile stresses 0.5 N/mm^2 will now occurs at bottom. (because when service/live/external load is applied, beam has tendency to bend due to which maximum tensile stress will be developed at bottom)

At bottom the stresses can be expressed as

$$\frac{0.8\, P_i}{A} + \frac{0.8\, P_i e}{Z_b} - \frac{M_S}{Z_b} = -0.5$$

$\frac{0.8\,P_i}{46400} + \frac{0.8\,P_i e}{4.52 \times 10^6} - \frac{55.22 \times 10^6}{4.52 \times 10^6} = -0.5$

$\frac{0.8\,P_i}{46400} + \frac{0.8\,P_i e}{4.52 \times 10^6} = 11.71$……..(2)

Solving equations (1) and (2) , find P_i and 'e'

$P_i = 290.166 \times 10^3$N

e = 130.86 mm

1 c

Eff span = 16 m

$f_t = 0$, $f_b = 12$ MPa

A = 1200×200 + 1000 × 240 = 480000mm^2

$\mathbf{y_t} = \frac{1200 \times 200 \times 100 + 1000 \times 240 \times (200+500)}{480000}$ **= 400 mm , y_b = 800 mm**

$\mathbf{I_{xx}} = \frac{1200 \times 200^3}{12} + (\mathbf{1200} \times \mathbf{200}) \times ((\mathbf{400} - \mathbf{100})^2) + \frac{240 \times 1000^3}{12} + (\mathbf{240} \times \mathbf{1000}) \times ((\mathbf{700} - \mathbf{400})^2)$

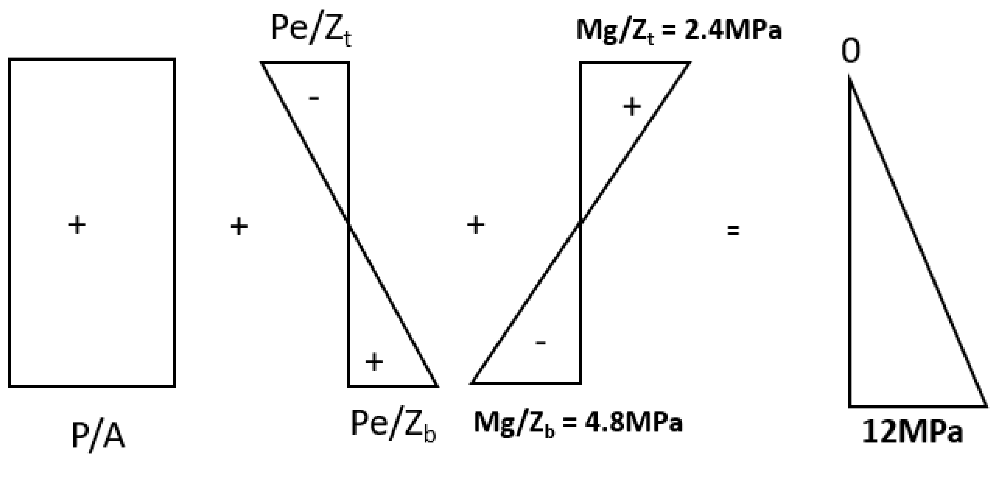
$= 6.4 \times 10^{10}$mm^4

$Z_t = \frac{I_{xx}}{y_t} = \frac{6.4 \times 10^{10}}{400} = 160 \times 10^6$mm^3

$Z_b = \frac{I_{xx}}{y_b} = \frac{6.4 \times 10^{10}}{800} = 80 \times 10^6$mm^3

Self-weight of the beam = 25 × **480000/1000^2 = 12 kN/m**

Moment due to self-weight, Mg = $\frac{12 \times 16^2}{8}$ = 384kNm



At top stresses can be

$\frac{P_i}{A} - \frac{P_i e}{Z_t} + \frac{M_g}{Z_t} = 0$, $\frac{P_i}{480000} - \frac{P_i e}{160 \times 10^6} + \frac{384 \times 10^6}{160 \times 10^6}$ = 0, 333.3 P – Pe = - 384 × 10 ^{6} …. (1)

At bottom stresses can be

$\frac{P_i}{A} + \frac{P_i e}{Z_t}$ - $\frac{384 \times 10^6}{160 \times 10^6} = 12$

166.67 P +Pe = 1.344 × 10 ^{9} …….(2)

Solving (1) and (2)

P = 1920kN, e = 533.33 mm

2 a

- ✓ **Load Balancing concept**
- The concept is useful in selecting the tendon profile, which can provide the most desirable system of forces in concrete.
- The **cable profile in a prestressed member** corresponds to the **shape of the bending moment diagram** resulting from the external loads.
- Thus, if the beam supports two concentrated loads, the cable should follow a trapezoidal profile.
- If the beam supports uniformly distributed loads, the corresponding tendon should follow a parabolic profile.
- It is possible to select suitable cable profiles in a prestressed concrete member such that the transverse component of the cable force balances the given type of external loads.

- Straight concentric cables induce only horizontal reactions or pure axial forces at the ends.
- A straight eccentric cable induces axial force plus external moment causing a hogging moment
- Triangular profiles induce inclined forces at the ends and vertical upward reaction at the centre (2Psin θ)
- A trapezoidal profile induced inclined forces at the end and two vertical upward reaction at the point of change in the angle
- A parabolic profile induces inclined forces at the end and an upward UDL

2 b

A = 300 × 750 = 225 × 10^3 mm^2

e = 275 mm @midspan

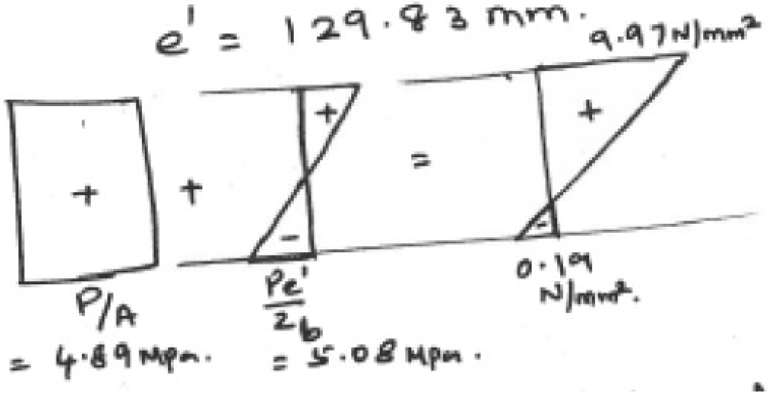
UDL = 30kN/m, l = 10 m

Total load = 30 +(0.3 × 0.75 × 25) = 35.625 kN/m

Total Moment, M = 35.625 × $10^2/8$ = 445.3125kNm

Zt = Zb = 300 × $750^2/6$ = 28.125 × 10^6 mm^3

$e' = \frac{M}{P} - e = \frac{445.3125 \times 10^6}{1100 \times 10^3} - 275 = 129.83\ mm$

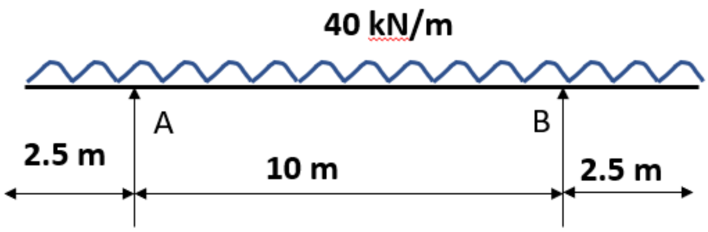


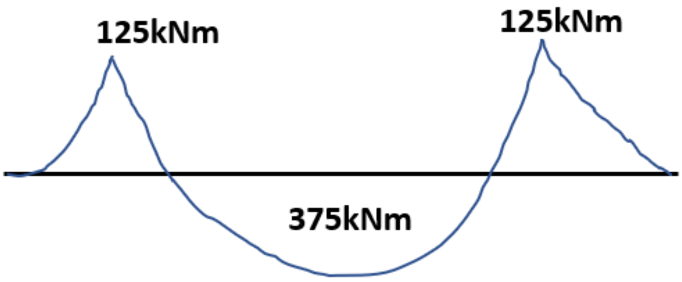
In above the problem, if effective PSF is required to balance the external load, P_e e = M , P_e = 445.3125 × 10^6/ 275×1000 = 1619.32kN

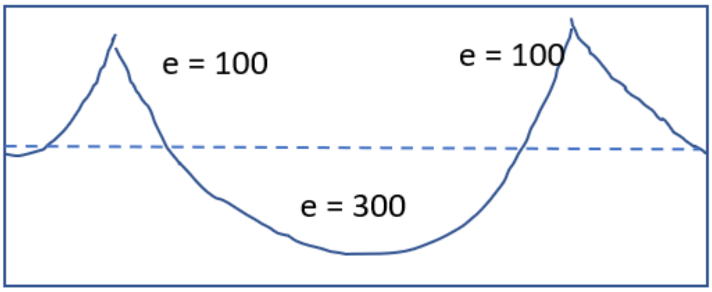
Case 2 To balance a total load of 50kN/m on the beam,

P × e = M , Here M = 50 × $10^2/8$ = 625kNm, P = 625/0.275 = 2272.73 kN

2 c







Ra = Rb = 300kN

BM @ supports Ms = 40 x $2.5^2/2$ = 125kNm

BM @ centre Mc = 300 x 5 – 40 x $7.5^2/2$ = 375 kNm

Eccentricity at 1) supports , e = Ms / P = 100 mm 2) centre = e = Mc / P = 300 mm

3 a

The **various reductions of the prestressing force** are termed as the losses in prestress. The losses are broadly classified into two groups, **immediate and time-dependent**. The **immediate losses** occur during prestressing of the tendons and the transfer of prestress to the concrete member. The **time-dependent losses** occur during the service life of the prestressed member.

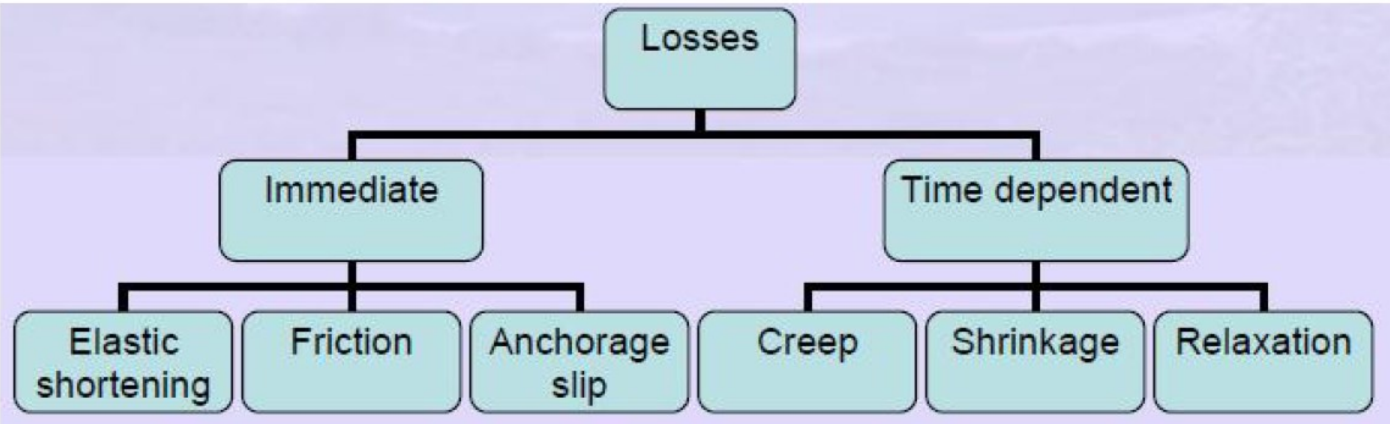
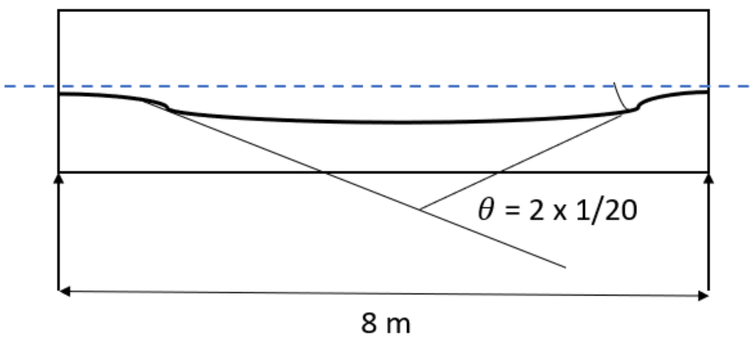


Table 5.1 *Types of losses of prestress*

S. No.	*Pretensioning*	*S. No.*	*Post-tensioning*
1.	Elastic deformation of concrete	1.	No loss due to elastic deformation if all the wires are simultaneously tensioned. If the wires are successively tensioned, there will be loss of prestress due to elastic deformation of concrete
2.	Relaxation of stress in steel	2.	Relaxation of stress in steel
3.	Shrinkage of concrete	3.	Shrinkage of concrete
4.	Creep of concrete	4.	Creep of concrete
		5.	Friction
		6.	Anchorage slip

3 b



Es = 210 kN/mm^2, P = 800 × 1200 = 960 kN, μ = 0.5, k = 0.0015/m, l = 8000 mm, δ_L = 2 mm, θ = change in slope = $2 \times \frac{1}{20} = 0.1$

Solution

If P_x = pre-stressing force (stress) in the cable at the farther end, $P_x = P_o e^{-(\mu\alpha + kx)}$

For small values of $(\mu\alpha + kx)$, we can write

$P_x = P[1 - (\mu\alpha + kx)]$

Loss of stress $= P_0(\mu\alpha + kx)$

a) Loss due to friction $= P_o(\mu\theta + kL) = 1200 \times (0.5 \times 0.1 + 0.0015 \times 8) = 74.4$ N/mm^2

b) Loss due to anchorage slip $= \frac{\delta L}{L} \times E_S = \frac{2}{8000} \times 2 \times 10^3 = 52.5$ N/mm^2

Total loss due to slip and friction = 126.9N/ mm^2

c) Final prestressing force $= (1200 - 126.9)800 = 858.48$kN

% loss $= \frac{126.9}{1200} \times 100 = 10.54$ %

3c

Solution.

Equation of a parabola is given by:

$$y = (4e/L^2)x(L - x)$$

Slope at ends (at $x = 0$) $= dy/dx = (4e/L^2)(L - 2x) = (4e/L)$

For Cable 1

$$\text{Slope at end} = \left(\frac{4 \times 10}{10 \times 100}\right) = 0.04$$

$\therefore$ Cumulative angle between tangents, $\alpha = (2 \times 0.04) = 0.08$ radians

For Cable 2

$$\text{Slope at end} = \left(\frac{4 \times 5}{10 \times 100}\right) = 0.02$$

$\therefore$ Cumulative angle between tangents, $\alpha = (2 \times 0.02) = 0.04$ radians
Initial prestressing force in each cable, $P_0 = (200 \times 1200) = 24{,}0000$ N
If P_x = prestressing force (stress) in the cable at the farther end,

$$P_x = P_o e^{-(\mu\alpha + kx)}$$

For small values of $(\mu\alpha + Kx)$, we can write

$$P_x = P_o[1 - (\mu\alpha + kx)]$$

$$\text{Loss of stress} = P_o(\mu\alpha + kx)$$

$$\text{Cable 1} = P_o(0.35 \times 0.08 + 0.0015 \times 10) = 0.043\ P_o$$

$$\text{Cable 2} = P_o(0.35 \times 0.04 + 0.0015 \times 10) = 0.029\ P_o$$

$$\text{Cable 3} = P_o(0 + 0.0015 \times 10) = 0.015\ P_o$$

If P_o = Initial stress = 1200 N/mm^2

Cable No.	*Loss of Stress* (N/mm^2)	*Percentage Loss*
1	51.6	4.3
2	34.8	2.9
3	18.0	1.5

4 a

A = 120 x 300 = 36000 mm^2 , Moment of Inertia, I = 120 x 300^3 / 12 = 270 x 10^6 mm^4

Span L = 6 m = 6000 mm

P = 180kN = 180 x 10^3 N

e = 50 mm

Modulus of elasticity of concrete, E_c = 38 x 10^3 N / mm^2

Self weight of beam/dead load, w_d = 24 x 0.12 x 0.3 = 0.864 kN/m = 0.864 N/mm

Upward Deflection due to initial prestress = δ_{pi} = - $\frac{P \times e \times L^2}{8 \times E_C \times I}$ = - 3.94 mm

Downward Deflection due to self weight/dead load = δ_d = + $\frac{5}{384 \times E_C \times I} w_d L^4$ = + 1.42 mm

i) Deflection due to prestress + self weight = $\delta_p + \delta_d$ = -3.94 + 1.42 = - 2.52 mm

Permissible upward deflection according to IS: 1343 = span/ 300 = 6000/300 = 20 mm.

Here deflection is -2.52 mm < 20 mm . Hence it is safe.

ii) Final deflection under prestress +self weight +imposed load or live load

w_l = 4 kN/m (given), E_c = 38 x 10^3 N / mm^2 , I = 270 x 10^6 mm^4 , L = 6000 mm

Deflection due to live load(UDL), δ_l = + $\frac{5}{384 \times E_C \times I} w_l L^4$ = + 6.57 mm

Upward deflection of the beam due to prestress after loss of 20 % (only 80 % of Prestressing force is effective) = 80 % $\times$ δ_p = 0.8 x - 3.94 = -3.152 mm

Final deflection under prestress +self weight + live load after the loss = -3.152 + 1.42 + 6.57 = 4.838 mm

iii) Long term deflection- (creep effects) – Use Formula by Lin

$$\alpha_f = \left[+\alpha_{il} - \alpha_{ip} \times \frac{P_t}{P_i}\right] \times (1 + \Phi)$$

initial deflection due to transverse loads(dead + live loads) α_{il} = δ_d + δ_l = +1.42 +6.57 = 7.99 mm

initial deflection due to prestressing force α_{ip} = δ_{pi} = -3.94 mm

$\frac{P_t}{P_i}$ or Loss ratio = 0.8 or 80 % , Creep coefficient , Φ = 1.8 , Then

$Long\ term\ deflection,\ \alpha_f$ =

[+7.99 – 3.94 x 0.8] x (1 + 1.8) = 13.54 mm

Check it with IS: 1343 code limit of span/ 250 = 6000/250 = 24 mm . It is safe against deflection since 13.54 mm < 24 mm

4 b

Self-weight of beam = w = 45 × 10 ^{3} × 1 × 24 = 1.08 kN/m

As = 7 × π × 7^2 /4= 269.39 mm^2 f_{si} = 1250 N/mm^2 , e = 60 mm , r = 86.6 mm, l = 10.5 m, Es = 210 kN/mm^2 Ec = 5000 × √ 45 = 33541.02 N/mm^2, I = $A_c r^2$ = 45 × 10^3 × 86.6^2 = 337.48 × 10^6 mm^4

Po = As f_{si} = 1250 × 269.39 = 336.73 kN

Downward deflection due to self-weight = $\frac{5}{384\,Ec\,I} w_d l^4 = \frac{5}{384 \times 33541.02 \times 337.4802 \times 10^6} \times 1.08 \times 9500^4$

= 10.12 mm

Upward deflection due to prestressing force = $\frac{5\,P\,e\,l^2}{48 E_c I} = \frac{5 \times\ 336.7 \times 60 \times 9500^2}{48 \times 33541.02 \times 337.4802 \times 10^6}$= 16.78 mm

Downward deflection due to live load = $\frac{5}{384\,Ec\,I} w_l l^4 = \frac{5}{384 \times 33541.02 \times 337.4802 \times 10^6} \times 4 \times 9500^4$

= 37.48 mm

Net deflection of the beam (self-weight + prestress) = 1.12 – 16.78 = -6.66 mm

Net deflection of the beam (self-weight + prestress + live load) = 1.12 – 16.78 + 37.48 = 30.82 mm

5 a

Given data: f_{ck} = 40 N/mm^2 , b = 400 mm, d = 800 mm , l = 16 m, A_{ps} = 2840 mm^2,
Effective prestress in the steel f_{pe} = 800 N/mm^2

As per IS 1343 – 2012 Page 51 , Annex D

> shall be ensured that the effective prestress, f_{pe} after all losses is not less than 0.45 f_{pu}, where f_{pu} is the characteristic tensile strength of tendon. Prestressing

f_{pe} = 0.45 × f_{pu} , 800 = 0.45 × f_{pu} , f_{pu} = 800/ 0.45 = 1777.7 N/mm^2

- STEP 1

Compute the effective reinforcement ratio

$$\frac{A_{ps} \times f_{pu}}{b \times d \times f_{ck}} = \frac{2840 \times 1777.7}{400 \times 800 \times 40} = 0.39 \approx 0.4$$

- STEP 2

From Table 11, take values of the ratios corresponding to $\frac{A_{ps} \times f_{pu}}{b \times d \times f_{ck}} \approx 0.4$ for post tensioned beam

$\frac{f_{pb}}{0.87 \times f_{pu}} = 0.75$, f_{pb} = 0.75 ×0.87 × 1600 =1160 N/mm^2 and $\frac{x_u}{d}$ = 0.653 , x_u = 0.653 × 800= 520mm

- STEP 3

Calculate ultimate moment of resistance of sections using IS 1343 recommendations

$$M_U = f_{pb} \times A_{ps} \times (d - 0.42 \times x_u)$$

$$= 1160 \times 2840 \times (800 - 0.42 \times 520) = 1916.02 \text{ kNm}$$

5 b

f_{ck} = 40 N/mm^2 and f_p = 1600 N/mm^2, $A_p = A_{ps}$ = 4700 mm^2 , Effective prestress in steel f_{pe} = 1000N/mm^2 , $\frac{l}{d}$ =20 m, d = 1600mm, D_f = 50 mm, $Web\ thickness$, b_w = 300 mm, $\boldsymbol{b_f}$ =1200 mm

- Assume $x_u > D_f$ (neutral axis may fall in the web portion)

  Calculate x_u, by putting **b**$_w$ and A_{pw} in $\frac{A_{pw} \times f_{pe}}{b_w \times d \times f_{ck}}$, $b_w = 300\ mm$

- **Area of prestressing in flange A$_{pf}$** $= \mathbf{0.45} \times \boldsymbol{f_{ck}} \times (\boldsymbol{b_f} - \boldsymbol{b_w}) \times \frac{\boldsymbol{D_f}}{\boldsymbol{f_p}}$

  $= \mathbf{0.45 \times 40 \times (1200 - 300) \times \frac{50}{1600}}$ = **506.25** mm^2

  Area of prestressing steel in web **A$_{pw}$ = (A$_{ps}$ - A$_{pf}$) = (** 4700 **- 506.25) = 4193.75** mm^2

- Find effective reinforcement ratio $\frac{A_{pw} \times f_{pe}}{b_w \times d \times f_{ck}} = \frac{4193.75 \times 1000}{300 \times 1600 \times 40}$ = 0.218 ≈ 0.20

- Table 12 , IS 1343 -1980, Then interpolate to get the ratios $\frac{f_{pu}}{f_{pe}}$ = 1.16 , f_{pu} = 1.16 $\times \mathbf{1000} = \mathbf{1160}$ and $\frac{x_u}{d}$ = 0.58 , x_u = 0.58 × 1600 = mm based on $\frac{A_{pw} \times f_{pe}}{b_w \times d \times f_{ck}} \approx$ 0.20 (Maximum ratio available in Table 12 is 0.20)

- Flexural strength of unbonded T section

  $M_U = f_{pu} \times A_{pw} \times (d - 0.42 \times x_u) + 0.45 \times f_{ck} \times (b - b_w) \times D_f \times (d - 0.5 \times D_f)$ =

  = 1160 × 4193.2 × (1600 – 0.42 × 928) + 0.45 × 40 × (1200 – 300) × 50 × (1600 – 0.5 × 50) kNm

  = 7163 kNm

6

Effective span = 15 m, Live load = 12 kN/m, f_{ck} = 50 N/mm^2 , f_{ct} = 41 N/mm^2, Loss ratio η = 0.85, f_p = 1500 N/mm^2, $Area\ of\ one\ cable, A_p = 12 \times \frac{\pi}{4} \times 7^2$ = 461.58 mm^2 , Area of the concrete section, **A** = b × d

- Step 1

Assume breadth of the section as b = 250 mm = 0.25 m , let 'd' be the depth of the section in metres

Self-weight of beam /gravity load = 25 × 0.25 × d = 6.25 × d

Moment due to self-weight/ gravity, $M_g = \frac{6.25\times d \times 20^2}{8} = 312.5 \times d$ kNm

Moment due to live load, $M_q = \frac{12.0 \times 20^2}{8}$ = 600 kNm

$Loss\ ratio \quad \eta$ = 0. 85

- Step 2

Use expression for $Z_b = \frac{M_q + (1-\eta) M_g}{(\eta f_{ct} - f_{tw})}$ … (1)

Also for rectangular section, $Z_b = \frac{b \times d^2}{6} = \frac{0.25 \times d^2}{6}$….(2)

Equate (1) = (2) , put $f_{ct} = 41000$ kN/m^2, tensile stress at working load, $f_{tw} = 0$ since it is Type 1 members, all tensile stresses are zero

$$\frac{M_q + (1-\eta) M_g}{(\eta f_{ct})} = \frac{0.25 \times d^2}{6}$$

$$\frac{600 + (1-0.85) \times 312.5 \times d}{(0.85 \times 41000)} = \frac{0.25 \times d^2}{6}$$

Solve for 'd' , d = 0.659m ≈ 0.700 m (upper rounding) -

Dimension of the section is 250 × 700 mm

- Step 3

Solve for $Z_t = Z_b = \frac{0.25 \times d^2}{6} = \frac{0.25 \times 0.7^2}{6} = 0.02 \quad m^3$

- Step 4

Calculate Stress at top fibre f_t

$f_t = f_{tt} - \frac{M_g}{Z_t} = 0 - \frac{312.5 \times 0.7}{0.02}$ = - 10937 kN/m^2 = -10.9 N/mm^2

- Step 5

Calculate Stress at bottom fibre f_b

$$f_b = \frac{1}{\eta}\left(f_{tw} + \frac{M_g}{Z_b} + \frac{M_q}{Z_b}\right) = \frac{1}{0.85}\left(0 + \frac{312.5\times0.7}{0.02} + \frac{600}{0.02}\right) = 48161 \text{ kN/m}^2 \quad = 48.16 \text{ N/mm}^2$$

- **Step 6**

$\boldsymbol{Prestressing\ Force\ ,\ P} = \frac{A\times(Z_t f_t + Z_b f_b)}{(Z_t + Z_b)}$, A = $0.\mathbf{25} \times \mathbf{0.7}$ = , $Z_b = Z_t$ = 0.02 , f_t = - 10937 kN/m^2 , f_b = 48161 kN/m^2

P = 3257.12 kN

- Step 7 calculate the eccentricity of the tendon

$$\boldsymbol{e} = \frac{(f_b - f_t)\times Z_t \times Z_b}{A\times(Z_t\times f_t + Z_b \times f_b)} = \frac{(48161-(-10937))\times0.02\times0.02}{A\times(Z_t\times f_t + Z_b \times f_b)} = 0.18 \text{ m} = 180 \text{ mm}$$

- Step 8

No of cables required =

$Prestressing\ \ Force$, P = 3257.12 kN = 3257000.12 N

Characteristic strength of tendon, f_p = 1500 N/mm^2

Characteristic strength of tendon, $f_p = \frac{Prestressing\ \ Force}{Total\ Area\ of\ cables}$ = 1500 = $\frac{3257000}{Total\ Area\ of\ cables}$,

Total $Area\ of\ cables = \frac{3257000}{1500}$ = 2171.41

Area of one cable = 12 × π/4 × 7^2 = 461

Number of cables =Total $Area\ of\ cables$ / Area of one cable = 2171.41/ 461

= 5 numbers of cables are needed

Design is over

7a

In the case of RCC members when subjected to super imposed loads, shear forces are developed at the loaded member. Due to these shear force and stresses are developed in the member. Shear stress will be at the neutral axis and is minimum at the extreme fibres. The shear stress at any depth in the cross section is calculated using the equation.

$$\tau = \frac{F\times A\times \bar{y}}{I\times b}$$

F is the shear force in KN

$A \times \bar{y}$ moment of area above the level at which shear stress is required about the neutral axis

I is Moment of inertia

b is the width of cross section at which shear stress is required

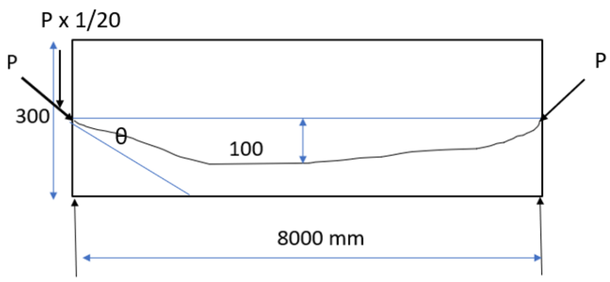
The effect of this shear stress is to induce the principal tensile stress on the diagonal planes. Due to diagonal tensile stress, diagonal cracks are developed at the support section. To reduce the diagonal cracks, the diagonal tension can be made compressive by following 3 ways

In general, there are **three ways of improving the shear resistance of structural concrete** members by prestressing techniques:

1. Horizontal or axial prestressing
2. Prestressing by inclined or sloping cables
3. Vertical or transverse prestressing

Axial prestressing reduces the principal stresses considerably when compared with the members without prestressing. Furher inaddition to axial prestressing, transverse or vertical prestressing is used it is possible to nullify the principal tension itself. In case the cables are placed as per the profile obtianed bya load balancing approach it results in the most desirable system of forces in concrte ie entire section of concrete will be subjected to uniform compressive state of stress at support.

7 b



Self-weight of the beam = (0.15 × 0.30 × 24) = 1.08 kN/m
Total load = (1.08 + 2.0) = 3.08 kN/m
Eccentricity of cable at the centre of span = 100 mm
Using the concept of load balancing, if P = effective prestressing force,

$$(P \times 100) = \left(\frac{3.08 \times 8000^2}{8}\right)$$

$$\therefore \quad P = 246400 \text{ N} = 246.4 \text{ kN}$$

Calculation of slope of the cable

Calculation of slope at support y = $\frac{4 \times e}{L^2}(Lx - x^2)$

$\frac{dy}{dx} = \frac{4 \times e}{L^2}(L - 2x)$

At support x =0, θ = $\frac{dy}{dx} = \frac{4 \times e}{L} = \frac{4 \times 100}{8000} = \frac{1}{20}$, (for small angle and zero shear let us take directly

sin θ = $\frac{1}{20}$)

Vertical component of prestressing force = (246.4 × 1/20) = 12.32 kN

Reaction at support due to dead and live loads = $\left(\frac{3.08 \times 8}{2}\right)$ = 12.32 kN

Hence, net shear force V at support = 0

Horizontal prestress at support = $\left(\frac{246400}{150 \times 300}\right)$ = 5.5 N/mm^2

Principal stress at support = 5.5 N/mm^2 (compression)

8 a

$$I = \frac{450 \times 1000^3}{12} - \frac{300 \times 700^3}{12} = 2.8975 \times 10^{10}\, mm^4$$

Calculation of slope at support y = $\frac{4 \times e}{L^2}(Lx - x^2)$

$\frac{dy}{dx} = \frac{4 \times e}{L^2}(L - 2x)$

At support x =0, θ = $\frac{dy}{dx} = \frac{4 \times e}{L} = \frac{4 \times 300}{20000}$ = 0.06 radians

θ in degrees = 0.06 × $\frac{180}{\pi}$ = 3.42°

self weight = 25 x .024 = 5.76 kN/m

Vertical component of prestressing force = P x sin 3.42 = 1250 × sin 3.42 =1250 kN

Horizontal component of prestressing force = P x cos 3.42 = 1250 × cos 3.42 = 74.96 kN

Total load = 25.76 kN/m

Shear force at support due to applied load = 25.76 x 20 /2 = 257.6 kN

Net shear force at support section = 257.6 – 74.96 = 182.64 kN

Shear stress

- At centroid

$$\tau = \frac{F \times A \times \bar{y}}{I \times b} = \frac{182640 \times (450 \times 150 \times 425 + 150 \times 350 \times \frac{350}{2})}{2.8975 \times 10^{10} \times 150} \quad \text{, (taking b = bw)}$$

$= 1.594 \text{ N/mm}^2$

- At junction of the web

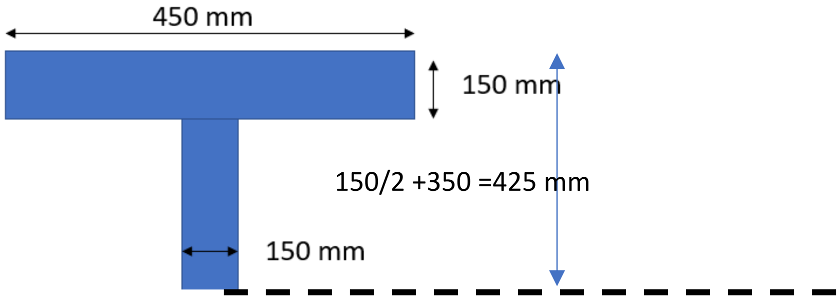
$$\tau = \frac{F \times A \times \bar{y}}{I \times b} = \frac{182640 \times (450 \times 150 \times (500 - \frac{150}{2})}{2.8975 \times 10^{10} \times 150} \quad \text{, (taking b = bw)}$$

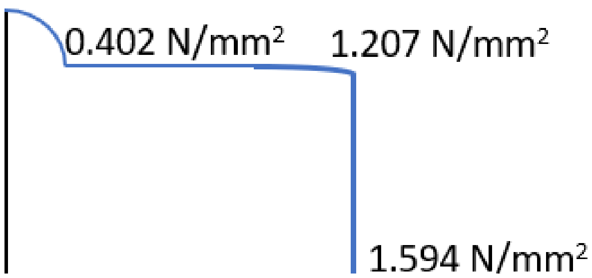
$= 1.207 \text{ N/mm}^2$

- At junction of the flange

$$\tau = \frac{F \times A \times \bar{y}}{I \times b} = \frac{182640 \times (450 \times 150 \times (500 - \frac{150}{2})}{2.8975 \times 10^{10} \times 450} \quad \text{, (taking b = bf)}$$

$= 0.402 \text{ N/mm}^2$





1. Principal tension along centrodial axis

$$f_{\substack{max\\min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$$

$$f_x = \frac{1250000}{240 \text{ x } 1000} = 5.21 \text{ N/mm}^2$$

$$f_{max,min} = \frac{5.21}{2} \pm \frac{1}{2}\sqrt{(5.21^2 + 4 \times 1.549^2)}$$

$= 2.605 \pm 3.504$

$f_{max} = 2.605 + 3.504 = + 5.6 \text{ N/mm}^2$ (compression)

f_{min} = 2.605 − 3.504 = - 0.449 N/mm^2 (tension)

2. Principal tension at the junction of flange

$$f_{\substack{max\\min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$$

$$f_x = \frac{1250000}{240 \text{ x } 1000} = 5.21 \text{ N/mm}^2$$

$$f_{max,min} = \frac{5.21}{2} \pm \frac{1}{2}\sqrt{(5.21^2 + 4 \times 0.402^{\,2})}$$

$= 2.605 \pm 2.63$

f_{max} = 2.605 + 2.63 = + 5.24 N/mm^2 (compression)

f_{min} = 2.605 – 2.63 = - 0.031 N/mm^2 (tension)

3. Principal tension at the junction of web

$$f_{\substack{max\\min}} = \left[\left(\frac{f_x + f_y}{2}\right) \pm \frac{1}{2}\sqrt{(f_x - f_y)^2 + 4\tau_v^2}\right]$$

$$f_x = \frac{1250000}{240 \text{ x } 1000} = 5.21 \text{ N/mm}^2$$

$$f_{max,min} = \frac{5.21}{2} \pm \frac{1}{2}\sqrt{(5.21^2 + 4 \times 1.207^{\,2})}$$

$= 2.605 \pm 2.87$

f_{max} = 2.605 + 2.87 = + 5.47 N/mm^2 (compression)

f_{min} = 2.605 – 2.87 = - 0.266 N/mm^2 (tension)

8 b

bw = 200 mm,D = 2000 mm,L =40 m,e = 750 mm @ centre,A = 0.88 x 10^6 mm^2, P = 1200 kN, Loss ratio = 0.8 , fy = 415, Peff = 0.8 x 1200 = 9600 kN, f_{ck} = 60 MPa

V = 2850 kN

f_t = 0.24 √ f$_{ck}$ = 1.86 N/mm^2

$$f_{cp} = \frac{Peff}{A} = \frac{9600000}{0.888 \times 10^{6}} = 10.91\ N/mm^2$$

$$\theta = \frac{dy}{dx} = \frac{4 \times e}{L} = \frac{4 \times 750}{40000} = 0.075 \text{ radians} \approx \sin\theta$$

$$V_C = V_{CO} = 0.676 \times b \times D \times \sqrt{(f_t{}^2 + 0.8 \times f_{cp} \times f_t)} + P_{eff}\ \sin\theta$$

$= 0.676 \times 200 \times 2000 \times \sqrt{(1.86^2 + 0.8 \times 10.91 \times 1.86)} + 9600 \times 0.075 = 1909.32$ kN < 2850kN

Let us assume 12 mm Φ and two legged stirrups and effective cover 100 mm

A_{sv} = 2 x π / 4 x 12^2 = 226.19 mm^2

$S_v = \frac{0.87 \times 226.19 \times 415 \times 1900}{(2850 - 1909.32) \times 1000}$ = 164.95 mm < 0.75 × d = 1425

Hence Ok

Use 12 mm Φ # two legged stirrups @ 150 mm c/c

9 a

Prestressed concrete contains tendons which are typically stressed to about 1000 MPa. These tendons need to be anchored at their ends in order to transfer (compressive) force to the concrete. In pretensioned concrete, the anchorage consists of a bonded length of tendon, in direct contact with the concrete. In post-tensioned concrete, an achorage plate is used, which bears onto the concrete over a relatively small area.The tendon is connnected to the plate either through wedges, button-heads or other methods. The plate itself then bears on the concrete. The plates employed for this are very much smaller than the area of concrete which is to be compressed. Therefore, a redistribution of stress occurs behind the anchorage plate as the compression trajectories spread out to form uniform stress patterns some distance into the concrete, according to St Venant's Principle. It is the distance over which this redistribution occurs that is of interest to the Engineer. This disturbed region is known as the Anchorage Zone. The state of stress in the anchorage zone is extremely complex. It consists of severely curved trajectories, perhaps interfering with 'secondary' stresses due to bearing supports. Therefore, it is in the Engineer's interest to ensure two things in this zone.

1. The zone must not crack at the serviceability limit state (this would allow the ingress of water, leading to possible corrosion problems), and

2. The zone must not fail at the ultimate limit state.

9b

The transverse reinforcement is provided in each principal direction based on the value of bursting force F_{bst}. This reinforcement is called end zone reinforcement or anchorage zone reinforcement or bursting links. The reinforcement is distributed within a length $0.1y_0$ to y_0 from an end of the member.

The amount of end zone reinforcement in each direction (A_{st}) can be calculated using

$$A_{st} = \frac{F_{bst}}{f_s} \text{ or } \frac{F_{bst}}{0.87 f_y}$$

where stress in steel 'f_s' is limitted to $0.87 f_y$,

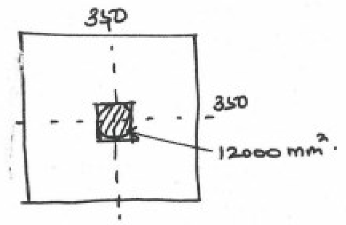
27 → when the cover is less than 50mm & f_s is limitted to a value corresponding to a strain of 0.001.

The end zone reinforcement is provided in several forms. Some of which are proprai tory of the construction forms. The forms are.

- Closed stirrups.
- mats or Links with loops and even
- Spiral reinforcements.

9 c

Given



Equivalent Square area = 12000 mm²

∴ side of the square plate = y_{po}

$= \sqrt{12000}$

$y_{po} = 109.54$ mm.

$y_o = 350$ mm.

$P_o = 850$ kN

Solution

IS 1343:2012 P-26 C-

$$\frac{F_{bst}}{P_o} = 0.32 - 0.3\frac{y_{po}}{y_o}$$

$$F_{bst} = 850\left[0.32 - 0.3 \times \frac{109.54}{350}\right] = 192.19 \text{ kN}$$

Area of end reinforcement

Assuming Fe415 steel for end reinforcement.

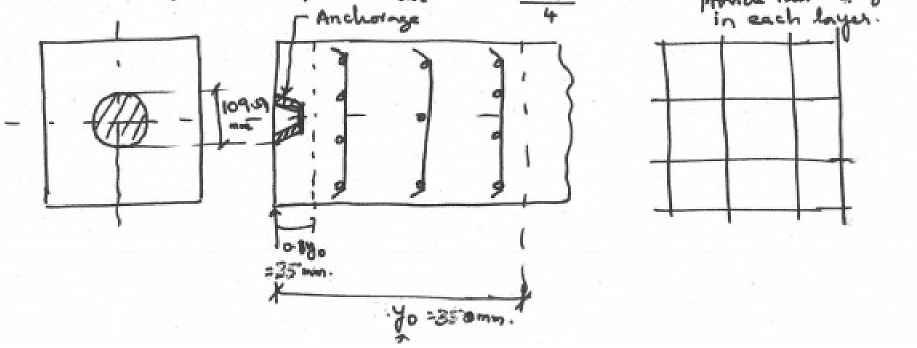
$$A_{st} = \frac{F_{bst}}{0.87 f_y}$$

$$A_{st} = \frac{192.19 \times 10^3}{0.87 \times 415} = 532.31 \text{ mm}^2.$$

Assuming 8 mm ϕ bars.

$$\text{No. of bars reqd} = \frac{A_{st}}{a_{st}} = \frac{532.31}{\frac{\pi (8)^2}{4}} = 10.58 \text{ nos} \simeq 11 \text{ nos.}$$

Provide this no. of bars in each layer.



10 a

In a composite construction, precast prestressed members are used in conjunction with the concrete cast in situ, so that the members behave as monolithic unit under service loads. Generally, the high-strength prestressed units are used in the tension zone while the concrete, which is cast in situ of relatively lower compressive strength, is used in the compression zone of the composite members. The composite action between the two components is achieved by roughening the surface of the prestressed unit on to which the concrete is cast in situ, thus giving a better frictional resistance, or by stirrups protruding from the prestressed unit into the added concrete, or by castellations on the surface of the prestressed unit adjoining the concrete which is cast in situ.

The phenomenon of differential shrinkage between the concrete cast in situ and the prestressed units also contributes to the monolithic action of the composite member.

The advantages in using precast prestressed units in association with the in situ concrete are:

1. Appreciable saving in the cost of steel in a composite member compared with a reinforced or prestressed concrete member.

2. Sizes of precast prestressed units can be reduced due to the effect of composite action.

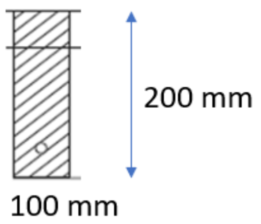
3. Low ratio of size of the precast unit to that of the whole composite member.

4. In many cases, precast prestressed units serve as supports and dispense with the form work for placement of in situ concrete.

5. Composite members are ideally suited for constructing bridge decks without the disruption of normal traffic.

10 b

1. **Section properties of the pretensioned beam**



A = 100 × 200 = 20000 mm^2

Z = $\left[\frac{100 \times 200^2}{6}\right] = 667 \times 10^3$ mm^3

P = 150 kN

Thus, stresses due to prestressing force becomes $= +\frac{2 \times P}{A} = \frac{2 \times 150000}{20000} = 15$ N/mm^2 at the bottom and zero at the top fibre

Effective prestress after losses = 0.85 × 15 = 12.8 N/mm^2

Dead load or Self-weight of the precast beam = 0.1 × 0.2 × 24 × 10^3= 480N/m

Dead load or Self- weight moment , $M_D = \frac{480 \times 5^2}{8} = 1500$Nm

Stresses at top and bottom fibre $= \pm \frac{M_D}{Z} = \frac{1500000}{667 \times 10^3} = \pm 2.25$ N/mm^2

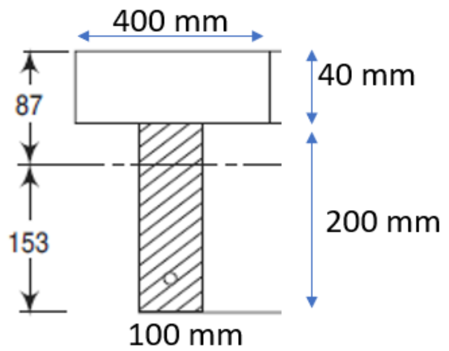
Always take density of PCC = 24 kN/m^2

Self-weight of insitu slab = $0.04 \times 0.4 \times 24 \times 10^3 = 384$ Nm

Moment due to slab weight = $\frac{384 \times 5^2}{8}$= 1200 Nm

Stresses due to slab weight in the precast section $= \frac{1200000}{667 \times 10^3} = \pm 1.8$ N/mm^2

2. Section properties of composite section



Distance of the centroid from the top fibre $= \frac{400 \times 40 \times 20 + 100 \times 200 \times (40+100)}{400 \times 40 + 100 \times 200}$ = 87 mm

Moment of inertia , I $= \frac{400 \times 40^3}{12} + 100 \times 40 \times (87 - 20)^2 + \frac{100 \times 200^3}{12} + 100 \times 200 \times (140 - 87)^2 = 1948 \times 10^5$ mm^4

Section moduli , $Z_t = \frac{1948 \times 10^5}{87} = 225 \times 10^4$ mm^3

$Z_b = \frac{1948 \times 10^5}{(153)} = 128 \times 10^4$ mm^3

Live load on the composite section = $0.4 \times 1.0 \times 8000 = 3200$ N/m

Maximum live load moment = $\frac{3200 \times 5^2}{8} = 10000 \times 10^3$ Nmm

3. Live load stresses in the composite section

At top $= \frac{M_L}{Z_t} = \frac{10000 \times 10^3}{225 \times 10^4} = 4.45$ N/mm^2 (compression)

At bottom $= \frac{M_L}{Z_b} = \frac{10000 \times 10^3}{128 \times 10^4} = 7.85$ N/mm^2 (tension)

If the pretensioned beam is propped, the self-weight of the slab acts on the composite section

Moment due to slab weight = 1200 Nm

Stress due to this moment in the composite section

At top $= \frac{M_d}{Z_t} = \frac{1200 \times 10^3}{225 \times 10^4} = 0.53$ N/mm^2 (compression)

At bottom $= \frac{M_d}{Z_b} = \frac{1200 \times 10^3}{128 \times 10^4} = 0.94$ N/mm^2 (tension)

The distribution of stresses for the various stages of loading for the propped and unpropped construction is shown in the figure below