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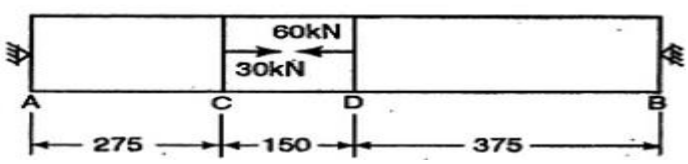
Internal Assesment Test – 1

Sub: Mechanics of Materials				Code: 18ME32
Date: 17/12/2021	Duration: 90 mins	Max Marks: 50	Sem: 3	Branch (sections): ME (A)

Answer any five full questions

		Marks	OBE CO	OBE RBT
1	(a) Sketch and explain stress-strain diagram for steel indicating all salient points and zones on it.	[05]	CO1	L2
	(b) Derive an expression for volumetric strain of a rectangular block subjected to axial load.	[05]	CO1	L2
2	Two vertical rods one of steel and the other of copper are each rigidly fixed at the top and 500mm apart. Diameters and lengths of each rod are 20mm and 4m respectively. A cross bar fixed to the rods at the lower ends carries a load of 5kN, such that the cross bar remains horizontal even after loading. Find the stress in each rod and the position of the load on the bar. Take Es = 2×10^5 N/mm^2 and Ec = 1×10^5 N/mm^2.	[10]	CO1	L3
3	A bar of 800mm length is attached rigidly at A and B as shown in Fig. 1. Forces of 30 kN and 60 kN act as shown on the bar. If E=200 MPa, determine the reactions at the two ends. If the bar diameter is 25 mm, find the stresses and change in length of each portion.  Figure 1	[10]	CO1	L3
4	Rails are laid such that there is no stress in them at 24°C. If the rails are 12.6 m long and maximum temperature expected is i) Estimate the minimum gap between two rails to be left so that temperature stresses do not develop. ii) Calculate the thermal stresses developed in the rails if a) No expansion joint is provided. b) If a 2 mm gap is provided for expansion. iii) If the stress developed is 20 MN/m^2, what is the gap left between the rails?	[10]	CO1	L3

	Coefficient of linear expansion, $\alpha = 12 \times 10^{-6}$/°C and Young's modulus E = 200 GPa.			
5	With usual notations derive torsion equation. Also state the assumptions in pure torsion theory.	[10]	CO5	L2
6	A hollow shaft of diameter ratio 3/8 is to transmit 375 kW at 100 rpm. The maximum torque being 20 % greater than the mean; the shear stress is not to exceed 60 N/mm^2 and the twist in a length of 4 m is not to exceed 2 degrees. Calculate its external and internal diameters. Take $G = 8 \times 10^4$ N/mm^2.	[10]	CO5	L3
7	A solid steel shaft has to transmit 75 kW at 200 rpm. Taking allowable shear stress as 70 N/mm^2, find the suitable diameter of the shaft, if the maximum torque transmitted in each revolution exceeds the mean by 30%. Also find the outer diameter of a hollow shaft to replace the solid shaft if the diameter ratio is 0.7.	[10]	CO5	L3

CI **CCI** **HOD**

Scheme Of Evaluation
Internal Assessment Test 1 – December 2021

Sub:	Mechanics of Materials							**Code:**	18ME32
Date:	/12/2021	Duration:	90mins	Max Marks:	50	**Sem:**	III	**Branch:**	ME

Note: Answer Any FIVE full questions

Question #	Description	Marks Distribution	Max Marks
1 a			
1 b			
2			
3			
4			
5			
6			
7			

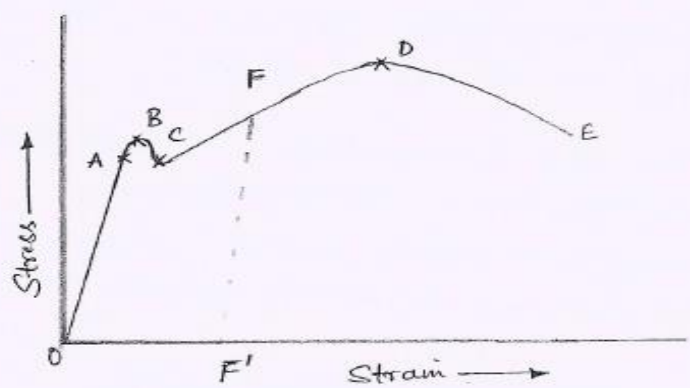
Internal Assesment Test – 1
Solutions

Sub:	Mechanics of Materials							**Code:**	18ME32
Date:	/12/2021	Duration:	90mins	Max Marks:	50	**Sem:**	III	**Branch:**	ME

1 [a]

Stress Strain relation:

Behaviour in Tension [Mild Steel]:



(a) Limit of Proportionality (A): It is the limiting value of the Stress up to which stress is proportional to strain.

(b) Elastic limit: This is the limiting value of stress up to which if the material is stressed and then released (unloaded) Strain disappears completely and the original length is regained. This point is slightly beyond the limit of proportionality.

(c) Upper yield point (B): This is the stress at which, load starts reducing and the extension increases. This phenomenon is called yielding of material.

(d) Lower yield point (C): At this stage the stress remains same but strain increases for some time.

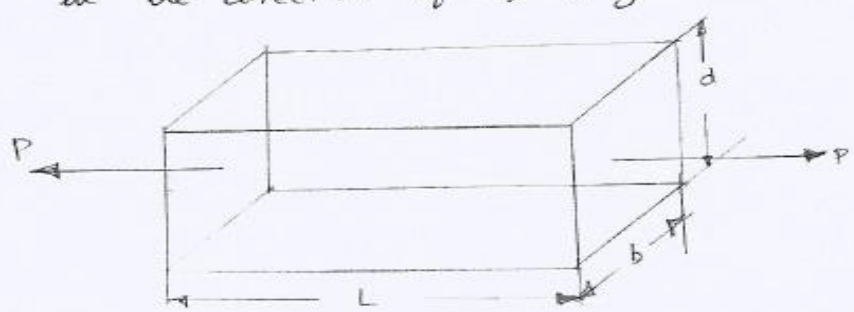
(e) Ultimate Stress (D): This is the maximum stress the material can resist. At this stage cross sectional area at a particular section starts reducing very fast. This is called neck formation. After this stage load resisted and hence the stress developed starts reducing.

(f) Breaking point (E): The stress at which finally the specimen fails is called breaking point.

1[b]

Volumetric strain of a Rectangular Bar which is subjected to an Axial load 'P' in the direction of its length.

Consider a rectangular bar of length 'L', width 'b' and depth 'd' which is subjected to an axial load 'P' in the direction of its length as shown in fig: below



Let, δL = Change in length

δb = Change in width and

δd = Change in depth

∴ Final length of the bar = $L + \delta L$

Final width of the bar = $b - \delta b$ (–ive sign due to decrease)

Final depth of the bar = $d + \delta d$ (–ive sign due to decrease)

Now Original volume of bar, $V = L \cdot b \cdot d$

Final volume = $(L + \delta L)(b - \delta b)(d + \delta d)$

$= Lbd + bd\,\delta L - Lb\,\delta d - Ld\,\delta b$

(Ignoring products of small quantities)

∴ Change in volume,

δv = Final volume – Original volume

$= (Lbd + bd\,\delta L - Lb\,\delta d - Ld\,\delta b) - Lbd$

$= bd\,\delta L - Lb\,\delta d - Ld\,\delta b$

$\therefore$ Volumetric strain,

$$e_v = \frac{\delta v}{V}$$

$$= \frac{bd\,\delta L - Lb\,\delta d - \delta b\,Ld}{Lbd}$$

$$= \frac{\delta L}{L} - \frac{\delta d}{d} - \frac{\delta b}{b} \quad \text{— (i)}$$

But $\frac{\delta L}{L}$ = Longitudinal strain and $\frac{\delta d}{d}$ (or) $\frac{\delta b}{b}$ are lateral strains

Substituting these values in the above equation, we get

e_v = longitudinal strain − 2× Lateral strain — (ii)

we know that, $\frac{\text{Lateral strain}}{\text{longitudinal strain}} = \mu$ [$\because$ poisson's ratio]

$\therefore$ Lateral strain = μ × longitudinal strain.

Substituting the value of lateral strain in equation (ii),

e_v = Longitudinal strain − 2 × μ longitudinal strain

e_v = longitudinal strain $(1 - 2\mu)$

$$= \frac{\delta L}{L}\left(1 - \frac{2}{m}\right) \qquad \left(\because \frac{1}{m} = \mu\right)$$

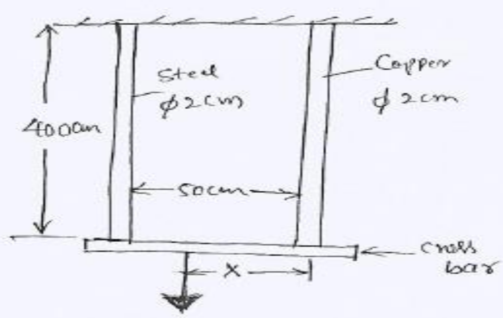
2.

Soln:

Given:

Distance between the rods
= 50 cm = 500 mm

Dia. of steel rod = Dia. of copper rod
= 2 cm = 20 mm



$\therefore$ Area of steel rod = Area of copper rod

$= \frac{\pi}{4} \times 20^2$

$\therefore A_S = A_C = 100\pi\ mm^2$

Length of each rod = 4 m = 4000 mm

Total load carried by rods, P = 5000 N

$E_S = 2\times10^5\ N/mm^2$, $E_C = 1\times10^5\ N/mm^2$

$\sigma_S = ?$ & $\sigma_C = ?$

Since the cross bar remains horizontal, the extensions of the steel and copper rods are equal. Also these rods have the same original length, hence the strains of these rods are equal.

$\therefore$ Strain in steel = Strain in copper.

$$\frac{\sigma_S}{E_S} = \frac{\sigma_C}{E_C}$$

$$\therefore \sigma_S = \frac{E_S}{E_C} \times \sigma_C = \frac{2\times10^5}{1\times10^5} = 2\times\sigma_C \quad \text{— (i)}$$

Total load = Load on Steel rod + Load on Copper rod.

$= 5000 N.$

$$5000 = \sigma_s \times A_s + \sigma_c \times A_c$$

$$= 2\sigma_c \times 100\pi + \sigma_c \times 100\pi$$

$$\sigma_c = \frac{5000}{300\pi} = \underline{\underline{5.305 \ N/mm^2}}$$

Substituting this value of σ_c in eqn (i)

$$\sigma_s = 2 \times \sigma_c = \underline{\underline{10.61 \ N/mm^2}}$$

(ii) Position of the load of 5000 N on cross bar.

Let, x = The distance of the 5000N load from the copper rod.

Load carried by each rod:

Load = Stress × Area.

Load carried by Steel

$$P_s = \sigma_s \times A_s$$
$$= 10.61 \times 100\pi$$
$$= \underline{\underline{3333 \ N}}$$

Load carried by copper rod.

$$P_c = \sigma_c \times A_c$$

(or)

$$P = P_s + P_c$$
$$P_c = P - P_s$$
$$= 5000 - 3333$$
$$\therefore P_c = \underline{\underline{1667 \ N}}$$

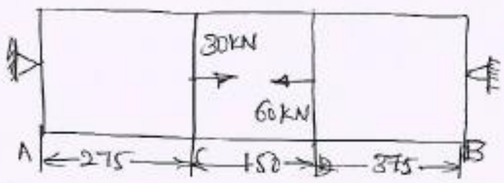
Now taking the moments about the Copper rod and equating the same we get.

$$5000 \times x = P_s \times 50$$

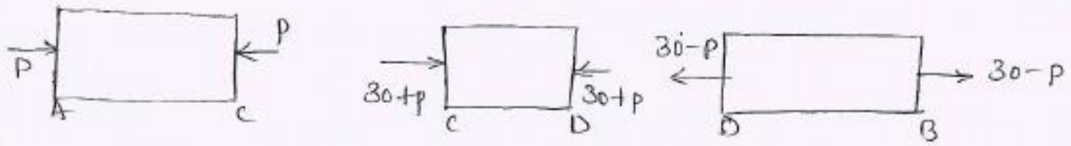
$$x = \frac{3333 \times 50}{5000}$$

$$\therefore x = \underline{\underline{33.33 \ cm}}$$

3.



Let 'p' be the reaction on the bar from support at A.
Then forces acting on the each portion of the bar are shown in fig. below.



Shortening of portion AC = $\frac{P \times 275}{AE}$ $\quad (\because \Delta = \frac{PL}{AE})$

Shortening of portion CD = $\frac{(30+P)150}{AE}$

Extension of portion DB = $\frac{(30-P)375}{AE}$

$\therefore$ Total extension = $\frac{1}{AE}[-P \times 275 - (30+P)150 + (30-P)375]$

As supports are unyielding, Total extension = 0.

$\therefore$

$$0 = -275P - 150(30+P) + (30-P)375$$

$$800P = 30 \times 375 - 150 \times 30$$

$$\therefore \; P = 8.4375 \text{ KN}$$

$\therefore$ Reaction of support 'A' is = 8.4375 KN (Ans)

and at support 'B' reaction is 30 − 8.4375 KN

= 21.5625 KN (Ans)

Now cross-sectional area, $A = \frac{\pi}{4} \times 25^2$

$= 490.8739 \text{ mm}^2$

Stress in portion AC = $\frac{8.4375 \times 10^3}{490.8739}$ $\quad (\because \sigma = P/A)$

$= 17.1887 \text{ N/mm}^2$ (Ans) (Comp)

Stress in portion CD = $\dfrac{(30+8.4375)\times10^3}{490.8739}$

$= 78.3042 \text{ N/mm}^2$ (Ans) (Comp)

Stress in portion DB = $\dfrac{(30-8.4375)\times10^3}{490.8739}$

$= 43.9268 \text{ N/mm}^2$ (Ans) (Tensile)

Now, $E = 200 \text{ GPa} = 200\times10^9\times\dfrac{1}{(1000)^2} \text{ N/mm}^2 = 200\times10^3 \text{ N/mm}^2$

$= 2\times10^5 \text{ N/mm}^2$

Shortening of Portion, AC = $\dfrac{8.4375\times10^3\times275}{490.8739\times2\times10^5} = 0.02363 \text{ mm}$ (Ans) $\left(\Delta = \dfrac{PL}{AE}\right)$

Shortening of portion, CD = $\dfrac{(30+8.4375)\times10^3\times150}{490.8739\times2\times10^5} = 0.05873 \text{ mm}$ (Ans)

~~Shortening of portion~~
Extension of portion, DB = $\dfrac{(30-8.4375)10^3\times375}{490.8739\times2\times10^5} = 0.08236 \text{ mm}$ (Ans)

4.

Soln:

(i) The free expansion of the rails $= \alpha t L$

$= 12\times10^{-6}\times(44-24)\times12.6\times1000$

$= 3.024 \text{ mm}$

(ii) Noting that $E = 2\times10^5 \text{ MN/m}^2 = 2\times10^5 \text{ N/mm}^2$

(a) If no expansion joint is provided, free expansion prevented is equal to 3.024 mm.

$dL = 3.024$

$\dfrac{PL}{AE} = 3.024$

$\dfrac{P}{A} = \dfrac{3.024\times2\times10^5}{12.6\times10^3}$

$\sigma = 48 \text{ N/mm}^2$ $(\sigma = \alpha E t)$

(b) If a gap of 2 mm is provided, free expansion prevented is

$dl = \alpha t L - \delta$

$= 3.04 - 2$

$= \underline{\underline{1.024 \text{ mm}}}$

$$\frac{PL}{AE} = 1.024$$

$$\frac{P}{A} = \frac{1.024 \times 2 \times 10^5}{126 \times 10^3}$$

$\therefore \ \sigma = \underline{\underline{16.254 \ N/mm^2}}$

(iii) If the stress developed is $= 20 MN/m^2 = 20 N/mm^2$, then

$$\frac{P}{A} = 20$$

If 'δ' is the gap, $\quad dL = \alpha t L - \delta$

$$\frac{PL}{AE} = \alpha t L - \delta$$

$$20 \times \frac{12.6 \times 1000}{2 \times 10^5} = (3.024 - \delta)$$

$$1.26 = 3.024 - \delta$$

$\therefore \ \delta = \underline{\underline{1.764 \text{ mm}}}$

5.

* Consider a shaft of length 'l', diameter 'd' fixed at one end and free at other end as shown in fig.
* Take any line PQ on the outer surface of the shaft. The line PQ is parallel to the longitudinal axis of the shaft.
* If torque 'T' is applied to the shaft at the free end, the line PQ is shifted to the new position PQ'.
* The angle between PQ and PQ' i.e $Q\hat{P}Q'$ is ϕ, which is the shear strain.
* Angle between OQ and OQ' in the end view i.e $Q\hat{O}Q'$ is the angle of twist 'θ' in length 'l'.

Let l = length of shaft

R = Radius of shaft

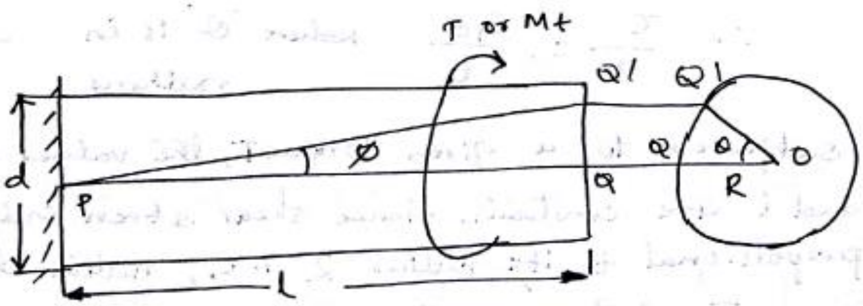
θ = Angle of twist

ϕ = Shear strain

G = Modulus of rigidity

M_t or T = Torque applied at the end Q

τ = Shear stress induced at the outermost surface.



Shear strain at the outer surface = Distortion per unit length

$$= \frac{\text{Distortion at the outer surface}}{\text{Length of shaft}}$$

$$= \frac{QQ'}{l} = \frac{QQ'}{PQ} = \tan\phi = \phi \quad (\because \phi \text{ is small})$$

$\therefore$ Shear strain at the outer surface $\phi = \frac{QQ'}{l}$

Also from the side view, Arc $QQ' = OQ \cdot \theta = R\theta$

i.e, $\phi l = R\phi$

$$\therefore \phi = \frac{R\theta}{l} \quad \text{— (i)}$$

For a given torque T and length of shaft, θ is constant and therefore shear strain is directly proportional to its radial distance from the centre of the section. Hence shear strain is zero at the centre and maximum at the outer surface.

$\therefore$ Shear strain at any other radius r, $\phi_1 = \frac{r\theta}{l}$ — (ii)

From Hooke's law, Modulus of rigidity = $\frac{\text{Shear stress}}{\text{Shear strain}}$

i.e., $G = \frac{\tau}{\phi} = \frac{\tau}{R\theta/l}$

$\therefore \frac{\tau}{R} = \frac{G\theta}{l}$ where θ is in radians — (iii)

For a shaft subjected to a given torque T, the values of G, θ and l are constants. Hence shear stress induced is directly proportional to its radius R i.e., radial distance from the centre of the section. Hence shear stress is zero at the centre and maximum at the outer surface. If 'q' is the shear stress at a radius 'r' from the centre of shaft, then

$$\frac{\tau}{R} = \frac{G\theta}{L} = \frac{q}{r} \quad \text{— (iv)}$$

Consider a solid circular shaft subjected to torque T. The maximum torque transmitted by a solid circular shaft is obtained from max. shear stress.

Now consider an elementary ring of thickness 'dr' at a radius 'r' from the centre of shaft as shown in fig.

Area of the elementary ring $\delta a = 2\pi r . dr$

From eqn (iv), $\frac{\tau}{R} = \frac{q}{r}$

$\therefore$ Shear stress at radius r, $q = \frac{\tau}{R} . r$

Shear force on the elemental ring δF = Shear stress in the ring × Area of the ring

$$\delta F = \frac{\tau}{R} . r . 2\pi r dr = \frac{\tau}{R} 2\pi r^2 dr$$

Torque on the elemental ring $\delta T = \delta F \times$ Dist. of the ring from the centre

$$= \left(\frac{\tau}{R} 2\pi r^2 dr\right) r$$

$$\therefore \delta T = \frac{\tau}{R} 2\pi r^3 dr$$

$\therefore$ Total torque or Total resisting torsional moment $T = \int_0^R \frac{\tau}{R} 2\pi r^3 dr$

$$= \frac{\tau}{R} 2\pi \int_0^R r^3 dr = \frac{\tau}{R} \cdot 2\pi \left(\frac{r^4}{4}\right)_0^R$$

$$= \frac{\tau}{R} 2\pi \left(\frac{R^4}{4}\right) = \left(\frac{\pi R^4}{2}\right) \frac{\tau}{R}$$

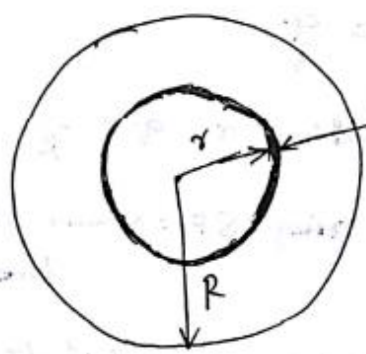
$$= \left(\frac{\pi d^4}{32}\right) \cdot \frac{\tau}{R} \qquad (\because R = d/2)$$

i.e $T = J_p \cdot \frac{\tau}{R}$ where, $J_p = \left(\frac{\pi d^4}{32}\right)$ = Polar MI of solid circular shaft.

$$\therefore \frac{T}{J_p} = \frac{\tau}{R} \quad \text{--- (v)}$$

From (iii) and (v)

$$\boxed{\frac{T}{J_p} = \frac{\tau}{R} = \frac{G\theta}{l}}$$



Assumptions in the theory of pure torsion:

The following assumptions are made in developing the equations for stresses and deformations in a bar subjected to pure torsion.

1. The material of shaft is homogeneous and isotropic
2. Normal cross-section of the shaft which are plane before twist remain plane after twist ie., no warping takes place.
3. Shear stress is proportional to shear strain and hence the material of the shaft obeys Hooke's law i.e., stresses are within elastic limit.
4. The twist along the shaft is uniform.
5. Circular section remains circular. Thus radii remain straight after torsion.
6. Torsion is uniform along the length.

6.

$d_o = 107.44$ mm

Data

Q.no 6: $d_i = \frac{3}{8} d_o$, $P = 375$ kW, $N = 100$ rpm, $T_{max} = 1.2\, T_{mean}$, $\tau = 60$ N/mm²,

$\theta = 2°$, $l = 4$ m $= 4000$ mm, $d_o = ?$, $d_i = ?$, $G = 8.5 \times 10^4$ N/mm²

We know that, Power transmitted, $P = \frac{2\pi N T}{60000}$

$$375 = \frac{2\pi \times 100 \times T_{mean}}{60000}$$

$$\therefore T_{mean} = 35810 \text{ Nm}$$

$$T_{max} = 1.2 \times T_{mean} = 1.2 \times 35810$$

$$= 42972 \text{ Nm}$$

or

$$T_{max} = 42972 \times 10^3 \text{ Nmm}$$

$\frac{3}{8} = 0.375$

$\therefore d_i = 0.375 d_o$

From strength criteria,

$$\frac{T}{J} = \frac{\tau}{R} \; ; \quad T = \frac{\pi}{16}\left(\frac{d_o^4 - d_i^4}{d_o}\right) \cdot \tau$$

$$(T_{max} = T) \quad 42972 \times 10^3 = \frac{\pi}{16} \frac{d_o^{4\,3}(1 - 0.375^4)}{d_o} \cdot 60$$

$$\therefore d_o^3 = \frac{42972 \times 10^3 \times 16}{\pi \times 0.98022 \times 60}$$

$$\therefore d_o = 154.96 \text{ mm} \approx 155 \text{ mm (Ans)}$$

$$d_i = \frac{3}{8} d_o = \frac{3}{8} \times 154.96 = 58.11 \text{ mm (Ans)}$$

(4)

rom rigidity criteria

$$\frac{T}{J} = \frac{G\theta}{l} \; ; \; \frac{T}{\frac{\pi}{32}(d_o^4 - d_i^4)} = \frac{G\theta}{l}$$

$$\therefore \frac{42972 \times 10^3}{\frac{\pi}{32}\left[d_o^4 - (0.375 d_o)^4\right]} = \frac{8.5 \times 10^4 \times 2 \times \pi/180}{4000}$$

$$\therefore d_o = 156.64\ mm \quad \text{(Ans)}$$

$$d_i = \frac{3}{8} \times 156.64 = 58.74\ mm \quad \text{(Ans)}$$

7.

7. Data: $P = 75\ kW$, $N = 200\ rpm$, $\tau = 70\ N/mm^2$, $T_{max} = 1.3\, T_{mean}$

$d_i = 0.7 d_o$

W.K.T, Power transmitted, $P = \frac{2\pi N T_{mean}}{60000} = \frac{2\pi \times 200 \times T_{mean}}{60000}$

$$\therefore T_{mean} = 3581\ Nm$$

$$T_{max} = 1.3 \times 3581 = 4655\ Nm \quad \text{(or)} \quad 4655 \times 10^3\ Nmm$$

Case(i): When a solid shaft is provided

$$T_{max} = \frac{\pi d^3}{16} \cdot \tau$$

$$\therefore d^3 = \frac{4655 \times 10^3 \times 16}{70\pi}$$

$$\therefore d = 69.7\ mm$$

Case (ii): when a hollow shaft of diameter ratio 0.7 is provided

$$T_{max} = \frac{\pi}{16\, d_o}(d_o^4 - d_i^4) \cdot \tau \; ; \; 4655 \times 10^3 = \frac{\pi}{16} \frac{(d_o^4 - (0.7 d_o)^4)}{d_o} \times 70$$

$$\therefore d_o = 76.38\ mm \text{ (Ans) and}$$

$$d_i = 0.7 \times 76.38 = 53.47\ mm \quad \text{(Ans)}$$