

Internal Assessment Test I – Nov 2021

Sub: Dynamics of Machines

Code: 18ME53

Date: 13/11/2021 Duration: 90 mins

Max Marks: 50 Sem: V

Branch: MECH

Note: Answer all questions.

		Marks	OBE	
			CO	RBT
1	Define the following i)Sensitiveness (ii) Isochronism (iii)Hunting of governor (iv)Effort of governor	8	CO3	L1
2	Derive an expression for equilibrium speed of governor	8	CO3	L2
3	Four masses 150, 250, 200 & 300kg are rotating in same plane at radii of 0.25m, 0.2m, 0.3m and 0.35m respectively. These angular locations are 40°, 120° & 250° from mass 150kg respectively measured in counter clockwise direction. Find the position and magnitude of balance mass required, if its radius of rotation is 0.25m.	12	CO2	L2
4	Explain Static and dynamic balancing	4	CO2	L2
5	Explain balancing of single rotating mass in different planes	6	CO2	L2
6	The mass of each ball of a Hartnell type governor is 1.4 kg. The length of ball arm of the bell-crank lever is 100 mm where as the lengths of arm towards sleeve is 50 mm. The distance of the fulcrum of bell-crank lever from the axis of rotation is 80 mm. the extreme radii of rotation of the balls are 75 mm and 112.5 mm. The maximum equilibrium speed is 6% greater than the minimum equilibrium speed which is 300 rev/min. determine i) Stiffness of the spring and ii) Equilibrium speed when the radius of rotation of the ball is 90 mm.	12	CO3	L3

## Solution for Internal Assessment Test I - Nov 2021

1.

SENSITIVENESS

It is defined as the ratio of the difference between the maximum & minimum equilibrium speeds to the mean equilibrium speed

$$\text{Mean Speed } N = \frac{N_1 + N_2}{2}$$

$$\therefore \text{Sensitiveness} = \frac{N_2 - N_1}{N} = \frac{N_2 - N_1}{\frac{N_1 + N_2}{2}} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

ISOCHRONOUS GOVERNOR

A governor is said to be isochronous when the equilibrium speed is constant (i.e. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction.

## HUNTING

A governor is said to be hunt if the speed of the engine fluctuates continuously above & below the mean speed. This is caused by a sensitive governor. In actual practice hunting is impossible in an isochronous governor because of friction of mechanism.

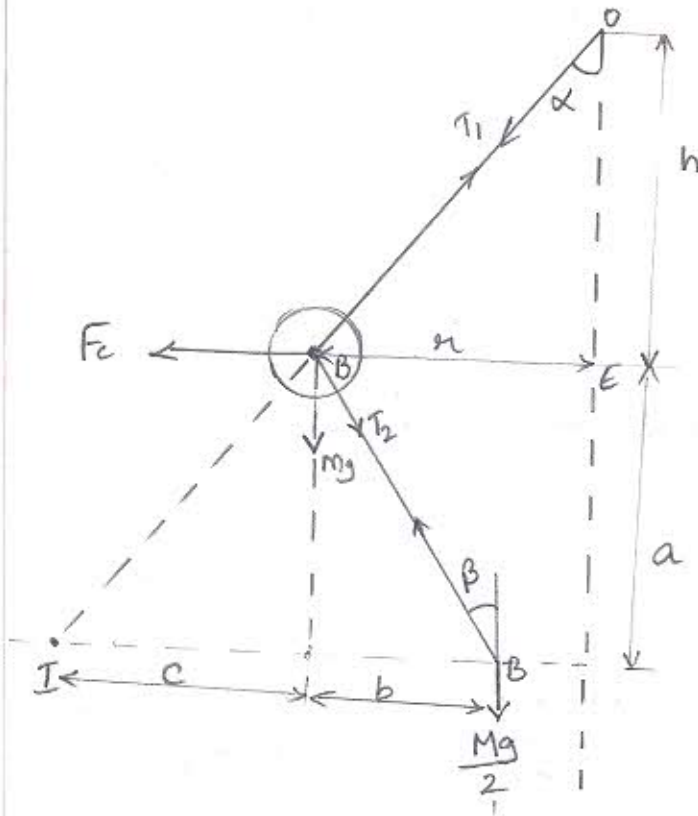
## EFFORT & POWER OF A GOVERNOR

The effort of a governor is the mean force exerted at the sleeve for a given percentage change of speed.

2

## Instantaneous Centre method.

In this method, equilibrium of forces acting on link AB is considered.



For equilibrium  $\Sigma F = 0$  ;  $\Sigma M = 0$

Taking moment about I.

$$F_c \cdot a = mg \cdot c + \frac{Mg}{2} [c + b] \quad \rightarrow \textcircled{1}$$

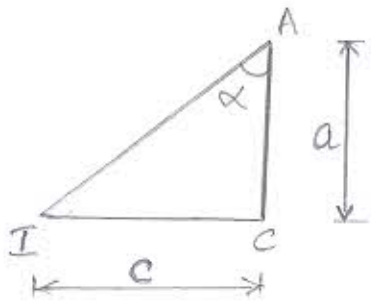
Centrifugal force  $F_c = m\omega^2 r$

Substituting this in eqn  $\textcircled{1}$

$$m\omega^2 r \cdot a = mg \cdot c + \frac{Mg}{2} [c + b]$$

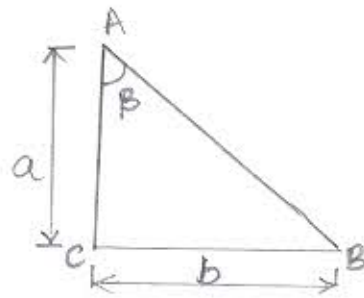
$$m\omega^2 r = mg \cdot \frac{c}{a} + \frac{Mg}{2} \left[ \frac{c}{a} + \frac{b}{a} \right] \quad \rightarrow \textcircled{2}$$

Consider  $\Delta^{lc} ACI$



$$\tan \alpha = \frac{c}{a} \rightarrow \textcircled{A}$$

Consider  $\Delta^{lc} ACB$



$$\tan \beta = \frac{b}{a} \rightarrow \textcircled{B}$$

Substituting  $\textcircled{A}$  &  $\textcircled{B}$  in eqn  $\textcircled{2}$  we get

$$m\omega^2 r = mg \cdot \tan \alpha + \frac{Mg}{2} [\tan \alpha + \tan \beta]$$

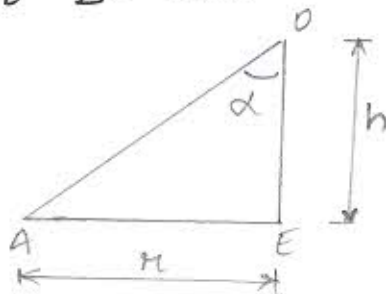
$$m\omega^2 r = \tan \alpha \left[ mg + \frac{Mg}{2} \left( 1 + \frac{\tan \beta}{\tan \alpha} \right) \right] \rightarrow \textcircled{3}$$

Denote  $\frac{\tan \beta}{\tan \alpha} = k$

Equation  $\textcircled{3}$  becomes

$$m\omega^2 r = \tan \alpha \left[ mg + \frac{Mg}{2} (1+k) \right] \rightarrow \textcircled{4}$$

Consider  $\Delta^{lc} OAE$



$$\tan \alpha = \frac{r}{h} \rightarrow \textcircled{C}$$

Substitute  $\textcircled{C}$  in eqn  $\textcircled{4}$  we get

$$m\omega^2 r = \frac{r}{h} \left[ mg + \frac{Mg}{2} (1+k) \right]$$

$$\omega^2 = \frac{\mu}{m\mu h} \left[ mg + \frac{Mg}{2} (1+k) \right]$$

$$= \frac{1}{mh} \left[ mg + \frac{Mg}{2} (1+k) \right]$$

$$\left( \frac{2\pi N}{60} \right)^2 = \frac{1}{mh} \left[ mg + \frac{Mg}{2} (1+k) \right]$$

$$= \frac{g}{h} \left[ m + \frac{M}{2} (1+k) \right]$$

$$N^2 = \frac{895}{h} \left[ m + \frac{M}{2} (1+k) \right]$$

- 3 Four masses 150, 250, 200 & 300 kg are rotating in same plane at radii of 0.25 m, 0.2 m, 0.3 m & 0.35 m resp. Their angular locations are  $40^\circ$ ,  $120^\circ$  &  $250^\circ$  from mass 150 kg respectively measured in counter clockwise direction. Find the position & magnitude of balance mass required, if its radius of rotation is 0.25 m.

Masses m (kg)	Radius of rotation r (m)	Centrifugal force $\div \omega^2$ mr (kg-m)	Angular positions $\theta$ (deg)	Horizontal Components H (mr cos $\theta$ ) kg-m	Vertical Components V (mr sin $\theta$ ) kg-m
150	0.25	37.5	0	37.5	0
250	0.2	50	40	38.3	32.14
200	0.3	60	120	-30	51.96
300	0.35	105	250	-35.9	-98.67

$$\sum H = 9.9$$

$$\sum V = -14.57$$

Resultant

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{9.9^2 + (-14.57)^2}$$

$$R = 17.61 \text{ kg-m}$$

$$\tan \theta = \frac{\sum V}{\sum H} = \frac{-14.57}{9.9} = -1.47172$$

$$\theta = -55.8^\circ$$

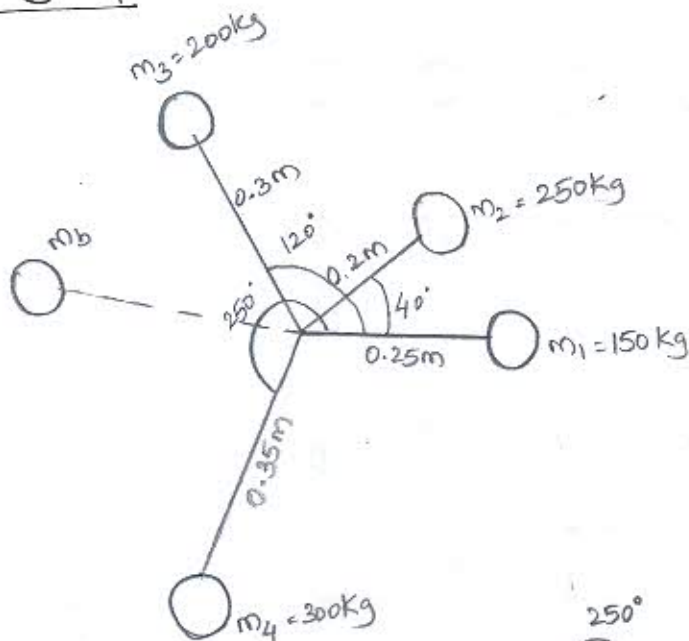
$$\theta_b = 180 + \theta = 180 - 55.8$$

$$\theta_b = 124.2^\circ$$

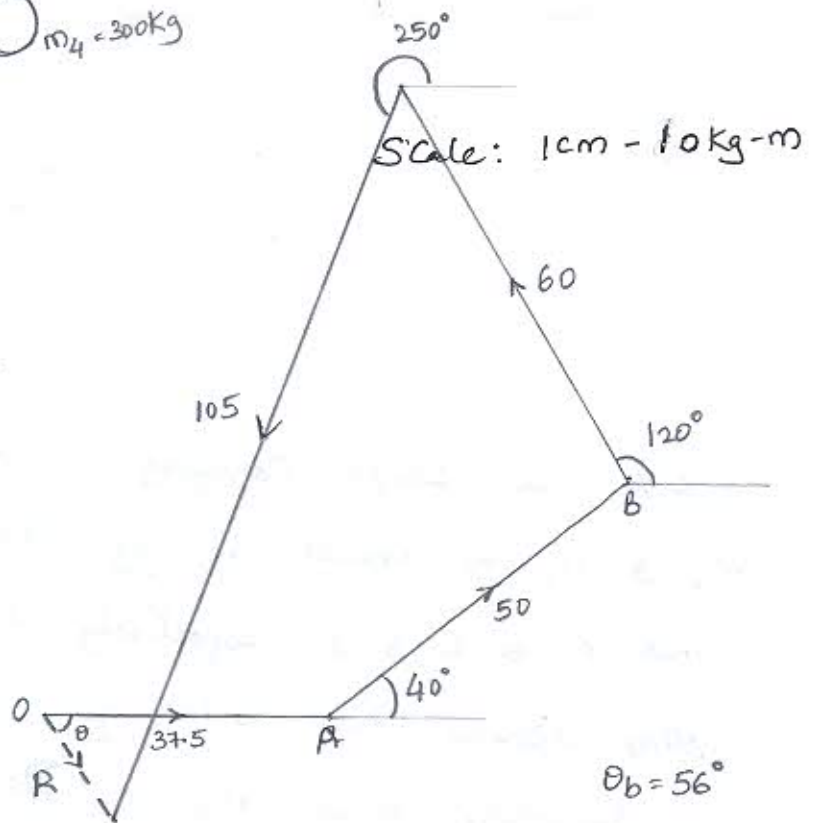


Graphical Method

Space diagram



Vector diagram :-



$m_b r_b = R = 18$

$m_b \cdot 0.25 = 18$

$m_b = 72 \text{ kg}$  - Balancing mass.



## STATIC AND DYNAMIC BALANCING

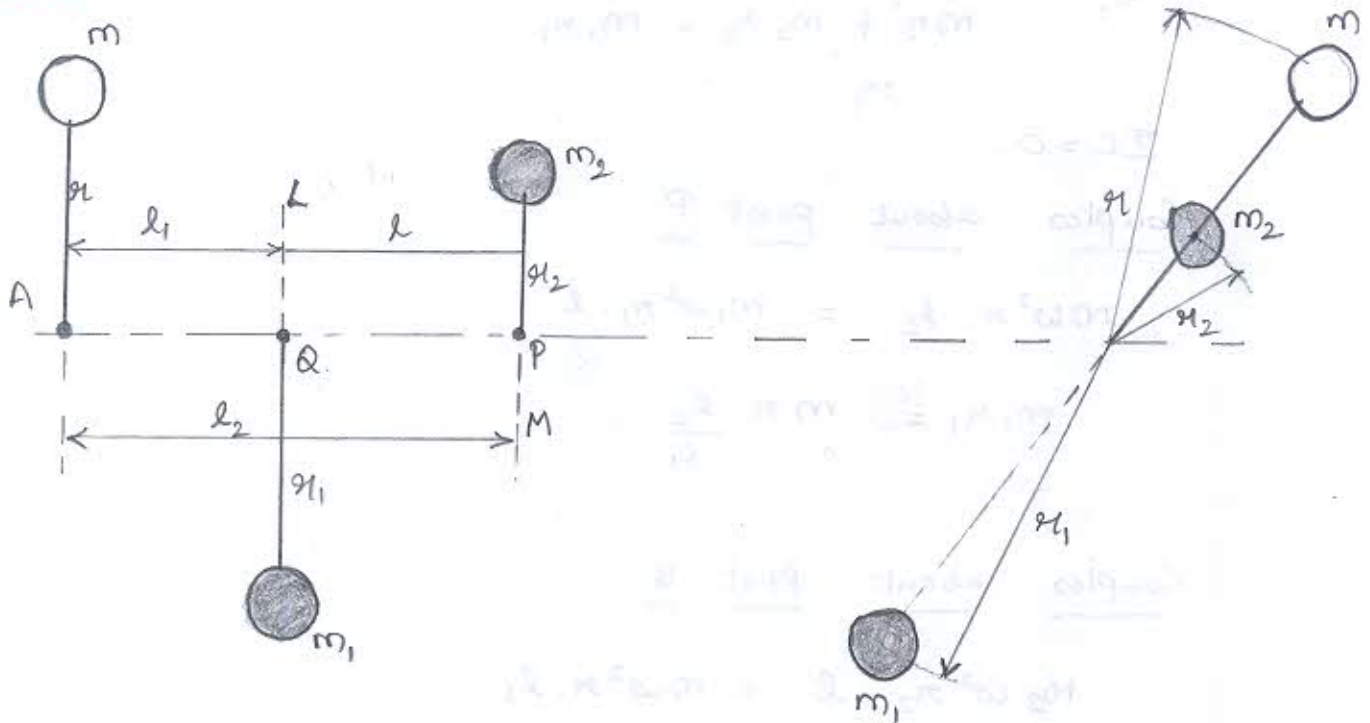
Static balance will be provided if the sum of the moments of weights about the axis of rotation is zero OR A rotor is said to be statically balanced if vector sum of centrifugal forces is zero.

A system of rotating masses is in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple i.e. for dynamic force

- i) Sum of Centrifugal forces must be equal to zero
- ii) Sum of Couple must be equal to zero.

## BALANCING OF SINGLE ROTATING MASS IN TWO DIFFERENT PLANES

CASE i) BOTH THE MASSES ARE ON SAME SIDE



Consider a disturbing mass  $m$  lying in plane A to be balanced by two rotating masses  $m_1$  &  $m_2$  lying in different planes L & M as shown.

Let  $r_1, r_1, r_2$  be the radii of rotation of masses in planes A, L & M respectively.

For perfect balancing, the following conditions to be satisfied

- 1) Sum of centrifugal forces must be equal to zero ( $\Sigma F = 0$ )
- 2) Sum of Couples about any plane must be equal to zero ( $\Sigma C = 0$ )



Consider a disturbing mass 'm' lying in plane A to be balanced by two rotating masses  $m_1$  &  $m_2$  lying on either side in different planes L & M as shown.

Let  $r$ ,  $r_1$ ,  $r_2$  be the radii of rotation of masses in planes A, L & M respectively.

$$\Sigma F = 0$$

$$m\omega^2 r = m_1\omega^2 r_1 + m_2\omega^2 r_2$$

$$mr = m_1 r_1 + m_2 r_2$$

$$\Sigma C = 0.$$

Couples at point P.

$$m\omega^2 r \cdot l_2 = m_1\omega^2 r_1 \cdot l$$

$$mr l_2 = m_1 r_1 l$$

$$m_1 r_1 = m r \frac{l_2}{l}$$

Couples at point Q

$$m\omega^2 r l_1 = m_2\omega^2 r_2 l$$

$$mr l_1 = m_2 r_2 l$$

$$m_2 r_2 = m r \frac{l_1}{l}$$

Given

$$m = 1.4 \text{ kg} ; x = 100 \text{ mm} = 0.1 \text{ m} ; y = 50 \text{ mm} = 0.05 \text{ m}$$

$$r_1 = 75 \text{ mm} = 0.075 \text{ m} ;$$

$$r_2 = 112.5 \text{ mm} = 0.1125 \text{ m} ; r = 0.09 \text{ m}$$

$$N_1 = 300 \text{ rpm} ; N_2 = 300 + \frac{6}{100} \times 300 = 318 \text{ rpm}.$$

$$S = 9 ; N = 9$$

Angular velocity :  $\omega_1 = \frac{2\pi N_1}{60} = \frac{2\pi(300)}{60} = 31.42 \text{ rad/s}$

Centrifugal force

$$F_{c1} = m \omega_1^2 r_1$$

$$= 1.4 (31.42)^2 \cdot 0.075$$

$$F_{c1} = 103.66 \text{ N}$$

Angular Velocity :  $\omega_2 = \frac{2\pi N_2}{60} = \frac{2\pi(318)}{60} = 33.3 \text{ rad/s}$

$$F_{c2} = m \omega_2^2 r_2 = 1.4 (33.3)^2 \cdot 0.1125$$

$$F_{c2} = 174.65 \text{ N}$$



### Stiffness of Spring

$$S = 2 \left[ \frac{F_{c2} - F_{c1}}{r_2 - r_1} \right] \left[ \frac{r}{y} \right]^2$$
$$= 2 \left[ \frac{174.65 - 103.66}{0.1125 - 0.075} \right] \left[ \frac{0.1}{0.05} \right]^2$$

$$S = 15.14 \times 10^3 \text{ N/m}$$

Centrifugal force at  $r = 0.09 \text{ m}$

$$S = 2 \left[ \frac{F_{c2} - F}{r_2 - r} \right] \left[ \frac{r}{y} \right]^2$$

$$15.14 \times 10^3 = 2 \left[ \frac{174.65 - F}{0.1125 - 0.09} \right] \left[ \frac{0.1}{0.05} \right]^2$$

$$F = 132.07 \text{ N}$$

Centrifugal force

$$F = m \omega^2 r$$

$$132.07 = 1.4 \left( \frac{2\pi N}{60} \right)^2 0.09$$

$$N = 309.16 \text{ rpm}$$