CBCS SCHEME

USN | | | | | | | | | | |

15ME81

Eighth Semester B.E. Degree Examination, July/August 2021
Operations Research

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions.

1 a. Define Operation Research. Discuss the scope of Operation Research. **(06 Marks)**

b. A firm manufactures 3 products A, B and C. Time to manufacture product A is twice for B and thrice for C and if the entire labour is engaged in making product A, 1600 units of this product can be produced. These products are to be produced in the ratio 3:4:5. There is demand for at least 300, 250 and 200 units of products A, B and C and the profit earned per unit if Rs.90, Rs.40 and Rs.30 respectively. Formulate the problem as a LPP. **(10 Marks)**

2 a. Discuss the assumptions made in LPP. **(06 Marks)**

b. Solve the following LPP graphically:

Maximize $Z = 2x_1 + 3x_2$

Subject to constraints $x_1 + x_2 \leq 30$,

$x_2 \geq 3$,

$x_2 \leq 12$,

$x_1 - x_2 \geq 0$,

$0 \leq x_1 \leq 20$ **(10 Marks)**

3 a. Explain the significance of following variables in LPP:

i) Slack variable

ii) Surplus variable

iii) Artificial variable. **(06 Marks)**

b. Solve by simplex method the following LPP:

Minimize $Z = x_1 - 3x_2 + 3x_3$

Subject to constraints $3x_1 - x_2 + 2x_3 \leq 7$,

$2x_1 + 4x_2 \geq -12$,

$-4x_1 + 3x_2 + 8x_3 \leq 10$,

$x_1, x_2, x_3 \geq 0$ **(10 Marks)**

4 a. What is Pseudo-optimal solution? **(06 Marks)**

b. Solve the following LPP by Big-M method

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

Subject to constraint $3x_1 + x_2 + 4x_3 \leq 600$,

$2x_1 + 4x_2 + 2x_3 \geq 480$,

$2x_1 + 3x_2 + 3x_3 = 540$,

$x_1, x_2, x_3 \geq 0$ **(10 Marks)**

5 a. Define the following with respect to transportation problem:
i) Basic feasible solution
ii) Optimal solution
iii) Degenerate basic feasible solution. **(06 Marks)**

b. For the following Transportation Problem a solution is given check it for optimality. If not, modify it to obtain a better solution (next best).

	D_1	D_2	D_3	D_4	Available units
S_1	6	1	$9_{(50)}$	$3_{(20)}$	70
S_2	$11_{(55)}$	5	2	8	55
S_3	$10_{(30)}$	$12_{(35)}$	4	$7_{(25)}$	90
Demand units	85	35	50	45	

(10 Marks)

6 The captain of a cricket team has to allot five middle batting positions to 5 batsmen. The average runs scored by each batsman at these positions are as follows:

Batsman	Batting Position				
	I	II	III	IV	V
P	40	40	35	25	50
Q	42	30	16	25	27
R	50	48	40	60	50
S	20	19	20	18	25
T	58	60	59	55	53

i) Find the assignment of batsman to positions which would give the maximum number of runs.
ii) If another batsman 'U' with the following average runs in batting position as given below:

Battery positions:	I	II	III	IV	V
Average runs scored:	45	52	38	50	49

is added to the team, should he be included to play in the team? If so, who will be replaced by him? **(16 Marks)**

7 a. Define:
i) Preceding activity
ii) Dummy activity
iii) Network
iv) Slack. **(06 Marks)**

b. Tasks A, B, C,….H, I constitute a project. The precedence relationships are A < D, A < E, B < F, D < F, C < G, C < H, F < I, G < I.

Task:	A	B	C	D	E	F	G	H	I
Time, days :	8	10	8	10	16	17	18	14	9

i) Draw the network
ii) Identify the critical path and duration. **(10 Marks)**

8 a. Discuss the operating characteristics of a queueing system. **(06 Marks)**

b. A typist at an office of a company receives on the average 20 letters/day for typing. The typist works 8 hours a day and it takes on the average 20 minutes to type a letter. The cost of a letter waiting to be mailed is 80 paise/hr and the cost of the equipment plus salary of the typist is Rs.45 per day.

i) What is the typists utilization rate?

ii) What is the average number of letters waiting to be typed?

iii) What is the average waiting time needed to have a letter typed?

iv) What is the total daily cost of waiting letters to be mailed. **(10 Marks)**

9 a. Define:

i) Strategy

ii) 2 person zero sum game

iii) Pay off matrix. **(06 Marks)**

b. Solve the following game by using principle of dominance:

		Player B					
		I	II	III	IV	V	VI
Player A	1	4	2	0	2	1	1
	2	4	3	1	3	2	2
	3	4	3	7	-5	1	2
	4	4	3	4	-1	2	2
	5	4	3	3	-2	2	2

(10 Marks)

10 a. Discuss any three priority rules of processing n jobs through one machine. **(06 Marks)**

b. Four jobs 1, 2, 3 and 4 are to be processed on each of the four machines. A, B, C and D in the order ABCD. The processing times in minutes are given in the table below. Find, for no passing the minimum elapsed time and idle time for each machine.

		Machines			
		A	B	C	D
Jobs	1	58	14	14	48
	2	30	10	18	32
	3	28	12	16	44
	4	64	16	12	42

(10 Marks)

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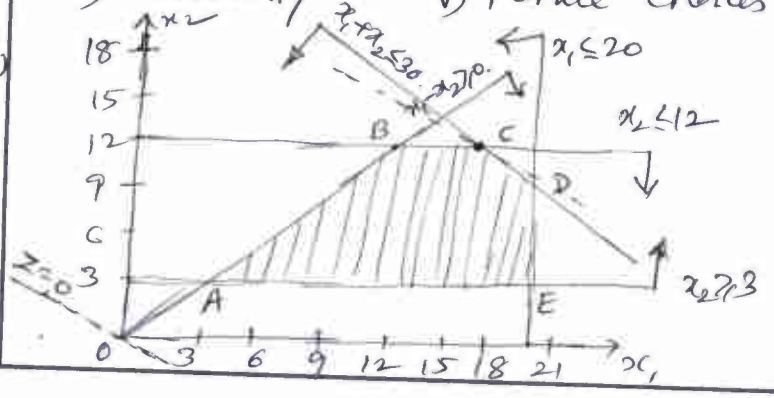
Scheme & Solutions

Subject Title : Operations Research. **Subject Code :** 15ME81 **Signature of Scrutinizer**

Question Number	Solution	Marks Allocated
1 a)	Definition — 1M, Scope – (Min 5 areas) – 5M.	1+5=6M
b)	Let x_1, x_2 & x_3 – Qty of A, B, & C resp. Objective function Maximize $Z = 90x_1 + 40x_2 + 30x_3$	-2M
	Constraints: Time → $tx_1 + \frac{t}{2}x_2 + \frac{t}{3}x_3 \le 1600t$, $\Rightarrow x_1 + \frac{x_2}{2} + \frac{x_3}{3} \le 1600$ $\Rightarrow 6x_1 + 3x_2 + 2x_3 \le 9,600$.	2M
	Market dem → $x_1 \ge 300$, $x_2 \ge 250$, $x_3 \ge 200$.	2M
	ratio of products → $x_1 : x_2 : x_3 = 3 : 4 : 5$ $\Rightarrow \frac{x_1}{3} = \frac{x_2}{4}$; $\frac{x_2}{4} = \frac{x_3}{5}$ i.e. $4x_1 - 3x_2 = 0$ & $5x_2 - 4x_3 = 0$	2M
	Writing the LPP in std format	2M
2 a)	i) Proportionality ii) Additivity iii) Continuity iv) Certainty v) Finite choices	6M
b)		

Subject Title : Operations Research Subject Code : 15ME81

Question Number	Solution	Marks Allocated
	Each constraint representation on Graph 1M each X 5	5M
	Z = 0 line representation	2M
	Marking the solution space	1M
	opt solution $x_1 = 18$, $x_2 = 12$ $Z_{max} = 72$	2M / 10M
Q.3a)	Significance of each var. 2M. 2M X 3	6M
b)	write the LPP in Std Simplex table format	

Minimize $Z = x_1 - 3x_2 + 3x_3 + 0S_1 + 0S_2 + 0S_3$

S.t $3x_1 - x_2 + 2x_3 + S_1 = 7$,

$-2x_1 + 4x_2 + S_2 = 12$,

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$,

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Put $x_1 = x_2 = x_3 = 0$ & Construct get bfs

$S_1 = 7$, $S_2 = 12$, $S_3 = 10$ $Z = 0$.

	C_j	1	-3	3	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	Q
0	S_1	3	-1	2	1	0	0	7	-7
0	S_2	-2	-4	0	0	0	0	12	-3
0	S_3	-4	(3) KE	8	0	0	1	10	10/3 ← KR
	Z_j	0	0	0	0	0	0	0	
	$C_j - Z_j$	1	-3	3	0	0	0		

↑ KC Initial soln 4M

	C_j	1	-3	3	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	Q
0	S_1	5/3	0	14/3	1	0	1/3	31/3	31/5 ← KR
0	S_2	-22/3	0	32/3	0	1	4/3	76/3	-38/11
-3	x_2	-4/3	1	8/3	0	0	1/3	10/3	-5/2
	Z_j	4	-3	-8	0	0	-1	-10	
	$C_j - Z_j$	-3	0	11	0	0	1		

↑ KC IInd solution 3M

Subject Title : Operation Research Subject Code : 15ME81

Question Number	Solution	Marks Allocated
	C_j: 1, -3, 3, 0, 0, 0	
	C_B | Basis | x_1 | x_2 | x_3 | S_1 | S_2 | S_3 | b	
	1 | x_1 | 1 | 0 | 14/5 | 3/5 | 0 | 1/5 | 31/5	
	0 | S_2 | 0 | 0 | 156/5 | 22/5 | 1 | 14/5 | 354/5	
	-3 | x_2 | 0 | 1 | 32/5 | 4/5 | 0 | 3/5 | 58/5	
	Z_j | 1 | -3 | -82/5 | -9/5 | 0 | -8/5 | -143/5	
	$C_j - Z_j$ | 0 | 0 | 97/5 | 9/5 | 0 | 8/5 | opt. s/n	
	optimal soln - $x_1 = \frac{31}{5}$, $x_2 = \frac{58}{5}$, $x_3 = 0$, $Z_{min} = -\frac{143}{5}$	3M / 10M
Q:4 a)	Explanation of Pseudo-optimal soln -	6M
b)	Introduce Slack, Surplus & Artificial var. and write the LPP in std Simplex table format Put $x_1 = x_2 = x_3 = S_2 = 0$ $S_1 = 600$, $A_1 = 480$, $A_2 = 540$, $Z = -1020M$	
	C_j: 2, 3, 4, 0, 0, -M, -M	
	C_B | Basis | x_1 | x_2 | x_3 | S_1 | S_2 | A_1 | A_2 | b | θ	
	0 | S_1 | 3 | 1 | 4 | 1 | 0 | 0 | 0 | 600 | 600	
	-M | A_1 | 2 | 4 | 2 | 0 | -1 | 1 | 0 | 480 | 120 ←	
	-M | A_2 | 2 | 3 | 3 | 0 | 0 | 0 | 1 | 540 | 180	
	Z_j | -4M | -7M | -5M | 0 | M | -M | -M | -1020M |	
	$C_j - Z_j$ | 2+4M | 3+7M | 4+5M | 0 | -M | 0 | 0 | Initial Soln |	4M
	C_j: 2, 3, 4, 0, 0, -M	
	C_B | Basis | x_1 | x_2 | x_3 | S_1 | S_2 | A_2 | b | θ	
	0 | S_1 | 5/2 | 0 | 7/2 | 1 | 1/4 | 0 | 480 | 960/7	
	3 | x_2 | 1/2 | 1 | 1/2 | 0 | -1/4 | 0 | 120 | 240	
	-M | A_2 | 1/2 | 0 | 3/2 | 0 | 3/4 | 1 | 180 | 120 ←	
	Z_j | $\frac{3-M}{2}$ | 3 | $\frac{3-3M}{2}$ | 0 | $\frac{-3+3M}{4}$ | -M | 360-180M |	
	$C_j - Z_j$ | $\frac{1+M}{2}$ | 0 | $\frac{5+3M}{2}$ | 0 | $\frac{3+3M}{4}$ | 0 | Second soln |	3M

Subject Title : OR Subject Code : 15ME81

Question Number	Solution	Marks Allocated
	C_j: 2 3 4 0 0	
	C_B Basis x_1 x_2 x_3 S_1 S_2 b	
	0 S_1 4/3 0 0 1 −3/2 60	
	3 x_2 1/3 1 0 0 −1/2 60	
	4 x_3 1/3 0 1 0 1/2 120	
	Z_j 7/3 3 4 0 1/2 660	
	$C_j - Z_j$ −1/3 0 0 0 −1/2 opt. soln	
	opt soln - $x_1=0$, $x_2=60$, $x_3=120$ $Z_{max}=660$.	3M / 10M
Q:5 a)	Each definition −2M X 3 = 6M	6M
b)	No. of allocations = r+c−1 = 3+4−1 = 6 ∴ soln can be tested for optimality	
	C_{ij} matrix for allocated cells	

u_i \ v_j	0	2	3	−3
6			9	3
11	11			
10	10	12		7

−2M

(u_i+v_j) for vacent cells

6	8		
	13	14	8
		13	

−2M

4M

$CEM = [C_{ij} - (u_i+v_j)]_{NM}$ Allocated cells

0	−7		
	−8	−12	0
		−9	

−ve Entries ∴ Solution is not optimal

2M

Obtain a better soln.

		150	201
155			
30	35		25

2M

bfs with closed Path

		25	45
30		25	
55	35		

2M

Second b.f.s

4M / 10M

Subject Title : Subject Code :

Question Number	Solution	Marks Allocated
Q:6	i) Converting the given Max. problem to Minimization type by subtracting all elements from the highest	

20	20	25	35	10
18	30	44	35	33
10	12	20	0	10
40	41	40	42	35
2	0	1	5	7

Minimization prob

Row reduction - 1M
Column red - 1M
Making assignments - 2M → 06M

10	10	14	25	[0]
[0]	12	25	17	15
10	12	19	[0]	10
5	6	4	7	✗
2	[0]	✗	5	7

Matrix after Row & column red^n. & made assignments.
No. of ass^s - 4 < 5
∴ non-optimal

6	6	10	21	[0]
[0]	12	25	17	19
10	12	19	[0]	14
1	2	[0]	3	✗
2	[0]	✗	5	11

Second b.f.s
5 asst's ⇒ optimal s/n

4 marks

Batsman : P Q R S T
Position : II I IV III II Total
Runs scored : 50 42 60 20 60 = 232 → 02M

ii) Include D Batsman in the table & make it 6×5 hence balance by adding a dummy column with runs scored Zero. & Convert into Minimization as earlier & Perform Row & column reduction and make assignments. 02M

	I	II	III	IV	V	VI – dummy
P	10	10	14	25	0	25
Q	0	12	26	17	15	17
R	10	12	20	0	10	35
S	5	6	4	7	0	0
T	2	0	0	5	7	35
D	7	0	13	2	3	27

Matrix after row & column red^n – 03M

Subject Title : Subject Code :

Question Number	Solution	Marks Allocated

10	10	14	25	[0]	25
[0]	12	25	17	15	17
10	12	19	[0]	10	35
5	6	4	7	X	[0]
2	10	[0]	5	7	35
7	[0]	13	2	3	27

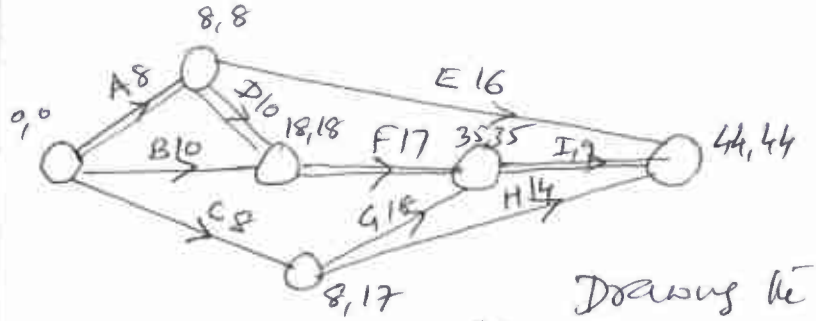
opt soln

Batsman	Partn	Runs
P	V	50
Q	I	42
R	IV	60
S	Dummy	0
T	III	59
U	I	52
		263

∴ Batsman U should be included (263 > 232) — 3M / 16M
he will replace S Batsman

7 a) Each defn - 1.5M × 4 = 6M — 6M

b)



Drawing the network — 4M
(TE) Forward pass calc — 2M
(TL) Reverse " — 2M
A-D-F-I (44 days) critical path — 2M / 10M

8 a) Queue length, system length, waiting time in the queue, Total time in the system, Utilization factor — 6M

b) λ = 20 letters/day, $\mu = \frac{1}{20} \times 8 \times 60$ = 24 letters/day — 2M

(i) Utilization rate = $\frac{\lambda}{\mu}$ = 0.833 — 2M

(ii) Av. no of letters waiting to be typed = $\frac{\lambda}{\mu} \cdot \frac{\lambda}{\mu - \lambda}$ = 4.17 — 2M

Subject Title : Subject Code :

Question Number	Solution	Marks Allocated
	iii) Avg waiting time needed to have a letter typed = waiting time in queue + time to type letter = time spent in system = $\frac{1}{\mu-\lambda} = \frac{1}{4}$ day = 2 hrs — 2M iv) Avg no of letters in system = $\frac{\lambda}{\mu-\lambda} = 5$. Total daily cost of waiting letters = $5 \times \left(\frac{80}{100} \times 8\right) + 45 = 77$ Rs. — 2M / 10M	
9 a)	Each defn - 2M × 3 = 6M ———	6M
b)	Row 2 dominates Row 1 Row 4 " Row 5 Hence delete R_1, R_5	

	I	II	III	IV	V	VI
2	4	3	1	3	2	2
3	4	3	7	-5	1	2
4	4	3	4	-1	2	2

— 2M

Colum I II & VI are dominated by III, IV & V
hence delete I, II & VI colu

	III	IV	V
2	1	3	2
3	7	-5	1
4	4	-1	2

— 2M

Avg of III & IV dominate V hence delete V

	III	IV
2	1	3
3	7	-5
4	4	-1

Avg of $R_2 R_3$ dominate R_4 hence delet R_4 — 2M

	III	IV		
2	1	3	12	12/14
3	7	-5	2	2/14
	8	6		
	8/14	6/14		

2M

opt strategy A : (0, 6/7, 1/7, 0, 0)
B : (0, 0, 4/7, 3/7, 0, 0)

$V = \frac{1 \times 8 + 3 \times 6}{8+6} = \frac{13}{7}$ — 2M / 10M

Subject Title : **Subject Code :**

Question Number	Solution	Marks Allocated
10 a)	one priority rule - 2 M each × 3 ——	6M
b)	Min (A, D) ≥ Max (B, C) 28, 32 ≥ 16, 18 ✓ Can be converted into equivalent 2 mc	1M

$G = A + B + C$

$H = B + C + D$

Job	Mc G	Mc H
1	86	76
2 ②	58	60
3 ①	56	72
4	92	70 ③

Opt. sequence → | 3 | 2 | 1 | 4 | — 4M

Seq.	Mc A	Mc B	Mc C	Mc D
3	0-28	28-40	40-56	56-100
2	28-58	58-68	68-86	100-132
1	58-116	116-130	130-144	144-192
4	116-180	180-196	196-208	208-250

3M

TET - 250 min (4 hr 10 min)

IT of mc
- A — 70 min
- B — 198 min
- C — 190 min
- 4 — 84 min

2M/10M