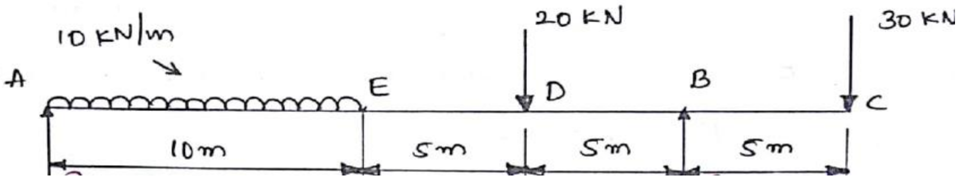
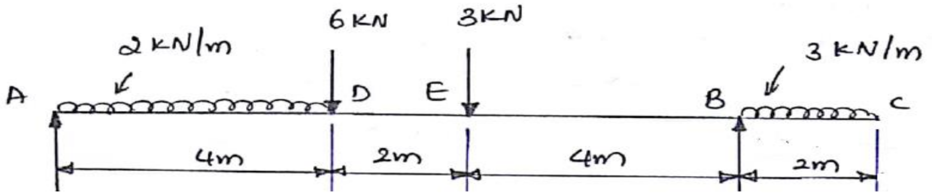
CMR INSTITUTE OF TECHNOLOGY

ROLL NO.

Internal Assesment Test – 2

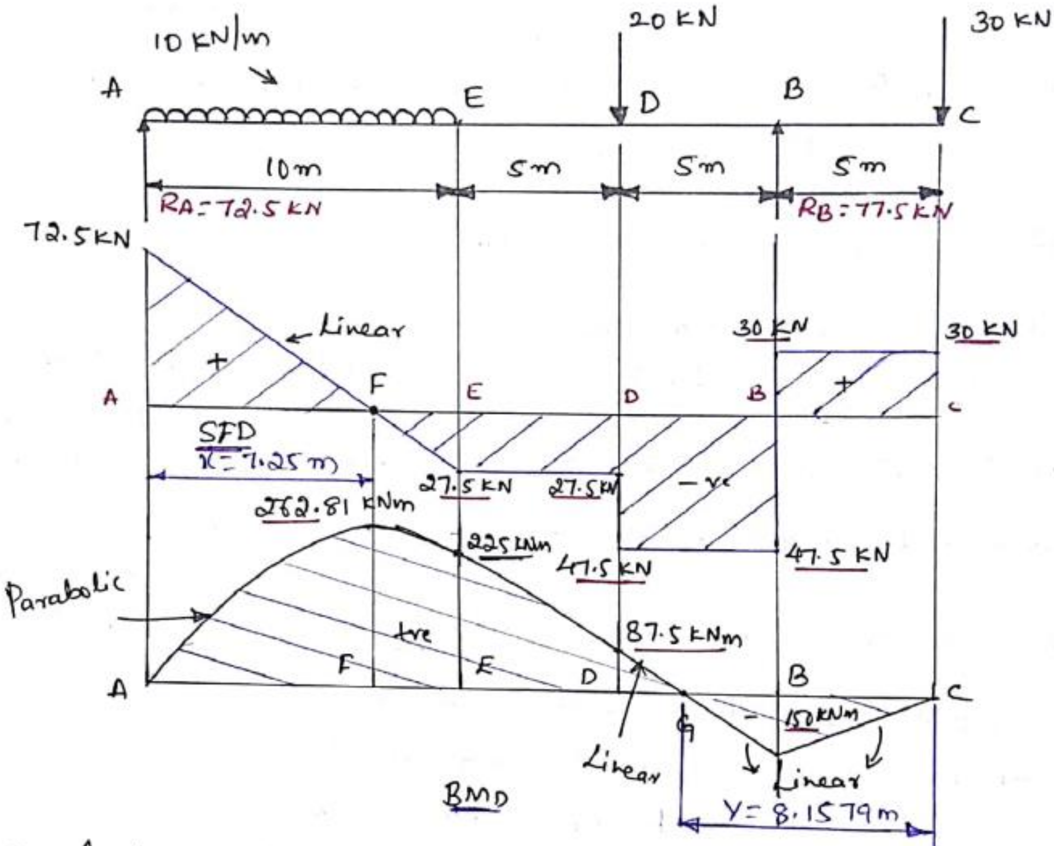
Sub: Mechanics of Materials				Code: 18ME52
Date: 25/01/2022	Duration: 90 mins	Max Marks: 50	Sem: 3	Branch (sections): ME

Answer one question from Part A and 3 questions from Part B

	PART A	Marks	OBE CO	OBE RBT
1	A beam 25 m long is supported at A and B and is loaded as shown in fig. 1. Draw SFD and BMD for the beam. Find the position and magnitude of the maximum bending moment. Also determine the point of contraflexure.  **Fig. 1**	[20]	CO4	L3
2	Draw SFD and BMD for the beam show in fig.2. Indicate all principal values.  **Fig. 2**	[20]	CO4	L3
	PART B			
3	With usual notations derive Lame's equation.	[10]	CO2	L2
4	(a) Derive an expression to find longitudinal stress in thin cylinder. (b) Derive an expression to find circumferential stress in thin cylinder.	[10]	CO2	L2
5	A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume. Take E=200 GPa and μ= 0.3.	[10]	CO2	L2
6	A thick cylindrical pipe of internal diameter 500 mm and 75 mm thick is subjected to an internal fluid pressure of 6 N/mm^2 and external pressure of 5 N/mm^2. Determine the maximum hoop and minimum stress developed and draw the variation of hoop stress and radial stress across the thickness of the cylinder.	[10]	CO2	L2

CI CCI HOD

1



To find reactions at supports (R_A & R_B):

Take moments about A

$$R_B \times 20 = (10\times10)(10/2) + 20\times15 + 30\times25$$

$$\therefore R_B = 77.5 \text{ KN (Ans)}$$

Total load: $(10\times10) + 20+30 = R_A + R_B$

$$\therefore R_A + 77.5 = 150 \; ; \quad \therefore R_A = 72.5 \text{ KN (Ans)}$$

Shear force diagram (SFD)

Shear force at A $F_A = R_A = +72.5$ KN

Shear force at E, $F_E = +72.5 - (10\times10) = -27.5$ KN

Between E and D, shear force is constant at -27.5 KN

$F_D = -27.5$ KN (without Point load)

Shear force at D, $F_D = -27.5 - 20 = -47.5$ KN (sudden variation due to Point load)

Between D and B, shear force is constant at $-47.5\ kN$

$F_B = -47.5\ kN$ (without point load)

S.F at B, $F_B = -47.5 + 77.5 = +30\ kN$

[Sudden variation due to point load]

Between 'B' and 'C' shear force is constant at $+30\ kN$

S.F at C, $F_C = +30\ kN$

Bending moment diagram (BMD)

Bending moment at A, $M_A = 0$

B.M at E, $M_E = 72.5 \times 10 - (10 \times 10)(10/2) = +225\ kNm$ (Parabolic)

B.M at D, $M_D = +72.5 \times 15 - (10 \times 10)(10/2 + 5) = +87.5\ kNm$

B.M at B, $M_B = +72.5 \times 20 - (10 \times 10)(10/2 + 10) - 20 \times 5$
$= -150\ kNm$

B.M at C, $M_C = 72.5 \times 25 - (10 \times 10)(\frac{10}{2} + 15) - 20 \times 10 + 77.5 \times 5 = 0$

TO find maximum bending moment.

At 'F', shear force is changing its sign.

shear force at F, $F_F = 72.5 - 10x$

$\therefore\ x = 7.25\ m$ (Ans)

Bending moment at F, $M_F = 72.5 \times 7.25 - (10 \times 7.25)(\frac{7.25}{2})$
$= +262.8125\ kNm$ (Ans).

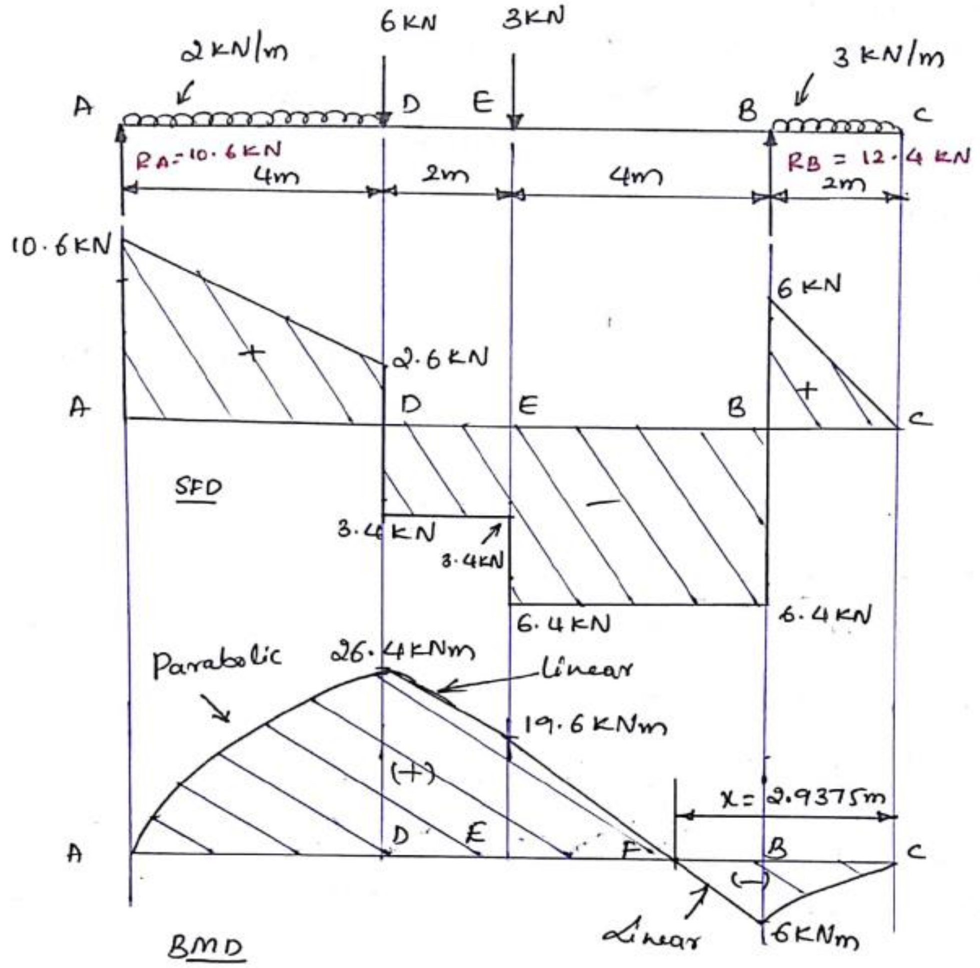
Point of contraflexure (It is the point where The BM changes its sign)

B.M at G, $M_G = -30y + 77.5(y - 5)$

$-30y + 77.5y - 387.5 = 0$

$\therefore\ y = 8.1579\ m$ (Ans)

$\therefore$ Point of contraflexure at 'G' is $8.1579\ m$ from c.



To find Reactions at supports (R_A & R_B).

Take moments about A

$$R_B \times 10 = (2\times4)(4/2) + 6\times4 + 3\times6 + (3\times2)(2/2 + 10)$$

$\therefore$ $R_B = 12.4\,KN$ (Ans)

Total load = $R_A + R_B$

$R_A + 12.4 = (2\times4) + 6 + 3 + (3\times2)$

$\therefore$ $R_A = 10.6\,KN$ (Ans)

Shear force diagram (SFD)

S.F at A, $F_A = +R_A = +10.6\,KN$

S.F at D, $F_B = R_A - 2\times4 = 2.6\,KN$

S.F at D, $F_D = 2.6 - 6 = -3.4\,KN$ [Sudden variation due to Point load at D]

Shear force remains constant between D and E

S.F at E, $F_E = -3.4\,kN$.

S.F at E, $F_E = -3.4 - 3 = -6.4\,kN$

[sudden variation due to Point load at E]

shear force remains constant between E and B

S.F at B, $F_B = -6.4\,kN$

S.F at B, $F_B = -6.4 + 12.4 = 6\,kN$

[sudden variation due to R_B]

S.F at C, $F_C = 6 - 3\times2 = 0$

Bending moment diagram (BMD)

B.M at A, $M_A = 0$

B.M at D, $M_D = R_A \times 4 - (2\times4)(4/2) = +26.4\,kNm$

B.M at E, $M_E = R_A\times6 - (2\times4)(4/2+2) - 6\times2 = +19.6\,kNm$

B.M at B, $M_B =$
$R_A\times10 - (2\times4)(\frac{4}{2}+6) - 6\times6 - 3\times4 = -6\,kNm$

B.M at C, $M_C = 0$
$R_A\times12 - (2\times4)(4/2+8) - 6\times8 - 3\times6 + R_B\times2 - (3\times2)(2/2) = 0$

Point of contraflexure:

Bending moment at F, $M_F = -(3\times2)(x - 2/2) + R_B(x-2)$

$0 = -6x + 6 + 12.4x - 24.8$

$0 = +6.4x - 18.8$

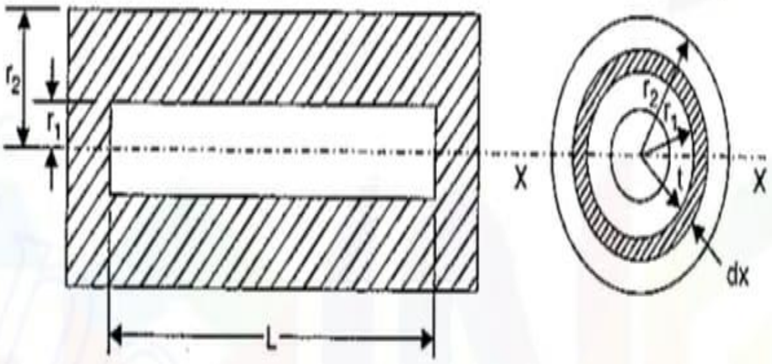
$6.4x = 18.8$

$\therefore x = \frac{18.8}{6.4}$

$\therefore x = 2.9375\,m$

18.2. STRESSES IN A THICK CYLINDRICAL SHELL

Fig. 18.1 (*a*) shows a thick cylinder subjected to a internal fluid pressure.



(*a*) (*b*)

Fig. 18.1

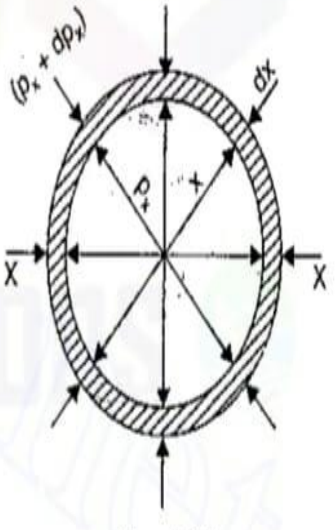


Fig. 18.2

Let r_2 = External radius of the cylinder,

r_1 = Internal radius of the cylinder, and

L = Length of cylinder.

Consider an elementary ring of the cylinder of radius x and thickness dx as shown in Fig. 18.1 (*b*) and 18.2.

Let p_x = Radial pressure on the inner surface of the ring

$p_x + dp_x$ = Radial pressure on the outer surface of the ring

σ_x = Hoop stress induced in the ring.

Take a longitudinal section x-x and consider the equilibrium of half of the ring of Fig. 18.2 as shown in Fig. 18.2 (a) or in Fig. 18.2 (b).

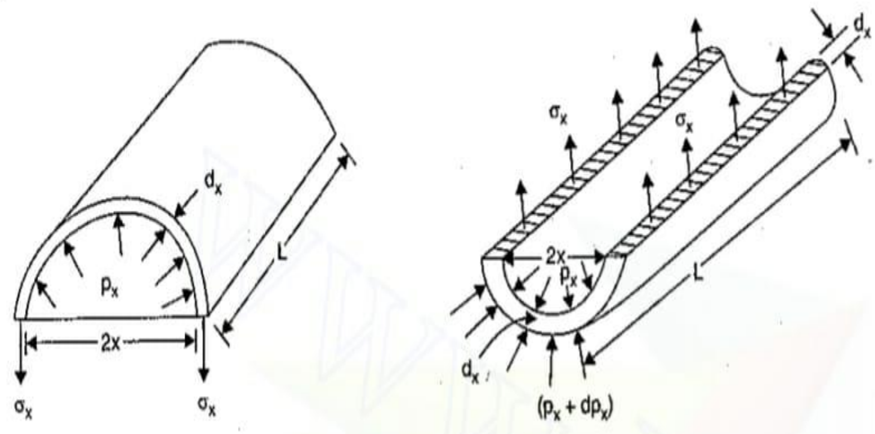


Fig. 18.2 (a) Fig. 18.2 (b)

Bursting force

$$= p_x (2xL) - (p_x + dp_x) \times 2(x + dx) \,.\, L$$
$$= 2L\,[p_x \,.\, x - (p_x \,.\, x + p_x \,.\, dx + x dp_x + dp_x \,.\, dx)]$$
$$= 2L\,[-p_x \,.\, dx - x \,.\, dp_x] \qquad \text{(Neglecting } dp_x \,.\, dx \text{ which is a small quantity)}$$
$$= -2L\,(p_x dx + x \,.\, dp_x) \qquad ...(i)$$

Resisting force = Hoop stress × Area on which it acts = $\sigma_x \times 2dx.\,L$ ...(ii)

Equating the resisting force to the bursting force, we get

$$\sigma_x \times 2dx \,.\, L = -2L\,(p_x \,.\, dx + x \,.\, dp_x)$$

or
$$\sigma_x = -p_x - x\,\frac{dp_x}{dx} \qquad ...(iii)$$

The longitudinal strain at any point in the section is constant and is independent of the radius. This means that cross-sections remain plane after straining and this is true for sections, remote from any end fixing. As longitudinal strain is constant, hence longitudinal stress will also be constant.

Let σ_2 = Longitudinal stress.

Hence at any point at a distance x from the centre, three principal stresses are acting : They are :

(i) the radial compressive stress, p_x

(ii) the hoop (or circumferential) tensile stress, σ_x

(iii) the longitudinal tensile stress σ_2.

The longitudinal strain (e_2) at this point is given by,

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E}$$

But longitudinal strain is constant.

$$\therefore \qquad \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E} = \text{constant}$$

But σ_2 is also constant, and for the material of the cylinder E and μ are constant.

$\therefore\ \ \sigma_x - p_x$ = constant

= $2a$ where a is constant

$$\therefore \qquad \sigma_x = p_x + 2a \qquad ...(iv)$$

Equating the two values of σ_x given by equations (*iii*) and (*iv*), we get

$$p_x + 2a = -p_x - x\frac{dp_x}{dx}$$

or

$$x \cdot \frac{dp_x}{dx} = -p_x - p_x - 2a = -2p_x - 2a$$

or

$$\frac{dp_x}{dx} = -\frac{2p_x}{x} - \frac{2a}{x} = \frac{-2(p_x + a)}{x}$$

or

$$\frac{dp_x}{(p_x + a)} = -\frac{2dx}{x}$$

Integrating the above equation, we get

$$\log_e (p_x + a) = -2\log_e x + \log_e b$$

where $\log_e b$ is a constant of integration.

The above equation can also be written as

$$\log_e (p_x + a) = -\log_e x^2 + \log_e b$$

$$= \log_e \frac{b}{x^2}$$

$$\therefore \qquad p_x + a = \frac{b}{x^2}$$

or

$$p_x = \frac{b}{x^2} - a \qquad ...(18.1)$$

Substituting the values of p_x in equation (*iv*), we get

$$\sigma_x = \frac{b}{x^2} - a + 2a = \frac{b}{x^2} + a \qquad ...(18.2)$$

The equation (18.1) gives the radial pressure p_x and equation (18.2) gives the hoop stress at any radius x. These two equations are called *Lame's equations*. The constants 'a' and 'b' are obtained from boundary conditions, which are :

(*i*) at $x = r_1$, $p_x = p_0$ or the pressure of fluid inside the cylinder, and

(*ii*) at $x = r_2$, $p_x = 0$ or atmosphere pressure.

After knowing the values of 'a' and 'b', the hoop stress can be calculated at any radius.

4(a)

EXPRESSION FOR LONGITUDINAL STRESS

Consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place along the section AB of Fig. (*a*).

The longitudinal stress (σ_2) developed in the material is obtained as :

Let p = Internal pressure of fluid stored in thin cylinder

d = Internal diameter of cylinder

t = Thickness of the cylinder

σ_2 = Longitudinal stress in the material.

The bursting will take place if the force due to fluid pressure acting on the ends of the cylinder is more than the resisting force due to longitudinal stress (σ_2) developed in the material as shown in Fig. 17.4 (*b*). In the limiting case, both the forces should be equal.

Force due to fluid pressure $= p \times$ Area on which p is acting

$$= p \times \frac{\pi}{4} d^2$$

Resisting force $= \sigma_2 \times$ Area on which σ_2 is acting

$$= \sigma_2 \times \pi d \times t$$

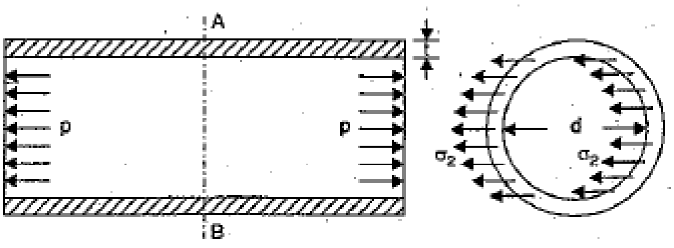
$\therefore$ Hence in the limiting case

Force due to fluid pressure = Resisting force

$$p \times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d \times t$$

$$\therefore \qquad \sigma_2 = \frac{p \times \frac{\pi}{4} d^2}{\pi d \times t} = \frac{pd}{4t}$$

The stress σ_2 is also tensile.



(a)

(b)
Longitudinal stress (σ_2) develop

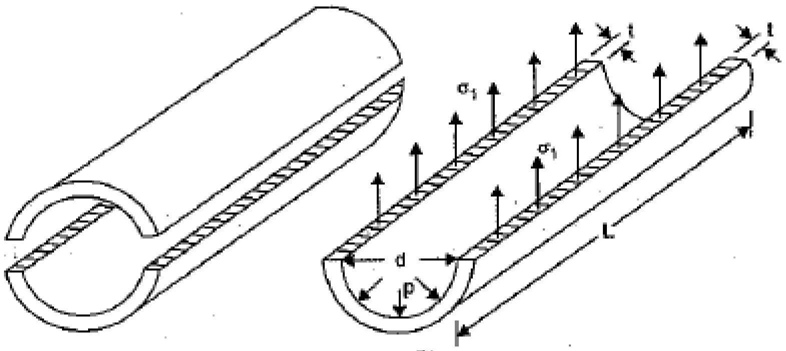
4(b)

EXPRESSION FOR CIRCUMFERENTIAL STRESS (OR HOOP STRESS)

Consider a thin cylindrical vessel subjected to an internal fluid pressure. The circumferential stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place as shown in Fig. (a).

The expression for hoop stress or circumferential stress (σ_1) is obtained as given below.

Let p = Internal pressure of fluid
d = Internal diameter of the cylinder
t = Thickness of the wall of the cylinder
σ_1 = Circumferential or hoop stress in the material.



(a) (b)

Fig. 17.3

The bursting will take place if the force due to fluid pressure is more than the resisting force due to circumferential stress set up in the material. In the limiting case, the two force should be equal.

Force due to fluid pressure $= p \times$ Area on which p is acting

$$= p \times (d \times L) \quad \text{...(i)}$$

($\because$ p is acting on projected area $d \times L$)

Force due to circumferential stress

$$= \sigma_1 \times \text{Area on which } \sigma_1 \text{ is acting}$$
$$= \sigma_1 \times (L \times t + L \times t)$$
$$= \sigma_1 \times 2Lt = 2\sigma_1 \times L \times t \quad \text{...(ii)}$$

Equating (i) and (ii), we get

$$p \times d \times L = 2\sigma_1 \times L \times t$$

$$\therefore \quad \sigma_1 = \frac{pd}{2t} \text{ (cancelling } L\text{)}$$

1. Circumferential stress, σ_c:

$$\sigma_C = (p\times d) / (2\times t)$$
$$= (1.2\times 1000) / (2\times 12)$$
$$= \underline{50\ N/mm^2 = 50\ MPa}\ \text{(Tensile)}.$$

2. Longitudinal stress, σ_L:

$$\sigma_L = (p\times d) / (4\times t)$$
$$= \sigma_C/2 = 50/2$$
$$= \underline{25\ N/mm^2}\ \underline{= 25\ MPa}\ \text{(Tensile)}.$$

3. Circumferential strain, ε_c:
$$\varepsilon_c = \frac{(p\times d)}{(4\times t)}\times\frac{(2-\mu)}{E}$$
$$= \frac{(1.2\times 1000)}{(4\times 12)}\times\frac{(2-0.3)}{200\times 10^3}$$
$$= \underline{2.125\times 10^{-04}}\ \text{(Increase)}$$

Change in diameter, $\delta d = \varepsilon_c \times d$
$$= 2.125\times 10^{-04}\times 1000 = \underline{0.2125}\ \text{mm (Increase)}.$$

4. Longitudinal strain:
$$\varepsilon_L = \frac{(p\times d)}{(4\times t)}\times\frac{(1-2\times\mu)}{E}$$
$$= \frac{(1.2\times 1000)}{(4\times 12)}\times\frac{(1-2\times 0.3)}{200\times 10^3}$$
$$= \underline{5\times 10^{-05}}\ \text{(Increase)}$$

Change in length = $\varepsilon_L\times L = 5\times 10^{-05}\times 3000 = \underline{0.15}$ mm (Increase).

Volumetric strain, $\frac{dv}{V}$:
$$\frac{dv}{V} = \frac{(p\times d)}{(4\times t)\times E}\times(5-4\times\mu)$$
$$= \frac{(1.2\times 1000)}{(4\times 12)\times 200\times 10^3}\times(5-4\times 0.3)$$
$$= 4.75\times 10^{-4}\ \text{(Increase)}$$

$$= 4.75\times 10^{-4}\times\frac{\pi}{4}\times 1000^2\times 3000$$
$$= 1.11919\times 10^6\ mm^3 = 1.11919\times 10^{-3}\ m^3$$
$$= \underline{1.11919\ \text{Litres}}.$$

Data:

$d_2 = 500\,mm$; $\therefore r_2 = 250\,mm$; $t = 75\,mm$ $\therefore r_1 = 250 + 75$
$= \underline{325\,mm}$

$p_x = 6\,N/mm^2$

Solⁿ: Internal pressure at any radius, x $p_x = \frac{b}{x^2} - a$

when $x = r_2 = 250\,mm$, $p_x = 6\,N/mm^2$

i.e $6 = \frac{b}{250^2} - a$ — (i)

when $x = r_1 = 325\,mm$, $p_x = 0$.

$0 = \frac{b}{325^2} - a$ — (ii)

Eqⁿ (i) − (ii) gives, $6 = \frac{b}{250^2} - \frac{b}{325^2}$

$\therefore \underline{b = 918478.261}$

Substituting 'b' in eqⁿ (i), $a = \underline{8.696}$

Hoop stress at any radius, x, $f_x = \frac{b}{x^2} + a$

$f_x = \frac{918478.261}{x^2} + 8.696$

Mean radius, $r_m = \frac{r_2 + r_1}{2} = \frac{250 + 325}{2} = \underline{287.5\,mm}$

when $x = r_2 = 250\,mm$; $f_x = f_{250}$

i.e., Hoop stress at the inner circumference

$$f_{250} = \frac{918478.261}{250^2} + 8.696 = \underline{23.4\ N/mm^2}$$

when $x = r_m = 287.5\,mm$, $f_x = f_{287.5}$

i.e., Hoop stress at the mean radius

$$f_{287.5} = \frac{918478.261}{287.5^2} + 8.696 = \underline{19.81\ N/mm^2}$$

when $x = r_1 = 325\,mm$, $f_x = f_{325}$

i.e., Hoop stress at the outer circumference

$$f_{325} = \frac{918478.261}{325^2} + 8.696 = \underline{17.4\ N/mm^2}$$

Radial pressure at the mean circumference

$$P_{287.5} = \frac{918478.261}{287.5^2} - 8.696 = \underline{2.416\ N/mm^2}$$