Modified.

CBCS SCHEME

USN | | | | | | | | | | |

17ME81

Eighth Semester B.E. Degree Examination, July/August 2021
Operations Research

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions.

1 a. List and explain the phases of operations research. **(08 Marks)**

b. A diet of a sick person must contain atleast 4,000 units of vitamins, 50 units of minerals and 1400 calories. Two foods are available at a cost of Rs.4 and Rs.3 per unit respectively for A and B types. If food A contains 200 units of vitamins, 1 unit of mineral and 40 calories and if food B contains 100 units of vitamins, 2 units of mineral and 40 calories, formulate this problem as LPP model and solve it by graphical method to find the least cost with minimum requirement of the ingredients. **(12 Marks)**

2 a. Discuss the scope of operations research. **(06 Marks)**

b. Use graphical method to solve the following LPP.

Maximize $Z = 2x_1 + 3x_2$

Subjected to constraints (i) $x_1 + x_2 \le 30$ (ii) $x_2 \ge 3$ (iii) $0 \le x_2 \le 12$

(iv) $0 \le x_1 \le 20$ (v) $x_1 - x_2 \ge 0$; $x_1, x_2 \ge 0$ **(14 Marks)**

3 a. What is the significance of introducing slack, surplus and artificial variables in LPP? **(04 Marks)**

b. Solve the following LPP by simplex method:

Maximize $Z = 4x_1 + 3x_2$

Subject to constraints (i) $2x_1 + x_2 \le 1000$ (ii) $x_1 + x_2 \le 800$ (iii) $x_1 \le 400$ (iv) $x_2 \le 700$ **(16 Marks)**

4 Solve the given problem using Big M method.

Maximize $Z = -2x_1 - x_2$

Subject to $3x_1 + x_2 = 3$, $4x_1 + 3x_2 \ge 6$, $x_1 + 2x_2 \le 4$, $x_1, x_2 \ge 0$ **(20 Marks)**

5 a. What is degeneracy in transportation problem? Discuss how it can be overcome. **(04 Marks)**

b. L & T Company needs 3, 3, 4 and 5 million cubic feet of fill at 4 earthen dam sites I, II, III and IV in Karnataka. It can transfer the fill from 3 mounds A, B and C where 2, 6 and 7 million cubic feet of fill is available respectively. Costs of transportation for one million cubic feet of fill from 3 mounds to the 4 sites in lakhs of rupees are given in the table below (Table.Q5(b)). Determine the optimal transportation plan which minimizes cost to company.

		Sites			
		I	II	III	IV
Mounds	A	15	10	17	18
	B	16	13	12	13
	C	12	17	20	11

Table.Q5(b) **(16 Marks)**

6 a. What do you understand by a balanced and unbalanced transportation problem? How an unbalanced problem is tackled? **(06 Marks)**

b. A product is produced by four factories A, B, C and D and their unit production costs in them are Rs.2, 3, 1 and 5 respectively. Their production capacities are factory A – 50 units, B – 70 units, C – 30, D – 50 units. These supply the products to four stores with their demands of 25, 35, 105 and 20 units respectively. Unit transportation cost from each factory to each store is given in Table.Q6(b). Determine the extent of deliveries from each factory to each store, so that total cost of production cum transportation is minimum.

		Stores			
		1	2	3	4
Factories	A	2	4	6	11
	B	10	8	7	5
	C	13	3	9	12
	D	4	6	8	3

Table.Q6(b)

(14 Marks)

7 A small project is composed of activities with their time estimates listed in Table.Q7.

a. Draw project network

b. Find expected duration and variance of each activity and its expected project length

c. What is the probability of completing project:

(i) Atleast 4 weeks earlier than expected.

(ii) If project is due in 19 weeks, what is the probability of meeting the due date?

Activity	t_o	t_m	t_p
1 - 2	1	1	7
1 - 3	1	4	7
1 - 4	2	2	8
2 - 5	1	1	1
3 - 5	2	5	14
4 - 6	2	5	8
5 - 6	3	6	15

Table.Q7

(20 Marks)

8 a. State and explain in brief Kendall's notation for representing queing models. **(06 Marks)**

b. A self service store employs one cashier at its counter. An average of 9 customers arrive every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution of arrival rate and exponential distribution of service rate find:

(i) Average number of customers in system

(ii) Average number of customers in queue

(iii) Average time a customer spends in system

(iv) Average time a customer waits before being served. **(14 Marks)**

9 a. Explain the following terms: (i) Pay off matrix (ii) Saddle point (iii) Fair game **(06 Marks)**

b. Use dominance rule to find the optimum strategies for both players.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{c} \begin{array}{cccccc} B_1 & B_2 & B_3 & B_4 & B_5 & B_6 \end{array} \\ \begin{bmatrix} 4 & 2 & 0 & 2 & 1 & 1 \\ 4 & 3 & 1 & 3 & 2 & 2 \\ 4 & 3 & 7 & -5 & 1 & 2 \\ 4 & 3 & 4 & -1 & 2 & 2 \\ 4 & 3 & 3 & -2 & 2 & 2 \end{bmatrix} \end{array}$$

(14 Marks)

10 a. State assumptions made while applying Johnson's rule to n jobs on 2 machines. **(06 Marks)**

b. Use graphical method to minimize the time required to process the jobs. Details of processing time (hrs) and sequence given below:

Job 1:	A – 4, C – 2, D – 6, E – 3, B – 2
Job 2:	C – 8, A – 3, D – 4, B – 2, E – 3

Find sequence of jobs on each machine and total elapsed time for both jobs. **(14 Marks)**

Re: Sir, regarding modification of scheme and solution(17ME81)

"virupaxi bagodi" <virupaxibagodi@yahoo.com> July 28, 2021 11:58 AM

To: boe@vtu.ac.in

Dear Sir,

I understand that the students in some colleges demanded more graph sheets (than what has been provide in the answer booklet) to solve a few questions with regard to 17ME81. The valuators may please consider the graphs drawn in ruled sheets of the answer booklet supplied by the university and award the marks.

The scheme and solutions of 17ME81 is approved.

Thanking you with warm regards.

Sincerely,
Dr. Virupaxi Bagodi
Principal
Government Engineering College,
TALAKAL - 583238, India
E-mail: virupaxibagodi@yahoo.com
Cell: +91 94 49 973293

> On Wednesday, 28 July, 2021, 11:51:33 am IST, virupaxi bagodi <virupaxibagodi@yahoo.com> wrote:
>
> Dear Sir,
>
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> Sincerely,
> Dr. Virupaxi Bagodi
> Principal
> Government Engineering College,
> TALAKAL - 583238, India
> E-mail: virupaxibagodi@yahoo.com
> Cell: +91 94 49 973293
>
> > On Wednesday, 28 July, 2021, 11:17:08 am IST, <boe@vtu.ac.in> wrote:

"APPROVED"
B.E
Registrar (Evaluation) 28/07/2021
Visvesvaraya Technological University
BELAGAVI - 590018

210617ME8130398

Visvesvaraya Technological University

Belagavi, Karnataka - 590 018

Signature of Scrutinizer

Scheme & Solutions

Subject Title : Operations Research **Subject Code :** 17ME81

Question Number	Solution	Marks Allocated
1 a	(i) Judgement phase :- includes identification of real life problem (b) selection of objective (c) measures of effectiveness (d) formulation — 2M 3 (ii) Research phase :- (iii) Action phase :- (a) Observation & data collection (b) Analysis of data & verification 3 (c) Prediction of results — 2M (d) Generalisation of results & method Action phase :- Making recomendation for implementing the decision ; depending on environment of problem, objectives ; assumptions of problem . — 2 M 2	6M 8M
1(b)	Solution :- Data summarised as a table	

(Food Type)	Unit Contents Vitamins	Minerals	Calories	Cost/unit
A	200	1	40	4 Rs
B	100	2	40	3 Rs
Minimum Requirement	4000	50	1,400	

x_1 – unit of food A , x_2 – units of food B

Minimise $Z = 4x_1 + 3x_2$

Constraints :- $200x_1 + 100x_2 \geq 4000$ (vitamins)

$x_1 + 2x_2 \geq 50$ (minerals)

$40x_1 + 40x_2 \geq 1,400$ (calories)

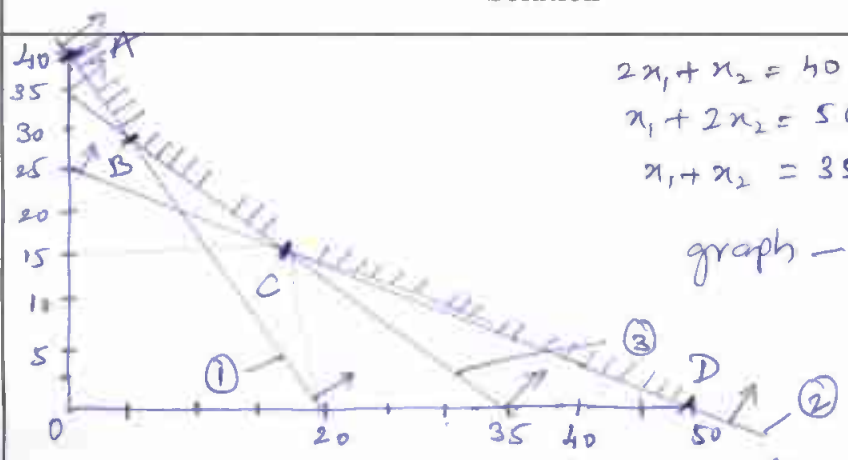
& $x_1, x_2 \geq 0$

— 04 marks

06

Subject Title : **Subject Code :**

Question Number	Solution	Marks Allocated



$2x_1 + x_2 = 40$ — (1)

$x_1 + 2x_2 = 50$ — (2)

$x_1 + x_2 = 35$ — (3)

graph — 5 marks

01M

06M + 04 = 10M

12

Co ordinates of extreme points

	coordinates	Objective
A	(0, 40)	4(0) + 3(40) = 120
→ B	(5, 30)	4(5) + 3(30) = 110
C	(20, 15)	4(20) + 3(15) = 125
D	(50, 0)	4(50) + 3(0) = 200

Minimum → $x_1 = 5$, $x_2 = 30$

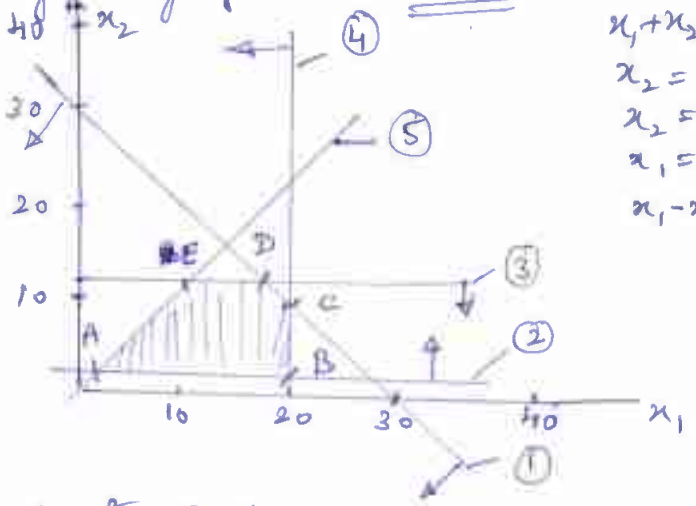
2(a) Atleast 6 fields of applications —

- Agriculture → Economy planning
- Defence → Hospitals
- Industry → Traffic control etc

each 1 × 6 = 6M.

06M

2(b) Plotting the graph — 06 Marks



$x_1 + x_2 = 30$ — (1)

$x_2 = 3$ — (2)

$x_2 = 12$ — (3)

$x_1 = 20$ — (4)

$x_1 - x_2 = 0$ — (5)

8+6

6+4 = 10M

14

Coordinates Table

	(x_1, x_2)	$Z = 2x_1 + 3x_2$
A	(3, 3)	→ 15
B	(20, 3)	→ 49
C	(20, 10)	→ 70
D	(18, 12)	→ 72
E	(12, 12)	→ 60

04M

max value Z = 72 at pt. D.

D(18, 12).

$x_1 = 18$, $x_2 = 12$

Subject Title : Subject Code :

Question Number	Solution	Marks Allocated
3(a)	Meaning of Slack — (1) Meaning of Surplus — (1) Meaning of Artificial variable — (1) Significance — (1)	04M.
(b)	Solution :- Initial Table	

	C_B	B	C_j	—	4	3	0	0	0	0	x_B/x_1
			b		x_1	x_2	S_1	S_2	S_3	S_4	
	0	S_1	1000		2	1	1	0	0	0	$1000/2 = 500$
	0	S_2	800		1	1	0	1	0	0	$800/1 = 800$
→	0	S_3	400		(1)	0	0	0	1	0	$400/1 = 400$ →
	0	S_4	700		0	1	0	0	0	1	—
$Z=0$			Z_j		0	0	0	0	0	0	
			$C_j - Z_j$		4 ↑	3	0	0	0	0	

x_1 is entering S_3 leaving

R_3 new $= R_3$ old $\div$ 1 (key element) $\quad R_{1(new)} \rightarrow R_1 old - 2R_3(new)$

$R_{2\,new} \rightarrow R_2(old) - R_3(new)$

Ist iteration.

	C_B	B	C_j	—	4	3	0	0	0	0	x_B/x_2	
					x_1	x_2	S_1	S_2	S_3	S_4		
	0	S_1	200	—	0	(1)	1	0	−2	0	$200/1 = 200$	→
	0	S_2	400	—	0	1	0	1	−1	0	$400/1 = 400$	
	4	x_1	400	—	1	0	0	0	1	0	$400/0 = —$	
	0	S_4	700	—	0	1	0	0	0	1	$700/1 = 700$	
$Z = 1600$			$C_j - Z_j$		0	3 ↑	0	0	−4	0		

x_2 → entering

$C_2 - Z_2 = 3 > 0$, (S_1 → leaving)

(not optimal)

4 Marks 6

II iteration

	C_B	B	C_j	—	4	3	0	0	0	0	x_B/S_3	
					x_1	x_2	S_1	S_2	S_3	S_4		
	3	x_2	200		0	1	1	0	−2	0	—	
	0	S_2	200		0	0	−1	1	(1)	0	$200/1 = 200$	→
	4	x_1	400		1	0	0	0	1	0	$400/1 = 400$	
	0	S_4	500		0	0	−1	0	2	1	$500/2 = 250$	
$Z = 3\times200 + 4\times400$			Z_j		4	3	3	0	−2	0		
$= 2{,}200$			$C_j - Z_j$		0	0	−3	0	2 ↑	0		

4 Marks → 6

Subject Title : Operations Research Subject Code : 17ME81

Question Number	Solution	Marks Allocated

$C_5 - Z_5 > 0$ in S_3 column ∴ go for further

III iteration

			C_j	4	3	0	0	0	0
C_B	B	x_B		x_1	x_2	S_1	S_2	S_3	S_4
3	x_2	600		0	1	−1	2	0	0
0	S_3	200		0	0	−1	1	1	0
4	x_1	200		1	0	1	−1	0	0
0	S_4	100		0	0	1	−2	0	1
$Z = 2,600$			Z_j	4	3	1	2	0	0
			$C_j - Z_j$	0	0	−1	−2	0	0

4M

Since all $C_j - Z_j \leq 0$, solution is optimal

$x_1 = 200$

$x_2 = 600$

6+6+4
4+4+4
= 12
16

(4) Solution :- Draw the initial table with all columns and rows upto $Z_j - C_j$ equation by using the standard form shown below

Max $Z = -2x_1 - x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

Subject to $x_1 + 2x_2 + S_1 = 4$ S_1 – slack

$4x_1 + 3x_2 - S_2 + A_1 = 6$

$3x_1 + x_2 + A_2 = 3$

where S_1 – slack variable

S_2 – surplus variable

A_1 – Artificial –n–

A_2 – Artificial variable.

writing for standard form – 04M

06

Subject Title : O R. **Subject Code :** 17ME81

Question Number	Solution	Marks Allocated

$C_j \rightarrow$ | | −2 | −1 | 0 | 0 | −M | −M | | |

B	C_B	x_1	x_2	S_1	S_2	A_1	A_2	x_B	x_B/x_1
S_1	0	1	2	1	0	0	0	4	$4/1=4$
A_1	−M	4	3	0	−1	1	0	6	$6/4=1.5$
A_2	−M	(3)	1	0	0	0	1	3	$3/3=1$ →
$Z_j - C_j$		$-7M+2$ ↑	$-4M+1$	0	M	0	0		

4M

5M

x_1 → enters, A_2 → leaves

S_1	0	0	5/3	1	0	0	−1/3	3	$3/5/3=5$
A_1	−M	0	(5/3)	0	−1	1	−4/3	2	$2/5/3=6/5$ →
x_1	−2	1	1/3	0	0	0	1/3	1	$1/1/3=3$
$Z_j - C_j$		0	$\frac{-5m+1}{3}$ ↑	0	M	0	$\frac{7m-2}{3}$		

4M

5M

S_1	0	0	0	1	4	−4	0	1	
x_2	−1	0	1	0	−3/5	3/5	−4/5	6/5	
x_1	−2	1	0	0	1/5	−1/5	3/5	3/5	
$Z_j - C_j$		0	0	0	1/5	M−1/5	M−2/5		

4M

4M

Since all $Z_j - C_j \geq 0$ solution is optimal with

$x_1 = 3/5$

$x_2 = 6/5$

$Z = -2(3/5) - 6/5 \quad Z = -12/5$

$= -\frac{6}{5} - \frac{6}{5} = \frac{-12}{5}$

6+5+5+4
4+4+4+4
= 16 Marks

20

Subject Title : OR Subject Code : 17ME81

Question Number	Solution	Marks Allocated
5(a)	Explaination of degeneracy — 02M Overcoming method for degeneracy — 02M	04M
5(b)	Solution:-	

Table I

	I	II	III	IV	Supply	Pty 1	Pty 2	Pty 3	Pty 4
A	15	10 (2)	17	18	20	5 (I)			
B	16	13 (1)	12 (4)	13 (1)	6/2/1	1	1	3	0
C	12 (3)	17	20	11 (4)	7/4/0	1	1	1	6 (IV)
Demand	3/0	3/1/0	4/0	5/1/0	15	=15			

Supply = Demand ∴ balanced

Pty 1	3	3	5	2	
Pty 2	4	4	8 (II)	2	
Pty 3	4 (III)	4 (V)		2 (VI)	

Solve by VAM method. — 06 Marks

Check for degeneracy, $m = 3$, $n = 4$,

$\therefore m+n-1 = 3+4-1 = 6$

no of allocations = 6. ∴ no degeneracy

allocations also independent positions shown in table I

Transportation cost = $10\times2 + 13\times1 + 12\times4 + 13\times1 + 12\times3 + 11\times4$

$= 174.$

Subject Title : Subject Code :

Question Number	Solution	Marks Allocated

Table 2

	I	II	III	IV	U_i
A		10			$u_1 = 11$
B		13	12	13	$u_2 = 14$
C	12			11	$u_3 = 12$
V_j	$V_1 = 0$	$V_2 = -1$	$V_3 = -2$	$V_4 = -1$	

— 2M

3M

(U_i, V_j) for allocated cells.

$u_1 + v_2 = 10$, $\Rightarrow$ Let $v_1 = 0$, $u_3 = 12$

$u_2 + v_2 = 13$ $\quad v_4 = 11 - 12 = -1$

$u_2 + v_3 = 12$ $\quad u_2 = 13 - v_4 = 14$

$u_2 + v_4 = 13$ $\quad v_3 = 12 - v_2 = 12 - 14 = -2$

$u_3 + v_1 = 12$ $\quad v_2 = 13 - 14 = -1$

$u_3 + v_4 = 11$ $\quad u_1 = 10 - (-1) = 11$

6+2+2+2 = 12 Marks

Table 3

	I	II	III	IV
A	11		9	10
B	14			
C		11	10	

— 2 M

3M

6+3+3+4

$u_i + v_j$ for vacant cells

4M

16

Table 4

— 2M

	I	II	III	IV	U_i
A	15−11		17−9	18−10	
B	16−14				
C		17−11	20−10		

Solution is optimal since all Δ_{ij} values are +ve

Net evaluation matrix $\Delta_{ij} = C_{ij} - (u_i + v_j)$

Subject Title : Subject Code :

Question Number	Solution	Marks Allocated
6(a)	Meaning of balanced and unbalanced problem – 2M Explaining to balance a problem – 2M	06 04M
6(b)	Formation of transportation and production cost matrix	

	1	2	3	4	Capacity
A	2+2	4+2	6+2	11+2	50
B	10+3	8+3	7+3	5+3	70
C	13+1	3+1	9+1	12+1	30
D	4+5	6+5	8+5	3+5	50
Demand	25	35	105	20	

02M

04

Total capacity 200, Demand = 185

Add 15 units on demand side to balance by creating dummy 'd'.

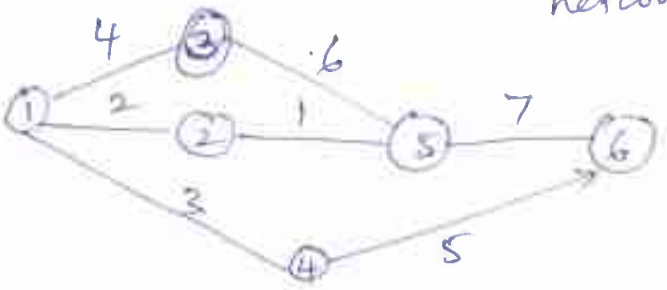
	1	2	3	4	d	Capacity
A	(25) 4	6 (5)	8 (20)	13	0	50/25/20/0
B	13	11	10 (70)	8	0	70/0
C	14	4 (30)	10	13	0	30/0
D	9	11	13 (15)	8 (20)	0 (15)	50/35/15/0
Demand	25	35	105	20	15	200 / 200

Solution by VAM method 06 Marks

Optimality check → 04 Marks

4+6+4= 14

Subject Title : OR. Subject Code : 17ME81

Question Number	Solution	Marks Allocated
7	(a) network  network — (06 Marks) 06	
	(b) critical path Examine all possible paths 1 – 2 – 5 – 6 → 2+1+7 = 10 weeks 1 – 3 – 5 – 6 — 4+6+7 = 17 weeks 1 – 4 – 6 — 3+5 = 8 weeks ∴ Critical path = 1-3-5-6 & 17 weeks Duration of project = 17 weeks — 02M 04 Variance = Sum of variance along Critical path $\sigma_c^2 = 1+4+4 = 9 \quad \sigma_c = 3$	
	(c) Std normal deviation $Z = \frac{\text{Due date} - \text{Expected date}}{\sqrt{\text{Variance}}}$ (i) atleast earlier = 17 – 4 = 13 weeks $\therefore Z = \frac{13-17}{\sigma_c} = \frac{-4}{3} = -1.33$ 06 $P(T_s \le 13) = P(Z \le -1.33)$ — 4M $= 0.5 - 0.4082$ $= 0.0918 = 9.18\%$ (ii) when due date = 19 weeks	

Question No.	Solution	Marks allotted

$$Z = \frac{19-17}{3} = 0.67$$

$\therefore$ Probability is $P(Z \leq 0.67) = 0.5 + \phi(0.67)$

$= 0.5 + 0.2486$

$= 0.7486$ — 4M

$= 74.86\%$ (6+4+6+4) 20

8(a) Kendall notations — 04 M

Explanations — 02 M — 06 M

8(b) Solution:- Arrival rate $\lambda = 9/5 = 1.8$ customers/min

Service rate $\mu = 10/5 = 2$ customers/min

— 02 M = 03

1. $L_s = \frac{\lambda}{\mu - \lambda} = \frac{1.8}{2-1.8} = 9$ — 2M = 03

2. $L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{(1.8)^2}{2(2-1.8)} = 8.1$ — 2M 03

(10M)

3. Average time customer spends in system $W_s = \frac{1}{\mu - \lambda} = \frac{1}{2-1.8} = 5$ min

— 2M = 03

14M

4. $W_q = \frac{\lambda}{\mu}\left(\frac{1}{\mu-\lambda}\right) = \frac{1.8}{2}\left(\frac{1}{2-1.8}\right)$

$= 4.5$ min

— 2M

Subject :- Operations Research Subject Code :- 17ME81

Question Number	Solution	Marks Allocated
9(a)	Explanation Pay of matrix — 02 M Saddle point — 02 M 2×3 — 06 M Fair game — 02 M	
(b)	Solution :- Player A's point of view row 1 & row 5 are dominated by 2 & 4 respectively delete 1 & 5 rows player B's point of view columns I & II are dominated by columns IV, V & VI, column VI is dominated by V ∴ I, II & VI are deleted. resulting in	

Player A	III	IV	V
2	1	3	2
3	7	-5	1
4	4	-1	2

Taking average of III + IV & comparing with V, V is dominating.

∴ V is deleted..

Player A	III	IV
2	1	3
3	7	-5
4	4	-1

row 4 is dominated by Average of 2 & 3, rows

$\frac{1+7}{2} = 4$, $\frac{3-5}{2} = \frac{-2}{2} = -1$

∴ Delete row 4

Player A	III	IV
2	1	3
3	7	-5

by arithmetic method, $P_1 = \frac{(-5-7)}{(4+5)-(3+7)} = 6/7$

$P_2 = 1 - 6/7 = 1/7$.

$q_1 = 4/7$, $q_2 = 1 - 4/7 = 3/7$.

Value of game $V = 13/7$.

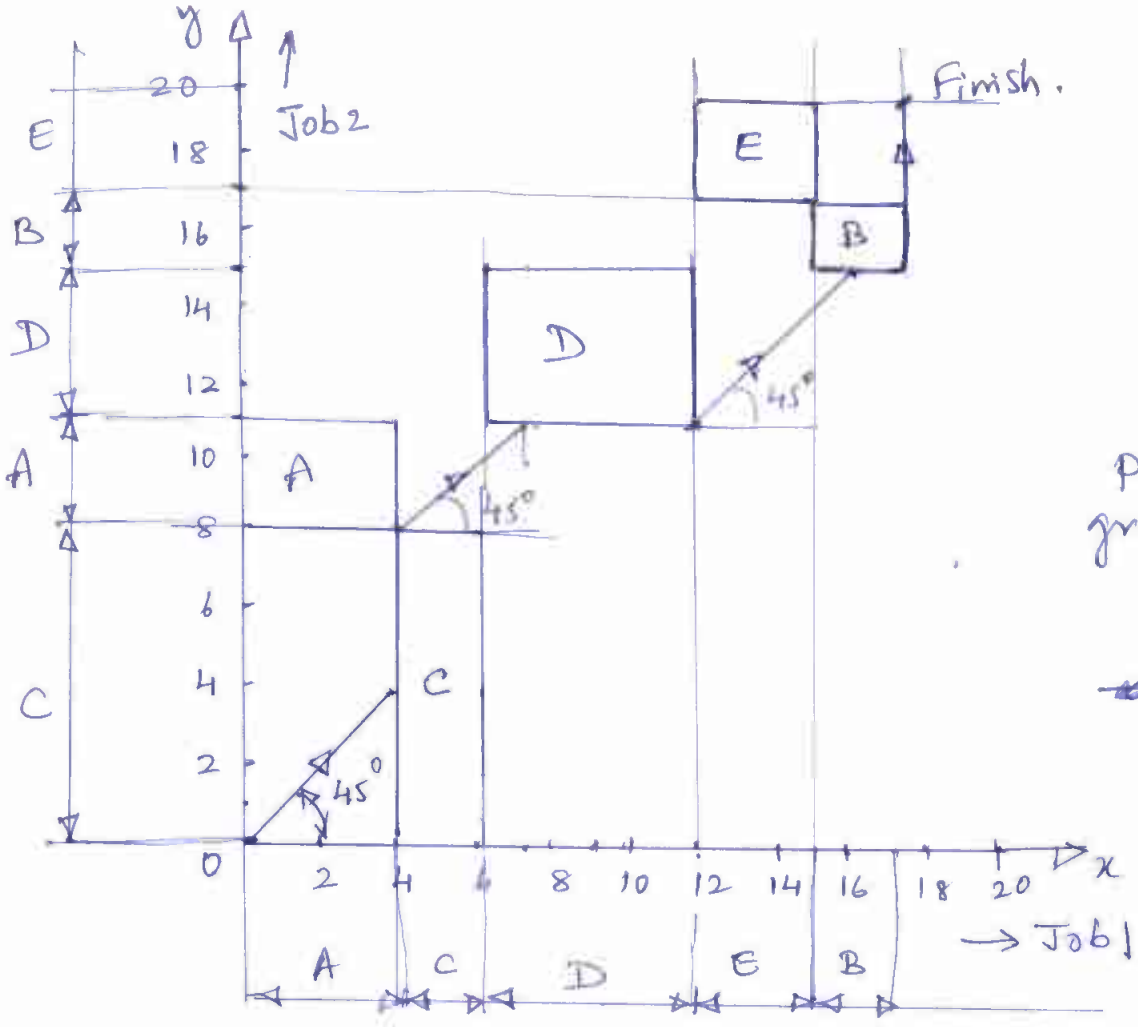
Optimal strategy for A :- $[0, 6/7, 1/7, 0, 0]$

for B :- $[0, 0, 4/7, 3/7, 0, 0]$

10(a) Atleast 6 assumptions : 6 × 1 = 6 M

P.T.O

10 (b) Solution



Plotting graph = 08 Marks

Total elapsed time to complete Job 1
= Processing time of Job 1 + idle time of J_1

Processing time of J_1 = 4 + 3 + 5 + 4 + 1 = 17 hrs

Idle time of J_1 = 4 + 5 = 9 hrs

Total time elapsed = 17 + 9 = 26 hrs for J_1

Total time to complete job J_2
= Processing time of J_2 + Idle time of J_2

Processing time = 4 + 4.3 + 4 + 5 = 20 hrs

Idle time = 5 + 1 = 6

Total time = 20 + 6 = 26 hrs for J_2

02 M

"APPROVED"
Ray B-E
Registrar (Evaluation)
Visvesvaraya Technological University
BELAGAVI - 590018

M/c	I st	II nd
A →	J_1	J_2
B	J_1	J_2
C	J_2	J_1
D	J_1	J_2
E	J_1	J_2

– Sequence of Jobs on each M/C – 02 M

6 + 2 + 2 = 10 M